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Physical Effects of Infrared Quark Eigenmodes in LQCD *

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A truncated determinant algorithm is used to study the physical effects of the quark eigenmodes associated with eigenvalues below 300 MeV. This initial study focuses on coarse lattices (with $O(a^2)$ improved gauge action), light internal quark masses and large physical volumes. Four bellweather full QCD processes are discussed: topological charge distributions, the eta prime propagator, string breaking as observed in the static energy and the rho decay into two pions.

1. Basics

In the truncated determinant approach (TDA) to full QCD, the quark determinant, $\mathcal{D}(A) = det(H) = det(\gamma_5(\not{D}(A) - m))$ is split-up gauge invariantly into an infrared part and an ultraviolet part[1].

$$\mathcal{D}(A) = \mathcal{D}_{IR}(A)\mathcal{D}_{UV}(A) \quad (1)$$

The ultraviolet part, $\mathcal{D}_{UV}$, can be accurately fit by a linear combination of a small number of Wilson loops[2]. The infrared part $\mathcal{D}_{IR}(A)$ is defined as the product of the lowest N_λ positive and negative eigenvalues of H. with $|\lambda_i| \le \Lambda_{QCD}$ (typically, $\simeq$ 300-400 MeV). The eigenvalues λ_a of H are gauge invariant and measure quark off-shellness. The cutoff is tuned to include as much as possible of the important low-energy chiral physics of the unquenched theory while leaving the fluctuations of $\ln \mathcal{D}_{IR}$ of order unity after each sweep updating all links with the pure gauge action (assuring a tolerable acceptance rate for the accept/reject stage)[3]. This procedure works well even for kappa values arbitrarily close to kappa critical.

Initial studies using TDA focus on the qualitative physical effects of the inclusion the infrared quark eigenmodes. For this purpose, coarse lattices with large physical volumes are appropriate.

*Talk presented by E. Eichten

Table 1
Coarse lattices studied.

Label	Volume; Fermions; Gauge Action
PW6	6^4; $n_{cut} = 840$ $\kappa = .2180$; Naive $\beta = 4.5$
RW6	6^4; $n_{cut} = 840$ $\kappa = .1950$; $O(a^2)$ $\beta = 6.8$
RW8	8^4; $n_{cut} = 1780$ $\kappa = .1950$; $O(a^2)$ $\beta = 6.8$

2. Bellwethers

The coarse lattices given in Table 1 are being studied on PC clusters. The $O(a^2)$ improved gauge action, $\beta = 6.8[1.0(plaq) - 0.08268(rect) - 0.01240(para)]$, was adjusted in Ref. [4] to have approximately the same lattice scale $a = .22fm$ as the naive gauge action at $\beta = 4.5$. The physical lattice size is 2.4 (fm) for PW6 and RW6 and 3.2 (fm) for RW8; $\kappa_c = .2190$ for PW6, and .1960 for RW6 and RW8; and eigenvalue cutoff scale is 560 MeV for PW6 and RW6, and 445 MeV for RW8. In addition, 10^3x20 lattices at $\beta = 5.7$, $c_{SW} = 1.57$ and $n_{cut} = 520$ ($E_{cut} = 460MeV$) are being generated on ACPMAPS.

As shown in Figure 1 it took approximately 10,000 full steps for the PW6 lattice configurations to equilibrate (reflecting critical slowing

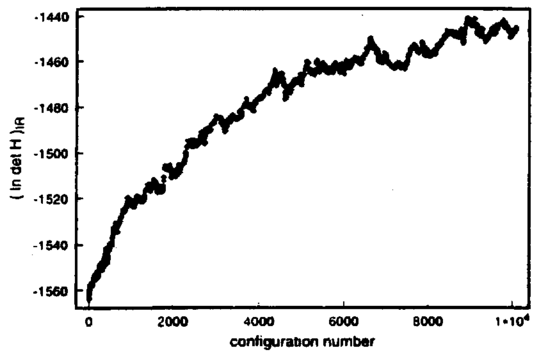

Figure 1. Reaching equilibrium for the PW6 lattices.

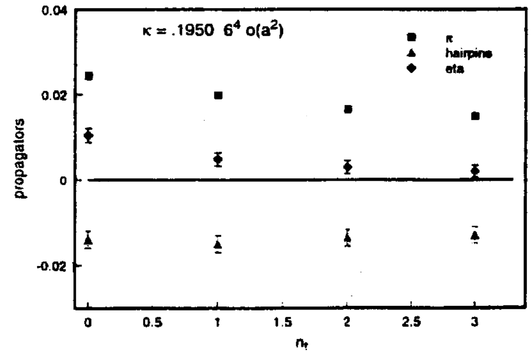

Figure 2. Eta prime propagator for the RW6 lattices. The valence term (squares), hairpins term (triangles) and total propagator (diamonds) are shown separately.

down).

Four bellwethers can be used characterize the physical differences between quenched and full QCD. They are discused (in order of increasing difficulty to observe in lattice calculations) in the following four subsections.

2.1. Topolopy

The topological charge, Q_{TOP}, can be expressed in terms of the eigenvalues of the Wilson-Dirac operator.

$$Q_{TOP} = \frac{1}{2\kappa}(1 - \frac{\kappa}{\kappa_c}) \sum_{i=1}^{N} \frac{1}{\lambda_i} \qquad (2)$$

This sum is quickly satuarated by the low eigenvalues.

In full QCD configurations with very small eigenvalues of H are suppressed by the quark determinant factor. In particular, non-zero topological charges must be suppressed in the chiral limit ($m_q \to 0$). Futhermore, the functional dependence of the topological charge distribution, P_Q, on the light quark mass m_q is predicted by the chiral analysis of Leutwyler and Smilga[5].

$$P_Q = I_Q(x)^2 - I_{Q+1}(x) I_{Q-1}(x) \qquad (3)$$

where

$$x = 1/2V f_\pi^2 m_\pi^2 = V m_q < \overline{\psi}\psi >, \qquad (4)$$

I_Q are modified Bessel Functions of order Q and V is the total space-time volume.

The quantitative agreeement with the expected behaviour of Eq. 3 has already been reported for the TDA method[1]. General agreement is also observed on the coarse lattices of the present studies.

2.2. Eta Prime Mass

The relation between the axial $U(1)$ anomaly and the η prime mass is well understood in full QCD. For two light quarks ($N_f = 2$), $m_\eta^2 = m_\pi^2 + m_0^2$ where $m_0^2 = 2N_f\chi/f_\pi^2$ and $V\chi \equiv < Q_{TOP}^2 >_{\text{quneched}}$. The full η propagator is the sum of a connected (valence quark) term and a disconnected (hairpins) term. Thus, in the continuum, the momentum space the full propagator can be written:

$$(p^2 + m_\pi^2 + m_0^2)^{-1} = (p^2 + m_\pi^2)^{-1} - m_0^2(p^2 + m_\pi^2)^{-1}(p^2 + m_\pi^2 + m_0^2)^{-1} \qquad (5)$$

These separate terms and their sum are shown in Figure 2 for the RW6 lattices. The cancellation between the valence and hairpin terms in the full propagator is evident.

2.3. Static Energy

To date no convincing evidence for string breaking in full QCD has been presented using calculations of the static energy alone. However string breaking has been seen using the TDA method in 2D QED[1] and by the usual methods

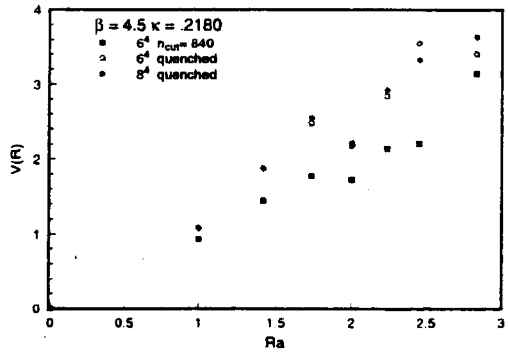

Figure 3. Comparison of the static energy between the RW6 lattices (solid squares) and unquenched 6^4 (open circles) (and 8^4 (solid circles)) lattices at $\beta = 4.5$. The static energy was extracted from the ratio of time slices $T = 2/T = 1$.

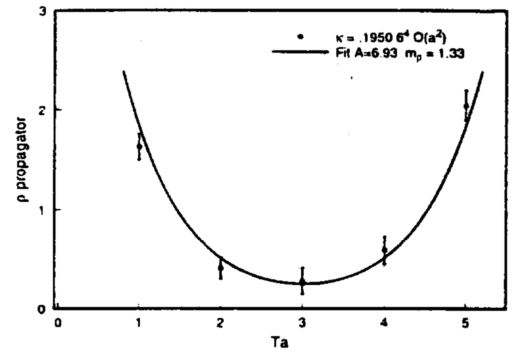

Figure 4. Rho propagator for the RW6 lattices.

in 3D QCD[6]. Studies of the $\bar{c}c$ and $\bar{b}b$ systens, lead to the expectation that virtual pair effects (below heavy-light meson pair production threshold) will soften the linear rise in the static energy, while above threshold the potential will flatten out (i.e.) the string will break. The heavy-light meson mass is $0.81 \pm .02$ for the RW6 lattices.

Figure 3 shows the static energy for 200 RW6 lattices versus the same number of unquenched 6^4 lattices at $\beta = 4.5$. There is evidence of the virtual pair effects but no hard evidence of string breaking is found. Seeing string breaking, will require more statistics (to study $T = 3/T = 2$ with small error bars) and also the study of the $RW8$ lattices.

2.4. Vector Meson Resonances

For the RW6 and RW8 lattices at $\kappa = .1950$, the rho mass is 1.33 and the pion mass is 0.205 (in lattice units); hence the π/ρ mass ratio is close to the physical value. For example, the rho propagator for the RW6 lattice is shown in Figure 4. However, since this is a P wave coupling, the physical volume of the lattice must be large enough that the decay is allowed with the first nonzero momentum, $p_{min} = \frac{2\pi}{Na}$. This requires a 10^4 lattice (RW10) or creating a rho with initial momentum p_{min}. Neither of these alternatives have been studied as yet.

3. Status

The present status of full QCD bellwethers is as follows:

- The behaviour of the topological charge distribution Q^2 as a function of light quark mass m_q[5] – Seen.
- The eta prime mass - $m^2_{\eta'} = m^2_{pi} + m^2_0$ – Seen.
- The static energy - string breaking. – In progress but needs more statistics.
- Vector meson resonances - $\rho \to \pi\pi$. – Yet to be studied in detail.

Results for these four bellwether processes on coarse lattices should be available within a few months.

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