Fermi National Accelerator Laboratory

FERMILAB-Conf-98/096

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March 1998

Published Proceedings of the *Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider*, Fermilab, Batavia, Illinois, November 6-9, 1997

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

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November 1997

Abstract. The possibilities of a ring cooler stage in a muon collider are explored. A basic design—along with some variations—are examined both with analytic calculations and simulation of the evolution of beam phase space and differences encountered between them are discussed.

INTRODUCTION

This report examines the possibility of using a racetrack–like ring accelerator in the cooling stage of a muon collider. The main merit of such a cooler compared with a linear one is its lower projected costs. The expense ratio linear/ring scales roughly as the number of turns needed to achieve effective cooling—a factor of some tens in a typical scenario.

The cooling in the 'usual' ring machine is shown in Fig. 1a. The beam is compressed in both X and X' directions because they are coupled by the β-function. Strong cooling is impossible in this case because reduction of the angular spread is limited by scattering in the absorber. A system with decreasing β–function can be used for cooling in a linear scheme as shown in Fig. 1b [1]. The same result can be obtained in a ring cooler if its transfer matrix is λD where λ is the cooling coefficient and D is a diagonal matrix (a lattice *without* β–function). In this case motions in X and X' directions are independent and the beam evolution from turn to turn looks like in Fig. 1b.

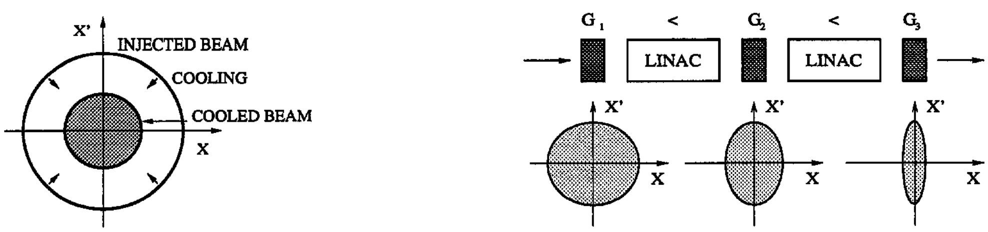

FIGURE 1. Beam cooling by a ring cooler (a) and linear cooler with decreasing β-function (b).

1) Work supported by the Universities Research Association, Inc., under contract DE-AC02-76CH0300 with the U. S. Department of Energy.

SCHEMATIC AND PARAMETERS OF THE COOLER

A schematic of the cooler is shown in Fig. 2. It includes two bending sections with wedge absorbers and two straight sections which house RF cavities and the main absorbers—assumed here to be beryllium (Fig. 3). Each bending section includes magnets with field index 0.5 for focusing and bending the beam. Betatron phase advances are 360° to get independent variation of coordinates and angles (which is why turns in two planes are used). Skew quadrupoles are placed on the edges and center of the bending section to suppress dispersion in the straight sections.

Each straight section includes three FODO cells with phase advance of 60° for X and 120° for Y in the leading part of the section, and two 90° FODO cells in the trailing part. This gives betatron transfer matrices $M_{x,y} = \pm\sqrt{\lambda} \times D$ per half-turn, and $M_{x,y} = \lambda \times D$ per turn, as desired. With an appropriate choice of length for the straight section, revolution frequency becomes independent of energy. Bunchers are installed to provide energy-time coupling so that the longitudinal transfer matrix is also of $\lambda \times D$ type. Some parameters of the cooler are listed in Table 1.

TABLE 1. Parameters of the Cooler

Muon energy (γ): min, center, max	1.5 – 1.647 – 1.875
Circumference	19.51 m
Revolution frequency	12.13 MHz
Bending radius	0.458 m
Length of straight sections	5.687 m
Energy rate in SS	7.0 MeV/m
Length of main absorber (Be)	11.2 cm
Angle of the wedge absorber	5.8 deg

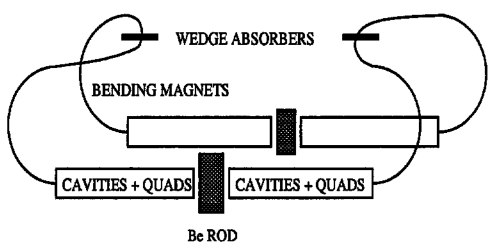

FIGURE 2. Schematic of the ring cooler.

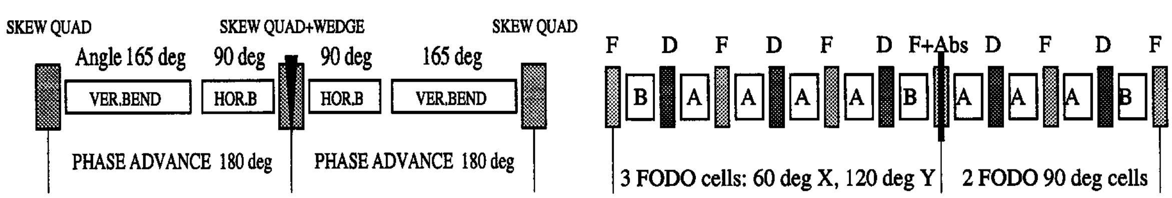

FIGURE 3. Bending and straight sections of ring cooler (F,D-quads,A,B-accelerating cavities and bunchers).

COOLING

Results of both analytical calculations and Monte Carlo simulations with the code SIMUCOOL [2] are presented. The simulations are limited to what happens to the muons during material traversals. Elsewhere they employ the same transfer matrices as in the analytical approach.

Evolution of rms sizes and invariant emittances of a cooled bunch as a function of turn–number, as determined by analytical calculation, are presented in Fig. 4a. The two lower (straight) lines represent energy– and angular spreads of the bunch. They remain constant because their initial values are chosen as the equilibrium ones. Also shown in Fig. 4a are the rms radius and length of the bunch, along with transverse and longitudinal invariant rms emittances and the 6–dimensional emittance (longitudinal emittance is the product of the lengths of the semi–axes $\Delta\gamma\Delta cT$). The 6–D emittance decreases approximately by a factor of 15,000 times in 32 turns—with equilibrium not yet achieved.

Results of a SIMUCOOL simulation for this cooler are presented in Fig. 5 which shows the initial and final distributions of longitudinal and transverse phase planes.

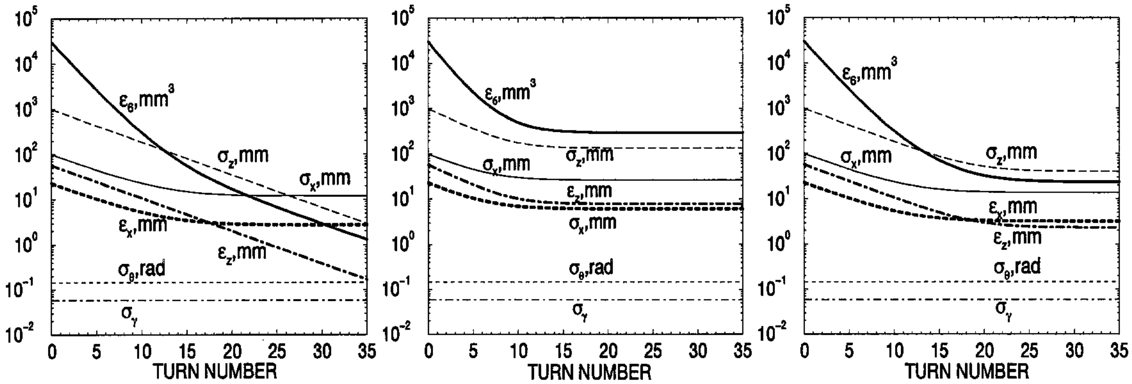

FIGURE 4. Cooling of the bunch: a–ideal wedge, b–wedge 32 mm, c–2.4mm.

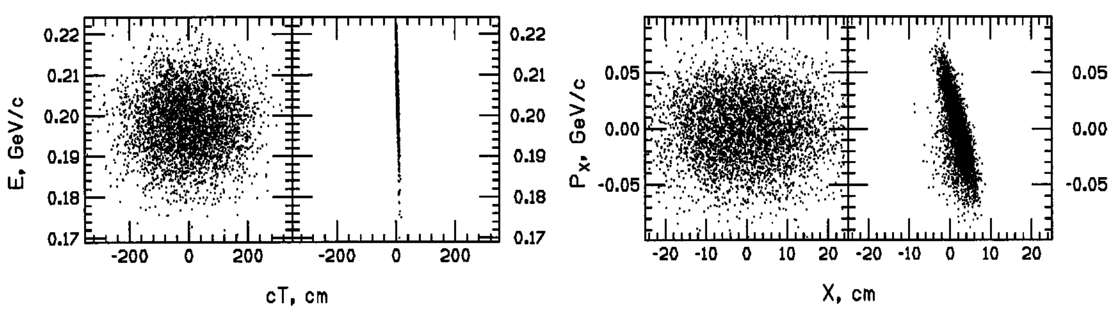

FIGURE 5. Initial and final phase space distributions for ideal wedge absorber: a–longitudinal, b–transverse.

Some differences between analysis and simulation are apparent in this study. Scattering in the simulation is less than in the analytical approach by a factor of 0.8 whereas straggling is 2.8 times larger. The (nonlinear) dependence of energy loss by the muon on X'^2 and Y'^2 may be responsible for the latter. About 51% of the muons decay over the 32 turns. Muon scattering and straggling in the wedge absorber are not taken into account in this case (ideal wedge).

To study a more realistic wedge absorber, its angle is fixed at about 5.8° which means its thickness at the center has to be about 32 mm at a radius of 310 mm (aperture of bending magnets). Evolution of bunch size and emittances in the cooler with such an absorber is presented on Figs. 4b and 6. Longitudinal and 6–D emittances are much larger than with the 'ideal' wedge absorber. An explanation for this is sketched in Fig. 7 where phase ellipses of the bunch in the $cT - X'$ plane are pictured first in the main absorber, then in the wedge, and again in the main absorber. In the wedge, the ellipse is turned because the time of passage between absorbers of a particle depends on its angle. The deviation from an upright position of the ellipse results in an increase of bunch length due to scattering in the wedge absorber. This effect appears unavoidable because the only way to suppress it would be to suppress the dispersion at the location of the wedge absorber—but then it is pointless to place a wedge there. But it may be decreased by moving the beam into the thin part of the wedge during cooling. This can be achieved, e.g., by making the accelerating cavities undercompensate the energy loss in absorbers.

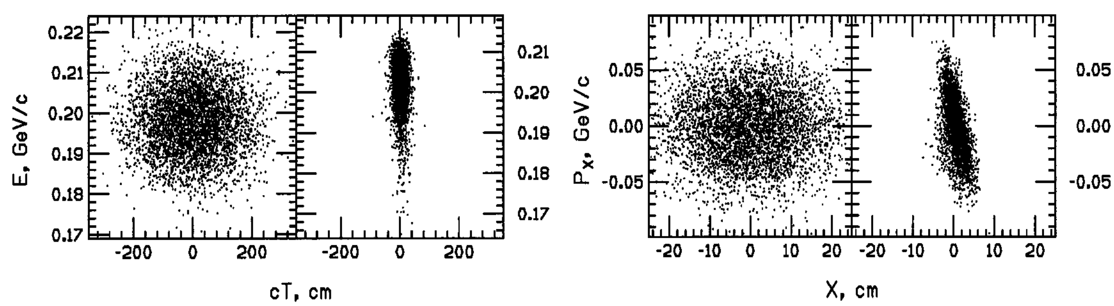

FIGURE 6. Initial and final phase space distributions for wedge absorber 32 mm of thickness : a–longitudinal, b–transverse.

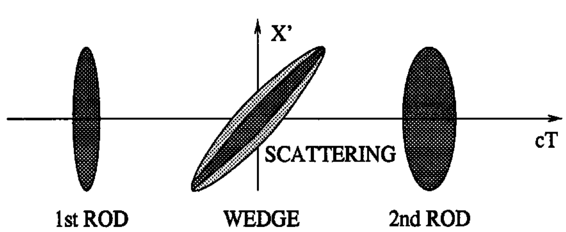

FIGURE 7. Lengthening of bunch due to scattering in wedge absorber.

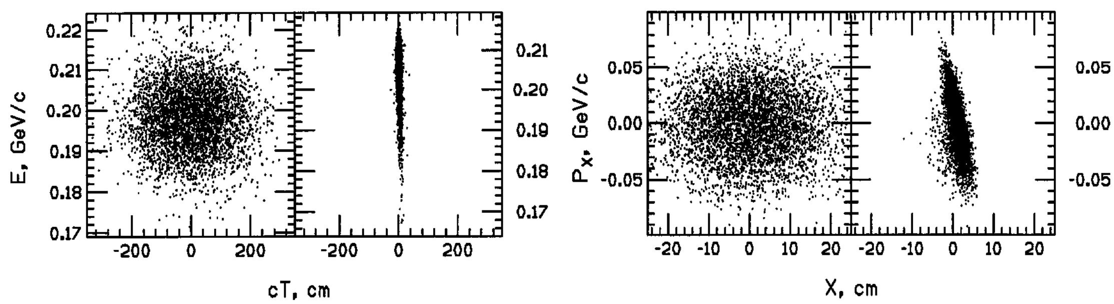

FIGURE 8. Initial and final phase space distributions for wedge absorber 2.4 mm of thickness: a–longitudinal, b–transverse.

In this way, the final (equilibrium) emittance of the bunch is only affected by scattering in the thinnest part of the absorber—2.4 mm thick in the present model. The results displayed in Figs. 4c and 8 show great improvement over the 32 mm thick wedge absorber case.

CONCLUSIONS

A ring cooler appears capable of cooling a muon beam satisfactorily both in transverse and longitudinal directions. The achievable emittance suggests its use as a precooler in a muon collider complex especially for effective bunch shortening which is necessary in any scenario. Near–term goals are:

- Explain differences of analytical calculation and simulation, leading to improvements in both methods and achieving maximal compatibility.
- Study the effect of different absorber materials on cooler performance.
- Run a full simulation of the cooler, i.e., replace the transfer matrices presently used by full tracking through magnets and cavities to investigate suppression of both chromatic and nonlinear effects and to study dynamic aperture.

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2. Van Ginneken, A., *Nucl. Inst. Meth.* **A362**, p. 213 (1995).