Fermi National Accelerator Laboratory

FERMILAB-Conf-98/132-T

Implications of a Minimal SO(10) Higgs Structure

C.H. Albright, K.S. Babu and S.M. Barr

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

May 1998

Published Proceedings of the *Neutrino 98 Conference*,
Takayama, Japan, June 4-9, 1998

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

FERMILAB-Conf-98/132-T
IASSNS-HEP-98-41
BA-98-23
hep-ph/9805266
April 1998

Implications of a Minimal $SO(10)$ Higgs Structure* †

Carl H. Albright[1]
Department of Physics, Northern Illinois University, DeKalb, IL 60115
and
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510

K.S. Babu[2]
School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540

S.M. Barr[3]
Bartol Research Institute, University of Delaware, Newark, DE 19716

Abstract

A minimal $SO(10)$ Higgs structure involving a single adjoint field along with spinors, vectors and singlets has been shown to break the $SO(10)$ gauge symmetry to the standard model while stabilizing the F-flat directions and solving the doublet-triplet splitting problem naturally. With this minimal set of Higgs fields, we show how to construct quark and lepton mass matrices which explain well the many features of the observed spectrum, including the Georgi-Jarlskog mass relations. A large $\nu_\mu - \nu_\tau$ mixing angle results naturally as observed in the atmospheric neutrino data. A particular model relying on a family symmetry has been constructed which realizes the desired mass matrices.

---

*Contribution submitted to the NEUTRINO 98 Conference to be held in Takayama, Japan, June 4-9, 1998.

†Email: [1]albright@fnal.gov, [2]babu@ias.edu, [3]smbarr@bartol.udel.edu

A brief discussion is given of the implications of a minimal $SO(10)$ Higgs structure that have been developed in a recent series of papers. Barr and Raby [1] have shown how this minimal set of Higgs fields breaks the $SO(10)$ gauge symmetry to the standard model while stabilizing the F-flat directions and thus solves the double-triplet splitting problem. Following this lead, the authors [2] have used this Higgs structure to construct quark and lepton mass matrices which are fairly tightly constrained with some interesting features emerging. Of special interest to this Conference is the large $\nu_\mu - \nu_\tau$ mixing angle resulting from the special textures of the Dirac matrices, as opposed to the more conventional large hierarchical structure for the Majorana neutrino matrix [3].

I. MINIMAL HIGGS STRUCTURE

We begin with a summary of the minimal $SO(10)$ Higgs structure [1] which solves the doublet-triplet splitting problem naturally rather than by fine-tuning. The Higgs fields which are involved consist of a pair of $\mathbf{10}$'s, one $\mathbf{45}$, two pairs of $\mathbf{16} + \overline{\mathbf{16}}$'s and four singlets. The Higgs superpotential is written

$$\begin{aligned} W &= T_1 A T_2 + M_T T_2^2 + W_A + W_C + W_{CA} + W_{TC} \\ W_A &= \mathrm{tr} A^4/M + M_A \mathrm{tr} A^2 \\ W_C &= X(\overline{C}C)^2/M_C^2 + f(X) \\ W_{CA} &= \overline{C}'(PA/M_1 + Z_1)C + \overline{C}(PA/M_2 + Z_2)C' \\ W_{TC} &= \lambda T_1 \overline{C}\overline{C} \end{aligned} \tag{1}$$

Here T_1 and T_2 label the two $\mathbf{10}$'s, A labels the $\mathbf{45}$, $C,\ \overline{C},\ C',\ \overline{C}'$ label the two pairs of $\mathbf{16} + \overline{\mathbf{16}}$'s, while $P,\ X,\ Z_1,\ Z_2$ label the four singlets.

The W_A terms produce the Dimopoulos - Wilczek mechanism [4] by generating a VEV for the single $\mathbf{45}$ in the $B-L$ direction. The $T_1 A T_2$ term gives superheavy masses to the color triplets in T_1 and T_2. The mass term $M_T T_2^2$ gives superheavy masses to the T_2 doublets as well. As a result of the presence of W_C, the $F_X = 0$ condition forces the C

and $\overline{C}$ pair to get VEVs in the $SU(5)$-singlet direction. The VEVs of A and C then break $SO(10)$ to the standard model. The term W_{CA} couples C and $\overline{C}$ to A and prevents the production of colored pseudo-goldstone bosons in the breaking of $SO(10)$. Since no GUT-scale VEVs are generated for C' and $\overline{C}'$, the Dimopoulos - Wilczek hierarchical form of $\langle A\rangle$ is not destabilized by the presence of W_{CA}, thus solving the doublet-triplet splitting problem. Finally, the presence of the term W_{TC} induces an electroweak breaking VEV for C' which mixes with that in T_1. Hence the two Higgs doublets appear in the combinations

$$\begin{aligned} H &= \mathbf{5}(T_1) \\ H' &= \overline{\mathbf{5}}(C')\cos\theta - \overline{\mathbf{5}}(T_1)\sin\theta. \end{aligned} \tag{2}$$

in terms of the $SU(5)$ representations present in T_1 and C'. The combination orthogonal to H' gets massive and drops out of the picture.

An important point to be made is that the above form of the Higgs superpotential can be uniquely obtained by the introduction of a $U(1)\times Z_2\times Z_2$ family symmetry [1] with the appropriate assignment for the charges of the Higgs fields as follows:

$$\begin{array}{llll} A(0^{+-}), & T_1(1^{++}), & T_2(-1^{+-}) & \\ C(\tfrac{1}{2}^{-+}), & \overline{C}(-\tfrac{1}{2}^{++}), & C'([\tfrac{1}{2}-p]^{++}), & \overline{C}'([-\tfrac{1}{2}-p]^{-+}) \\ X(0^{++}), & P(p^{+-}), & Z_1(p^{++}), & Z_2(p^{++}) \end{array} \tag{3}$$

II. FERMION MASS MATRICES FROM THE MINIMAL SET OF HIGGS FIELDS

We can then attempt to construct fermion mass matrices from the VEVs appearing in the minimal set of Higgs fields. The VEVs in question appear at the GUT scale and at the electroweak scale as follows:

$$\begin{array}{ll} \Lambda_G: & \langle A\rangle,\ \langle C\rangle,\ \langle\overline{C}\rangle,\ \langle P\rangle,\ \langle X\rangle,\ \langle Z_1\rangle,\ \langle Z_2\rangle \\ \Lambda_{ew}: & \langle T_1\rangle,\ \langle C'\rangle \end{array} \tag{4}$$

Note that since the VEVs of the doublets of the T_1 $SO(10)$ $\mathbf{10}$ appear in the $SU(5)$ $\mathbf{5}+\overline{\mathbf{5}}$ pair, $\langle T_1\rangle$ couples symmetrically in family space to all members of a pair of $\mathbf{16}$ fermions, whether up or down quarks, neutrinos or charged leptons. On the other hand, since the C' VEV of the doublet appears only in the $SU(5)$ $\overline{\mathbf{5}}$ of the $\mathbf{16}$, this VEV couples only to the down quarks and charged leptons in a $\mathbf{16}$ and $\mathbf{10}$ fermion pair and asymmetrically at that. All the GUT scale VEVs except $\langle A \rangle$ are $SU(5)$ singlets, with $\langle A \rangle$ of the single $SO(10)$ $\mathbf{45}$ assigning an antisymmetric $B - L$ quantum number of magnitude 1/3 or 1 to the quarks and leptons, respectively.

Yukawa coupling unification at the GUT scale suggests as usual the coupling of $\langle T_1 \rangle$ to the third generation quarks and leptons according to $\mathbf{16}_3\mathbf{16}_3 T_1$. Now, however, because of the linear combination appearing in (2), the top-to-bottom quark mass ratio at the GUT scale assumes the form:

$$m_t^o/m_b^o = \tan\beta/\sin\theta \tag{5}$$

in terms of the $\langle \overline{\mathbf{5}}(T_1) \rangle - \langle \overline{\mathbf{5}}(C') \rangle$ mixing angle θ. Hence $\tan\beta$ can assume any value in the range 2 - 55.

The Georgi-Jarlskog relations [5], $m_s^0 \cong m_\mu^0/3$ and $m_d^0 \cong 3m_e^0$, together with the minimal Higgs structure then suggest the following textures for the Dirac mass matrices [2]:

$$\begin{aligned}
U^0 &= \begin{pmatrix} 0 & \sigma & 0 \\ \sigma & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m, \qquad
N^0 = \begin{pmatrix} 0 & \sigma & 0 \\ \sigma & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m, \\
D^0 &= \begin{pmatrix} 0 & \sigma + \sigma' & 0 \\ \sigma + \sigma' & 0 & \rho + \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} \tilde{m}, \quad
L^0 = \begin{pmatrix} 0 & \sigma + \sigma' & 0 \\ \sigma + \sigma' & 0 & -\epsilon \\ 0 & \rho + \epsilon & 1 \end{pmatrix} \tilde{m},
\end{aligned} \tag{6}$$

where the matrices are written so that the left-handed antifermions multiply them from the left and the left-handed fermions from the right. The 2 - 3 sector of the above matrices is essentially uniquely determined. Here the ϵ terms arise from the $B - L$ VEVs, $\langle A \rangle$, of the antisymmetric $\mathbf{45}$, while the ρ terms arise from the $\langle C' \rangle$ VEV. The 1 - 2 sector has more uncertainty. We have made the simplest choices here; for example, the σ terms may arise from $\langle T_1 \rangle$ Higgs VEVs after integrating out superheavy **16** fermions, while the σ' terms appear after integrating out superheavy **10** fermions.

If we assume that $\rho \gg \epsilon \gg \sigma' \gg \sigma$, by diagonalizing the matrices we find:

$$\begin{aligned}
& m_b^0/m_\tau^0 \cong 1 - \tfrac{2}{3}\tfrac{\rho}{\rho^2+1}(\epsilon\cos\alpha), \\
& m_\mu^0/m_\tau^0 \cong \epsilon\tfrac{\rho}{\rho^2+1}\left(1 - \tfrac{\rho^2-1}{\rho(\rho^2+1)}(\epsilon\cos\alpha)\right), \\
& m_s^0/m_b^0 \cong \tfrac{1}{3}\epsilon\tfrac{\rho}{\rho^2+1}\left(1 - \tfrac{1}{3}\tfrac{\rho^2-1}{\rho(\rho^2+1)}(\epsilon\cos\alpha)\right), \\
& m_c^0/m_t^0 \cong \epsilon^2/9, \\
& V_{cb}^0 \cong \tfrac{1}{3}\epsilon\tfrac{\rho^2}{\rho^2+1}\left(1 + \tfrac{2}{3}\tfrac{1}{\rho(\rho^2+1)}(\epsilon\cos\alpha)\right), \\
& m_d^0/m_e^0 = 3(1 + \tfrac{2}{3\rho}\epsilon\cos\alpha), \\
& |V_{us}^0| = |\sqrt{\tfrac{m_d^0}{m_s^0}}\tfrac{1}{(\rho^2+1)^{1/4}} - \sqrt{\tfrac{m_u^0}{m_c^0}}e^{i\phi}|, \\
& |V_{ub}^0| \simeq |\sqrt{\tfrac{m_d^0}{m_s^0}}\tfrac{m_s^0}{m_b^0}\tfrac{\rho}{(\rho^2+1)^{1/4}} - \sqrt{\tfrac{m_u^0}{m_c^0}}e^{i\phi}(\sqrt{\tfrac{m_c^0}{m_t^0}} - \tfrac{m_s^0}{m_b^0}\tfrac{1}{\rho})|.
\end{aligned} \tag{7}$$

Here α is the relative phase between ϵ and ρ, while ϕ is the relative phase between σ and σ'. In addition to the Georgi-Jarlskog relations [5], we observe that $m_b^0 \simeq m_\tau^0$; V_{cb}^0, m_μ^0/m_τ^0 and $m_s^0/m_b^0 \sim O(\epsilon)$; while $m_c^0/m_t^0 \sim O(\epsilon^2)$.

Of special interest is the issue of neutrino masses and mixings. The light neutrino mass matrix is given by $M_\nu = -N^T M_R^{-1} N$, in terms of the Dirac neutrino matrix and the superheavy right-handed Majorana neutrino mass matrix. If we simply take M_R diagonal and similar to the identity matrix, a large mixing emerges by virtue of the form of the Dirac matrices N^0 and L^0 in Eq. (6) as indicated below. In fact, the mixing will generally be very large, unless the form of M_R is fine-tuned. As a result of the asymmetrical ρ contributions appearing in D^0 and L^0, we can then understand why V_{cb} mixing is small in the quark sector while the $\nu_\mu - \nu_\tau$ mixing is large in the neutrino sector. The atmospheric anomaly [6] can thus be understood without resorting to a very hierarchical form for the Majorana matrix.

III. NUMERICAL RESULTS

In order to obtain numerical comparisons with experiment, the fermion masses and mixings have been evolved [2] from the unification scale, M_G, to the supersymmetry scale $M_{SUSY} \sim m_t$, by making use of 2-loop MSSM β functions and from M_{SUSY} to the running mass scales with the use of 3-loop QCD and 1-loop QED or EW beta functions. We find the known quark mass and mixing data is best fitted with $\tan\beta \simeq 30$. For this value, and the known m_μ, m_τ and V_{cb}, the two parameters ρ and ϵ are found to be

$$\rho = 1.73(1 - \Delta_{cb}), \qquad \epsilon = 0.136(1 - 0.5\Delta_{cb}), \tag{8}$$

in terms of the chargino loop correction $\Delta_{cb} \simeq -0.05$ for V_{cb}.

The following predictions then emerge with $\cos\alpha = 1$:

- Good agreement with the experimental value for $m_b(m_b) = 5.0(1+\Delta_b)$ GeV is reached with the combined gluino and chargino loop correction $\Delta_b \cong -0.15$.

- With $\Delta_s \simeq \Delta_b \cong -0.15$, $m_s(1GeV) = 176(1+\Delta_s) = 150$ MeV compared with 180 ± 50 MeV.

- We find $m_c(m_c) = (1.05 \pm 0.11)(1 - \Delta_{cb}) \sim (1.10 \pm 0.11)$ GeV, in reasonable agreement with the experimental value of (1.27 ± 0.1) GeV.

- For a non-hierarchical diagonal form for M_R, we find $\sin^2 2\theta_{\mu\tau} \simeq 0.7$. This large neutrino mixing occurs not because of a hierarchy in the right-handed Majorana neutrino mass matrix but rather because of the asymmetrical form appearing in the charged lepton mass matrix as a result of the minimal Higgs structure assumed.

- For the form of the first generation contributions to the mass matrices given in (6), acceptable results for $|V_{us}|$ and $|V_{ub}|$ emerge with the phase $\phi \sim 180^o$. The leptonic mixings $|(U_\nu)_{e\nu_2}|$ and $|(U_\nu)_{e\nu_3}|$ are small and consistent with the small angle MSW solution for the solar neutrinos, but their precise values are sensitive to the assumed structure of M_R.

In [2], detailed results have been obtained for a broader range of the input parameters $\rho,\ \epsilon,\ \cos\alpha$ and ϕ.

IV. SPECIFIC $SO(10)$ SUPERSYMMETRIC GRAND UNIFIED MODEL

It is of interest to construct a specific $SO(10)$ supersymmetric grand unified model which leads to the textures for the mass matrices postulated in Eq. (6). This has been accomplished in [2] for the second and third generation contributions which are essentially uniquely determined. The first generation contributions, being higher order, are less well determined and are subject to further study as are the contributions to the right-handed Majorana matrix.

Considering only the second and third generations, we are led to the following Yukawa superpotential,

$$\begin{aligned} W_{Yukawa} &= \mathbf{16}_3\mathbf{16}_3 T_1 \\ &+ \mathbf{16}\overline{\mathbf{16}}P + \mathbf{16}_3\overline{\mathbf{16}}A + \mathbf{16}_2\mathbf{16}T_1 \\ &+ \mathbf{10}\mathbf{10}'\overline{C}C/M_P + \mathbf{16}_2\mathbf{10}C + \mathbf{16}_3\mathbf{10}'C'. \end{aligned} \tag{9}$$

In addition to the two light fermion families, one pair of $\mathbf{16}+\overline{\mathbf{16}}$ and one pair of $\mathbf{10}+\mathbf{10}'$ fermions have been introduced which get superheavy as a result of the interactions present in Eq. (9). By making use of the previous $U(1)\times Z_2\times Z_2$ family assignments for the Higgs fields given in Eq. (3), the above terms for the Yukawa superpotential are uniquely obtained if we extend the following $U(1)\times Z_2\times Z_2$ assignments to the fermions:

$$\begin{aligned} &\mathbf{16}_3(-\tfrac{1}{2}^{++}), \quad &&\mathbf{16}_2([-\tfrac{1}{2}+p]^{++}), \\ &\mathbf{16}(-\tfrac{1}{2}^{++}), \quad &&\overline{\mathbf{16}}(\tfrac{1}{2}^{++}) \\ &\mathbf{10}(-p^{-+}), \quad &&\mathbf{10}'(p^{++}) \end{aligned} \tag{10}$$

The desired 22, 23, 32 and 33 entries in the Dirac matrices of Eq. (6) are then obtained with the Yukawa interactions in Eq. (9) by integrating out the superheavy fermions introduced above. The relevant diagrams are pictured in Fig. 1 where the asymmetrical nature of the contributions is readily apparent.

In summary, we have shown that with the minimal set of $SO(10)$ Higgs fields introduced in Eq. (1) to solve the doublet-triplet splitting problem, fermion mass matrices can be constructed which explain well the known quark mass and mixing data and lead to the suggestion of large $\nu_\mu - \nu_\tau$ mixing responsible for the atmospheric neutrino anomaly. Unlike previous studies, this large neutrino mixing arises not from a large hierarchy in the right-handed Majorana matrix but rather as a result of the skewed spinor $\mathbf{16}'$ Higgs and antisymmetrical $B-L$ adjoint $\mathbf{45}$ contributions to the Dirac matrices.

This work was supported in part by the Department of Energy Grant Nos. DE-FG02-91ER-40626, and DE-FG02-90ER-40542. CHA thanks the Fermilab Theoretical Physics Department for its kind hospitality.

REFERENCES

[1] S.M. Barr and S. Raby, Phys. Rev. Lett. **79**, 4748 (1997).

[2] C.H. Albright, K.S. Babu and S.M. Barr, Report NO. FERMILAB-Pub-98/052-T, IASSNS-HEP-98-14, BA-98-06.

[3] B. Brahmachari and R.N. Mohapatra,hep-ph/9710371; J. Sato and T. Yanagida, hep-ph/9710516; M. Drees, S. Pakvasa, X. Tata, and T. ter Veldhuis, hep-ph/9712392; M. Bando, T. Kugo, and K. Yoshioka hep-ph/9710417.

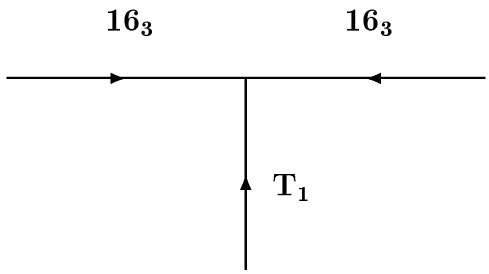
[4] S. Dimopoulos and F. Wilczek, Report No. NSF-ITP-82-07 (1981), in *The unity of fundamental interactions*, Proceedings of the 19th Course of the International School of Subnuclear Physics, Erice, Italy, 1981, ed. A. Zichichi (Plenum Press, New York, 1983).

[5] H. Georgi and C. Jarlskog, Phys. Lett. **B86**, 297 (1979).

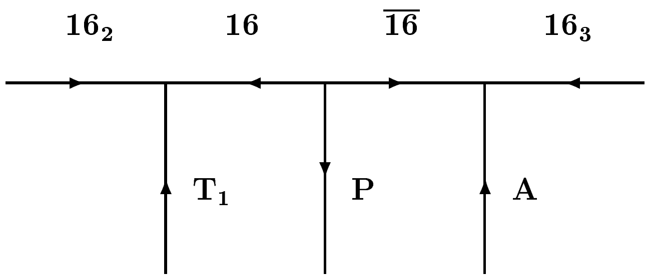
[6] K.S. Hirata *et al.*, Phys. Lett. B **205**, 416 (1988); K.S. Hirata *et al.*, Phys. Lett. B **280**, 146 (1992); Y. Fukuda *et al.*, Phys. Lett. B **335**, 237 (1994); D. Caspar *et al.*, Phys. Rev. Lett. **66**, 2561 (1991); R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992); Nucl. Phys. B (Proc. Suppl.) **38**, 331 (1995); T. Kafka, Nucl. Phys. B (Proc. Suppl.) **35**, 427 (1994); M. Goodman, *ibid.* **38**, 337 (1995); W.W.M. Allison *et al.*, Phys. Lett. B **391**, 491 (1997).

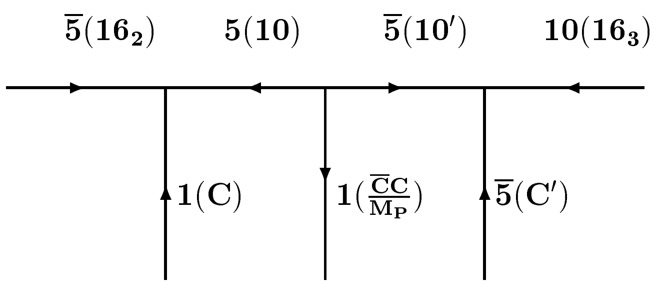
Fig. 1. Diagrams that generate the 33, 23, 32 and 22 entries in the quark and lepton mass matrices of Eq. (6). The second diagram of the 23 entry appears only for the down quark mass matrix. A similar diagram in reverse order would appear for the 32 entry of the charged lepton mass matrix.

33 :

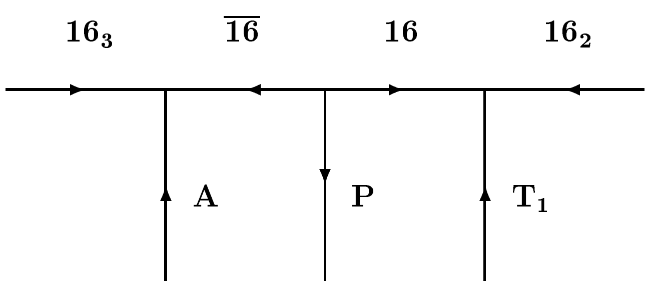


23 :





32 :



22 : (None)