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Weak Matrix Elements in the Large N_c Limit

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Abstract

The matrix elements of the four quark operators needed to predict many weak interaction processes can be evaluated using the large N_c limit of quantum chromodynamics. At leading order in the large N_c expansion, the weak matrix elements of four quark operators factorize into independent matrix elements of two quark operators, a common approximation being used today. At next leading order, the weak matrix elements acquire the leading scale and scheme dependence expected for these matrix elements in full QCD. We will discuss methods to evaluate these matrix elements which involve matching perturbative QCD calculations at short distance to nonperturbative hadronic matrix elements at long distance.

The large N_c expansion for quantum chromodynamics was formulated by 't Hooft [1] and has been used by many authors to study nonperturbative effects in QCD. The large N_c expansion is based on 't Hooft's observation that the perturbation series could be reorganized by considering the limit, $\alpha_s \to 0, N_c \to \infty, \alpha_s * N_c - fixed$. The leading order of this expansion involves only planar diagrams of quarks and gluons, and all diagrams with internal quark loops are suppressed. This approximation is very similar to the quenched version of QCD used in many lattice computations where quark loops are also suppressed while allowing nonplanar gluon interactions. The large N_c limit of QCD is nonperturbative as all orders of $\alpha_s * N_c$ must be included at the leading order of the large N_c expansion. The theory is expected to be a theory of hadronic bound states with color confinement and dynamical chiral symmetry breaking. From the topology of diagrams contributing to the large N_c limit of QCD, the theory is expected to consist of infinite towers of weakly-interacting, color-singlet mesons with all spins and flavors dictated by the quark substructure. The scattering amplitudes are order $1/N_c$ and can be viewed as tree diagrams of the effective meson theory. Higher order diagrams in the large N_c expansion involve the

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insertion of internal quark loops or nonplanar gluon interactions. At the meson level, the higher order diagrams correspond to a systematic loop expansion of the effective meson theory. The topological structure of these diagrams is very suggestive of an hadronic string picture for large N_c QCD.

The theoretical description of many weak processes requires knowledge of the hadronic matrix elements of weak currents and chiral densities. In addition, nonleptonic weak decays usually require the knowledge of four quark operators constructed from products of these currents and densities. The large N_c expansion can be used to analyze the leading behavior of the matrix elements of these operators. The weak operators are usually written in terms of the products of color singlet bilinear operators,

$$O_i = (\overline{\psi}\Gamma_i\psi)(\overline{\psi}\Gamma_i\psi) \ . \tag{0.1}$$

At leading order (LO) of the large N_c expansion, the hadronic matrix elements factorize. Each color singlet bilinear operator couples independently to the hadrons in the external states.

$$\langle O_i\rangle_{factorized} = \langle\left(\overline{\psi}\Gamma_i\psi\right)\rangle\langle\left(\overline{\psi}\Gamma_i\psi\right)\rangle \tag{0.2}$$

Higher order terms in the large N_c expansion involve both factorized and nonfactorized contributions to the weak matrix elements. Next leading order (NLO) terms include the addition of internal quark loop contributions to the factorized matrix elements and the generation of leading nonfactorizing contributions to the weak matrix elements. These are both one loop diagrams in the effective meson theory.

At NLO, the nonfactorized amplitudes have a particularly simple structure in the effective meson theory. The matrix element may be written as a momentum integral over a two current correlation function,

$$\langle O_i\rangle_{nonfactorized} = \int dk\ A_{\Gamma_i\Gamma_i}(k, -k; p_1, \cdots, p_N) \tag{0.3}$$

where k is the momentum flowing through the color singlet bilinear operators and $p_1, ..., p_N$ are the momenta of the external hadrons. We can use our knowledge of these two current correlation functions to evaluate the NLO contributions to weak matrix elements [2]. If all the external states are at low energy, the low momentum part of the integral requires only the low energy behavior of the two current correlation function. However, many experiments directly measure these current correlation functions. For meson external states, the lowest energy contributions are summarized in terms an effective chiral Lagrangian. The parameters of the effective chiral Lagrangian are nonperturbative quantities in QCD and have been determined by systematic phenomenological analysis [3]. At higher loop momenta, additional hadronic states must be included such as the vector and axial-vector mesons. In principle, the full intergral can be obtained from the complete tree-level correlation function of the LO meson theory. However, the high momentum behavior can also be obtained through the use of operator product expansion,

$$A_{\Gamma_i\Gamma_i}(k,-k;p_1,\cdots,p_N M) \rightarrow C_{\Gamma_i\Gamma_i;O_j}(k) * \langle O_j\rangle(p_1,\cdots,p_N) \qquad (0.4)$$

where the coefficient function, $C_{\Gamma_i\Gamma_i;O_j}$, can be computed using the large N_c version of perturbative QCD, PQCD. It is a remarkable fact that all of weak mixing processes are nonleading in the large N_c expansion. This implies that the coefficient function begins to receive contributions only at NLO. Therefore, the NLO calculation of the current correlation function requires only knowledge of the LO operator matrix element of the operators appearing in Eq.(4), and these are determined from parameters of the LO effective meson theory.

Since we are able to establish the precise behavior of the integrand appearing in Eq.(3) both at low energies and at high energies, we can hope to estimate the full integral by interpolating the results at moderate momentum scales. Of course, the accuracy of this interpolation can be improved by including more states or higher derivative terms in the low energy effective meson theory or by computing higher order terms in the PQCD expansion of the short distance theory. In principle, the matching between the effective meson theory and the PQCD expansion of the short distance theory can be improved to arbitrary accuracy. Comparisons with the conventional definitions of weak matrix elements require knowledge of the particular regularization schemes, NDR or HV, used to define the weak operators in PQCD. Hence, a second short distance matching must be made between the integral expression of Eq.(3) and the particular scheme used to reqularize the short distance behavior. This second matching can be done purely at the quark level as it depends solely on the short distance physics. Using this short distance matching, the high momentum part of the integral of Eq.(3) can be properly subtracted to obtain the full NLO contribution to the weak matrix elements in any renormalization scheme.

This method can be applied to any of the conventional four quark operators used to analyze nonleptonic weak interactions, $Q_1, Q_2, \cdots, Q_6, \cdots, Q_8, \cdots$. At NLO in the large N_c expansion, the weak matrix elements computed by the above method will have the appropriate scale and scheme dependence. These matrix elements can be used with any other analysis of the physical short distance physics which determines the physical coefficient functions. Scale and scheme dependence will properly cancel between the coefficient functions and the operator matrix elements, at least to NLO in the large N_c expansion. The precision of the NLO weak matrix elements determined through these methods depends on a number of factors:

- the phenomenological determination of the effective meson theory
  - chiral Lagrangians - O(p^2), O(p^4), O(p^6), ...
  - inclusion of heavy states - vector mesons, scalars, ...
  - models - effective NJL models, chiral quark model, ...
- the long distance - short distance matching conditions
- the short distance expansion of planar QCD

- the scheme dependent matching conditions of PQCD

The methods described above have been applied to a number of problems requiring knowledge of hadronic matrix elements. Applications include the $\pi^+ - \pi^0$ electromagnetic mass difference, the $\Delta I = 1/2$ Rule in nonleptonic weak decays, the B_K parameter in $K - \bar{K}$ mixing and the weak matrix elements needed for determining ϵ'/ϵ in the CP violating Kaon decays.

•$\pi^+ - \pi^0$ mass difference. This calculation involves the insertion of explicit one photon exchange processes which have the same structure of the insertion four quark operators in weak processes. The inserted vertex is nonlocal due to the photon propagator and does not require the scheme dependent short distance matching of the weak matrix elements. The matching between the long distance meson physics and the short distance quark physics involving chiral condensates is still required. Using the lowest order chiral Lagrangian, O(p^2), and the conventional short distance expansion gives an estimate of the mass difference good to about 15%-20%. Here the matching between the long distance physics of pointlike pions and the short distance quark physics is determined by the scale where the integrands coincide. This matching occurs in the range of 600-800 MeV although the two different approximations to the integrand have much different energy dependence. We are able to improve our knowledge of the long distance physics by including the contributions of vector and axial-vector mesons. The matching now becomes excellent at any scale above 600-800 MeV and is related to the known meson saturation of the Weinberg sum rules. The mass difference calculation is now good to about 5% [4].

•$\Delta I = 1/2$ Rule in Kaon decays. The CP conserving weak decays of Kaons is known to obey the $\Delta I = 1/2$ rule. The $\Delta I = 1/2$ amplitude is enhanced from the factorized matrix element and the $\Delta I = 3/2$ amplitude is suppressed. Part of this enhancement/suppression can be explained in terms of the conventional weak mixing [5] involving weak operators, Q_1 and Q_2 and a charm penguin contribution related to the Q_6 operator. Using the large N_c approach, the weak matrix elements can be evaluated and additional enhancements/suppressions are observed. Using an O(p^2) chiral Lagrangian, Buras, Bardeen and Gerard [6] were able to explain about 75% of the observed enhancement of the $\Delta I = 1/2$ amplitude and an additional suppression of the $\Delta I = 3/2$ amplitude. Using an O(p^4) chiral Lagrangian, Hambye et al [7], claim to observe the full enhancement of the $\Delta I = 1/2$ amplitude. Using an extended NJL model to improve the long distance approximation and the short distance-long distance matching conditions, Bijnens et al [8] also claim to see the full enhancement of the $\Delta I = 1/2$ amplitude. These calculations are sensitive to the precise method for calculating the long distance contributions and the matching procedure used to connect the long and short distance calculations. The present calculations do not include the full scheme dependence arising from the short distance matching conditions.

•$K^0 - \bar{K}^0$ mixing, B_K. The $K^0 - \bar{K}^0$ mixing arises from loops involving the top quark. Integrating out the top quark generates a unique $\Delta S = 2$ four quark operator. Perturbative QCD can be used to evolve the effective weak operator to a low energy scale. The large N_c method can then be used to evaluate the low energy matrix elements. A number of predictions for the B_K parameter have been made using various approximations for the long distance meson physics:

- $O(p^2)$ chiral Lagrangian [9] $\hat{B}_K \sim 0.7 \pm 0.$
- $O(p^2)$ chiral Lagrangian + vector mesons [10] $\hat{B}_K \sim 0.75 \pm 0.1$
- ENJL model [8] $\hat{B}_K \sim 0.69 \pm 0.1$
- $O(p^4)$ chiral Lagrangian [7] $\hat{B}_K \sim 0.6 \pm 0.1$.

• CP Violation and ϵ'/ϵ. CP Violation observed at low energy is expected to be generated by loop effects at a high mass scale. Top quark loops contribute to CP violation through complex phases associated with the effective four quark operators at low energy, particularly the operators Q_6 and Q_8. In the chiral Lagrangian approach, $O(p^4)$ terms are required for the Q_6 matrix element to be nonzero. Leading terms in the nonfactorized amplitudes cancel infrared singularities of the factorized amplitudes. The large N_c expansion method has been applied to these matrix elements, and the matrix elements of the electropenguin operator, Q_8, were found to be suppressed by the nonfactorizing contributions while the gluopenquin operator, Q_6, may receive a modest enhancement [11]. Both of these effects tend to increase theoretical estimates of ϵ'/ϵ.

The large N_c expansion permits a consistent evaluation of the weak matrix elements for a number of important physical processes. The method combines our knowledge of perturbative short distance processes with the nonperturbative contributions contained in the effective meson theories or ENJL models used to describe the long distance physics. At NLO the method is subject to considerable improvement. The effective meson physics could be extended by considering additional meson states or improved models which evolve the long distance physics to higher energy scales. This could improve matching of the long distance and short distance physics which is now at a rather crude level for most processes. Also, the scheme dependence of the weak matrix elements requires a specific calculation of the short distance matching of the momentum integral of the current correlation function to particular regularization scheme in PQCD. At this point, the method is restricted to NLO in the large N_c expansion as terms $O(1/N_c^2)$ can not be controlled. At some point, an hadronic string theory might be used to obtain a complete picture of the meson amplitudes at higher order. Many of the issues discussed in this short talk will be covered in more detail in other contributions to this conference.

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