Fermi National Accelerator Laboratory

FERMILAB-Conf-97/348-T

Decay Constants of Heavy-Light Pseudoscalar Mesons in the Fermilab Formalism

Sinéad M. Ryan

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

Jim N. Simone
Loomis Laboratory of Physics
1110 W. Green Street, Urbana, Illinois 61801-3080

November 1997

Published Proceedings of *Lattice '97*, Edinburgh, Scotland, July 22-26, 1997

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Disclaimer

Distribution

Decay constants of heavy-light pseudoscalar mesons in the Fermilab formalism

Sinéad M. Ryan[a*] and Jim N. Simone[b]

[a]Theoretical Physics Group, Fermilab, P.O. Box 500, Batavia, Il 60510, U.S.A.

[b]Loomis Laboratory of Physics, 1110 W. Green Street, Urbana, Il 61801-3080, U.S.A.

We present new results from the Fermilab weak matrix-element project. The decay constants of heavy-light pseudoscalar mesons are calculated using the Fermilab formalism for heavy quarks and a tadpole-improved SW action for light quarks. Improvement allows the bottom quark to be simulated directly on the lattice removing the need for large extrapolations. The dependence on the lattice spacing is discussed for $5.7 \leq \beta \leq 6.1$.

1. Introduction

In this paper we report results for the decay constants, $f_{\rm P}$, of heavy-light mesons. More details of this calculation will appear in a future paper [1]. We have results at four lattice spacings, three of which are used in a continuum extrapolation. The pseudoscalar decay constant, $f_{\rm P}$, parameterises the matrix element on the lattice by

$$\langle 0|\overline{Q}\gamma_\mu\gamma_5 q|{\rm P}\rangle = Z_A^{-1} f_{\rm P}\sqrt{m_{\rm P}}. \quad (1)$$

The techniques for determining $f_{\rm P}$ on a lattice are well established and here we attempt to quantify the systematic errors inherent in such a calculation.

The details of the lattices used in our simulations are in Table 1. We have three heavy quarks and four light quarks, four heavies and five lights and five heavies and five lights at $\beta = 6.1, 5.9$ and 5.7 respectively. We have also completed a simulation in the static approximation for all three lattices.

β	Vol.	# cfgs.	$a^{-1}(f_K)$ (GeV)
6.1	$24^3\times 48$	100	2.29^{+19}_{-13}
5.9	$16^3\times 32$	300	1.67^{+8}_{-2}
5.7	$12^3\times 24$	350	1.10^{+4}_{-2}

Table 1. Lattice details

*talk presented by S. Ryan.

2. The Fermion Action

For light quarks the action is tadpole improved, Sheikholeslami-Wohlert (SW), where the tadpole-improvement factor, u_0, is determined from the plaquette. Heavy quarks are treated in the Fermilab formalism [3,2], using a tadpole-improved, SW action, as for light quarks. The bare quark masses for charm and bottom, κ_c and κ_b are tuned from the dispersion relations in -onia states such that the kinetic mass, m_2 matches the physical mass of interest since for simulations involving heavy quarks it is the kinetic and not the rest mass which provides the relevant mass scale. To obtain full $\mathcal{O}(a)$ improvement of matrix elements we use $\mathcal{O}(a)$-improved currents. This is achieved by rotating the fermion fields with the factor

$$\hat{\psi} = \left(1 + a d_1(m_0, g^2)\vec{\gamma}\cdot\vec{D} + \mathcal{O}(a^2)\right)\psi', \quad (2)$$

where the coefficient d_1 is calculated at tadpole-improved tree-level ; and by a field renormalisation [3] which at tadpole-improved tree-level is,

$$\psi = \sqrt{2\kappa}\exp(a\frac{m_1}{2})\hat{\psi} = \sqrt{1-6u_0\kappa}\;\hat{\psi}. \quad (3)$$

The current renormalisation is determined from the light quark perturbative expression: $Z_A = 1 - 0.61\alpha_V(q^*)$ [4].

3. Details of the analysis

To reliably extract the ground state masses and amplitudes of the quantities of interest smearing functions, from a study of Coulomb-gauge pseudoscalar wavefunctions, are used. These are created for 1S and 2S sources and the fitting procedure uses a matrix of correlators *eg.* $\delta - \delta$, 1S-1S, 2S-2S... with multiexponential fits. We tested for excited state contamination by checking the consistency of one-, two- and three-state fits.

Since the quantity which is immediately extracted from the amplitude of such fits is $f_{\rm P}\sqrt{m_{\rm P}}$ we choose to extrapolate this to the continuum limit and then use the appropriate experimental pseudoscalar meson mass to arrive at a value for $f_{\rm P}$. This avoids any ambiguities and errors in the determination of $m_{\rm P}$ [5].

The scale for this lattice calculation is set from the decay constant f_K. This is the known quantity which most resembles the heavy-light decay constants and in $f_{\rm P}/f_K$ one can hope that some numerical errors will cancel. It is chosen in preference to f_π because it is determined by interpolation not extrapolation and thus has smaller statistical errors. Examples of our continuum extrapolations to determine f_{B_s}, f_{D_s} and the ratios f_{B_s}/f_B and f_{D_s}/f_D are in Figures 1 and 2[¶].

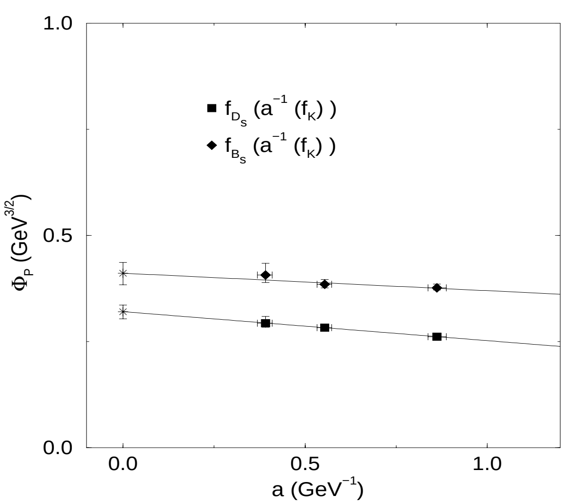

Figure 1. $f_{B_s}\sqrt{M}$ and $f_{D_s}\sqrt{M}$ extrapolated to the continuum limit.

¶In these and all future Figures we define $\Phi_{\rm P} = f_{\rm P}\sqrt{M}$.

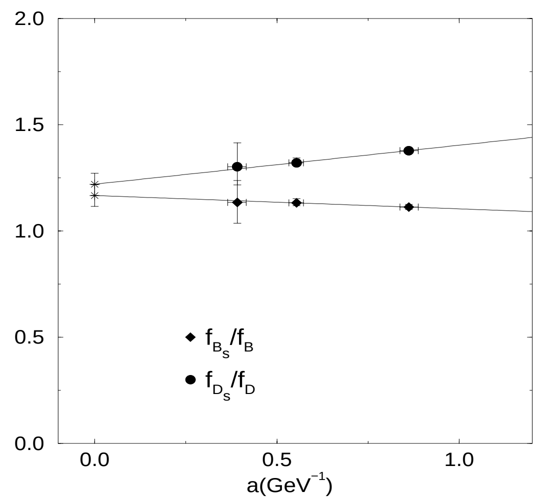

Figure 2. The extrapolated ratios of f_{B_s}/f_B and f_{D_s}/f_D.

4. Systematic Errors

There are a number of systematic uncertainties inherent in any lattice calculation of $f_{\rm P}$ and to make a realistic assessment of the accuracy of any final result these must be considered. One well known and possibly large effect of quenching the theory is the uncertainty in setting the scale. In this study we chose two quantities, f_K and the spin-averaged 1P-1S splitting from Charmonium and examined the effect on the extrapolation of using these to set the scale. The bare quark masses, at charm and bottom are tuned from -onia so we looked at the effect of tuning from a heavy-light system with the light quark at strange. We assign a 5% uncertainty to our results from the perturbative expression for Z_A and evaluate it at $q^* = 1/a$. An additional uncertainty, encompassing the remaining a-dependence in $f_{\rm P}\sqrt{M_{\rm P}}$ and the extra constraint of having three points which requires a linear fit to the data, is included. This is summarised in Table 2 which shows the shift in the central value of $f_{\rm P}$ on using different analyses. Our final numbers are quoted below. The central value is determined using linear fits to the

	f_B	f_{B_s}	f_D	f_{D_s}	f_{B_s}/f_B	f_{D_s}/f_D
a^{-1}(1P-1S)	$^{+29}_{-\ 0}$	$^{+39}_{-\ 0}$	$^{+41}_{-\ 0}$	$^{+51}_{-\ 0}$	–	–
a-dependence	$^{+\ 8}_{-\ 6}$	$^{+\ 2}_{-\ 2}$	$^{+20}_{-36}$	$^{+21}_{-30}$	$^{+.02}_{-.09}$	$^{+.15}_{-.05}$
$Z_A(q^*)$	$^{+\ 9}_{-\ 9}$	$^{+\ 9}_{-\ 9}$	$^{+\ 9}_{-\ 9}$	$^{+11}_{-11}$	–	–
κ_b(Bs), κ_c(Ds)	$^{+\ 0}_{-\ 5}$	$^{+\ 1}_{-\ 0}$	$^{+\ 0}_{-\ 0}$	$^{+\ 1}_{-\ 0}$	$^{+.02}_{-\ 0}$	$^{+\ 0}_{-.01}$
interpolation	$^{+\ 3}_{-\ 0}$	$^{+\ 1}_{-\ 0}$	$^{+\ 0}_{-\ 0}$	$^{+\ 0}_{-\ 0}$	$^{+02}_{-\ 0}$	$^{+\ 0}_{-\ 0}$

Table 2. A breakdown of the systematic errors (in MeV) for some of the quantites calculated.

light quark data and $a^{-1}(f_K)$. The first error is statistical, the second, the uncertainty in setting the scale and the third the remaining systematic error from Table 2. All values are in MeV.

$$f_B = 156\,^{+13}_{-14}\,^{+29}_{-\ 0}\,^{+\ 9}_{-\ 9} \quad , \quad f_{B_s} = 177\,^{+11}_{-12}\,^{+39}_{-\ 0}\,^{+13}_{-11}$$

$$f_D = 183\,^{+12}_{-13}\,^{+41}_{-\ 0}\,^{+\ 9}_{-25} \quad , \quad f_{D_s} = 229\,^{+10}_{-11}\,^{+51}_{-\ 0}\,^{+\ 3}_{-19}$$

$$\frac{f_{B_s}}{f_B} = 1.17\,^{+5}_{-5}\,^{+0}_{-0}\,^{+3}_{-3} \quad , \quad \frac{f_{D_s}}{f_D} = 1.22\,^{+5}_{-5}\,^{+0}_{-0}\,^{+8}_{-8}$$

$$f_B^{\rm stat} = 186\,^{+25}_{-25}\,^{+37}_{-\ 0}\,^{+24}_{-\ 9} \quad , \quad f_{B_s}^{\rm stat} = 219\,^{+21}_{-21}\,^{+35}_{-\ 0}\,^{+\ 9}_{-\ 9}$$

And finally the continuum plot of $f_{\rm P}\sqrt{M_{\rm P}}$ is shown in Figure 3. The points are our results at the charm quark mass, the bottom quark mass and in the static approximation.

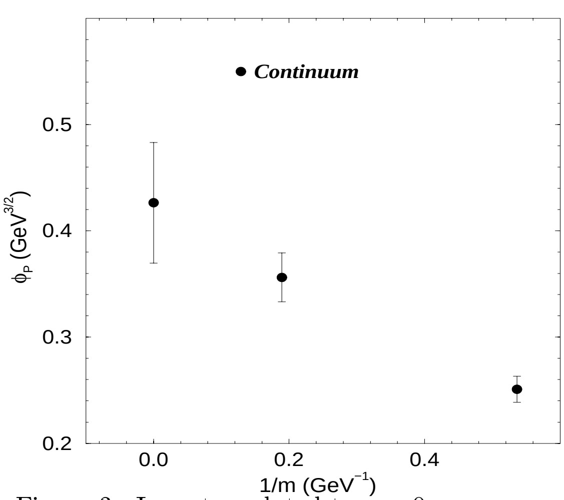

Figure 3. $\Phi_{\rm P}$ extrapolated to $a = 0$.

5. Conclusions

This calculation of decay constants using a tadpole-improved action and interpreted in the Fermilab formalism is very encouraging. The results are in agreement with those of other groups [6,7] and a preliminary estimate of the systematic uncertainties has been made. Final results for these and further calculations will be presented in greater detail in a forthcoming paper [1].

6. Acknowledgements

Fermilab is operated by Universities Research Association, Inc. for the U.S. Department of Energy.

REFERENCES

1. A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie, S.M. Ryan, J.N. Simone, *B and D meson decay constants*, FNAL Pub-97/322-T.
2. J.N. Simone, Review talk at Lattice'95, hep-lat/9601017.
3. A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie, Phys. Rev. **D55**, 3933 (1996).
4. A. Borelli, C. Pittori, R. Frezzotti and E. Gabrielli, Nucl. Phys. **B409**, 382 (1993); This is a preliminary estimate, we hope to use a calculation by JLQCD [8].
5. S. Collins *et al*, Nucl. Phys. **B**, Proc. Suppl. **47** (1996).
6. T. Onogi, these proceedings.
7. A. Ali Khan, these proceedings.
8. K. Ishikawa, these proceedings.