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IMPEDANCE SCALING AND IMPEDANCE CONTROL

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Abstract

When a machine becomes really large, such as the Very Large Hadron Collider (VLHC),[1] of which the circumference could reach the order of megameters, beam instability could be an essential bottleneck. This paper studies the scaling of the instability threshold *vs.* machine size when the coupling impedance scales in a "normal" way. It is shown that the beam would be intrinsically unstable for the VLHC. As a possible solution to this problem, it is proposed to introduce local impedance inserts for controlling the machine impedance. In the longitudinal plane, this could be done by using a heavily detuned rf cavity (*e.g.,* a biconical structure), which could provide large imaginary impedance with the right sign (*i.e.,* inductive or capacitive) while keeping the real part small. In the transverse direction, a carefully designed variation of the cross section of a beam pipe could generate negative impedance that would partially compensate the transverse impedance in one plane.

I. INTRODUCTION

In a recent workshop LHC96 at Montreux, Switzerland, a survey was conducted for the measured impedance of existing machines and expected impedance of some new machines.[2] It is interesting to note that the impedance, $Z_{\|}/n$, has been lowered by about an order of magnitude since 1980s, thanks to a number of low impedance design features in new machines (*e.g.,* a uniform beam pipe cross section, rf shielding of vacuum components, tapered transitions, *etc.*). Nowadays one believes that for high energy machines, in which space charge contribution is negligible, $Z_{\|}/n$ can be held at around 1 ohm or below, no matter how big the machine is. However, the transverse impedance $Z_{\perp}$ scales almost linearly with the machine size due to accumulation effects of discontinuities in the beam environment. This raises the question whether the beam could become intrinsically unstable in a large machine.

II. INSTABILITY SCALING

A. The transverse mode coupling instability

Although this type of instability has been well studied and clearly observed in electron machines, it has never been seen in any proton machine. But still, one always wants to keep the beam current below the threshold. The threshold current per bunch is:

$$I_{th} = \frac{2\nu_s(E/e)}{\beta_{av}(\mathrm{Im}Z_{\perp})} \cdot \frac{4\sigma_b}{R} \tag{1}$$

in which ν_s is the synchrotron tune, E the particle energy, e the electron charge, β_{av} the average β-function, $\mathrm{Im}Z_{\perp}$ the imaginary part of transverse impedance, σ_b the *rms* bunch length and R the machine radius. In a relativistic case, the following relations hold:

$$\nu_s\sigma_b = R\eta \cdot \left(\frac{\sigma_E}{E}\right) \tag{2}$$

$$\eta = \frac{1}{\gamma_t^2} \tag{3}$$

where σ_E/E is the relative *rms* energy spread, η the slip factor and γ_t the transition γ. Plugging into (1) one gets:

$$I_{th} \propto \frac{\frac{R}{\gamma_t^2}\cdot E}{\beta_{av}(ImZ_{\perp})\cdot R} \cdot \left(\frac{\sigma_E}{E}\right) = \frac{E}{\beta_{av}(ImZ_{\perp})\cdot \gamma_t^2} \cdot \left(\frac{\sigma_E}{E}\right)$$

A *large* machine means large R and high E. The scaling goes as:

$$\begin{aligned} E &\propto R \\ \gamma_t &\propto \sqrt{R} \\ \beta_{av} &\propto \sqrt{R} \\ \mathrm{Im}Z_{\perp} &\propto R \end{aligned}$$

Thus, the threshold bunch current decreases when the machine size increases:

$$I_{th} \propto R^{-3/2} \cdot \left(\frac{\sigma_E}{E}\right) \tag{4}$$

This scaling can also be expressed in terms of particle numbers. In view of

$$\begin{aligned} I_{th} &= N_b \cdot f_0 \cdot e = N_b \cdot \frac{c}{2\pi R} \cdot e \\ &\propto \frac{N_b}{R} \end{aligned}$$

in which N_b is the number of particles per bunch, f_0 the revolution frequency and c the velocity of light, the maximum N_b would be scaled as:

$$N_b \propto R^{-1/2} \cdot \left(\frac{\sigma_E}{E}\right) \tag{5}$$

Therefore, for large machines (*e.g.,* 10^6 meters in circumference), the transverse mode coupling could become an intrinsic bottleneck limiting the beam intensity and luminosity.

B. The resistive wall instability

The growth rate of resistive wall instability is:

$$\begin{aligned} \frac{1}{\tau_w} &= \frac{cr_pN_bM}{2\pi\gamma\nu_\beta b^3}\sqrt{\frac{2\rho}{\mu\omega}} \\ &\propto \frac{M}{E\nu_\beta} \cdot \sqrt{\frac{1}{\omega_0}} \cdot N_b \end{aligned} \tag{6}$$

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in which r_p is the classical radius of proton, M number of bunches, γ the relativistic energy of the particle, ν_β the betatron tune, b the beam tube radius, ρ the wall resistivity, μ the vacuum permeability and ω the angular frequency, which equals the product of fractional tune and angular revolution frequency ω_0. By using the following scaling:

$$\begin{aligned} M &\propto R \\ \nu_\beta &\propto \sqrt{R} \\ \omega_0 &\propto R^{-1} \end{aligned}$$

one gets:

$$\begin{aligned} \frac{1}{\tau_w} &\propto \frac{R}{R\sqrt{R}} \cdot \sqrt{R} \cdot N_b \\ &\propto N_b \end{aligned} \tag{7}$$

It is seen that the growth rate is *independent* (to the extent that the scaling holds) of machine size. In other words, for given bunch intensity, bunch spacing and beam pipe (material and cross section), large and small machines have more or less the *same* growth time of the resistive wall instability.

However, when expressing the growth time in terms of turn numbers:

$$n_w = f_0 \tau_w = \frac{c}{2\pi R} \tau_w$$

one obtains:

$$n_w \propto R^{-1} \cdot N_b^{-1} \tag{8}$$

Thus, large machines would require powerful feedback systems (*e.g.,* criss-crossing feedback and one-turn correction scheme).

C. The longitudinal microwave instability

The longitudinal impedance of a machine, $|Z_\parallel/n|$, is more or less independent of the machine size. So is the microwave instability threshold current:

$$I_{th}^{\text{peak}} = \frac{2\pi\eta}{|Z_\parallel/n|} \left(\frac{E}{e}\right) \left(\frac{\sigma_E}{E}\right)^2 \propto R^0 \tag{9}$$

III. IMPEDANCE CONTROL

A. Longitudinal impedance

There have been some discussions about using ferrite tori, which provides large inductive part of impedance, to compensate the capacitive part of impedance from, say, space charge. The problem is that the ferrite also introduces large additional real part of impedance, which may hurt the beam as well. Hence, the trick is how to get large reactance (either inductive or capacitive, whichever is needed) while keeping resistance under control. This can be achieved by using a heavily detuned rf cavity. When such a cavity is represented by a lumped RLC circuit, its impedance is:

$$\begin{aligned} Z(\omega) &= R_s \cos\theta \exp(-j\theta) && (10) \\ \theta &= 2Q\frac{\Delta\Omega}{\Omega_0} && (11) \end{aligned}$$

where R_s is the shunt impedance, Q the quality factor, Ω_0 the resonance frequency, and $\Delta\Omega$ the amount of detuning. From Eq. (10), it is seen that, for any given $\mathrm{Im}Z$, the $\mathrm{Re}Z$ is double valued: one for $\theta < 45°$, another for $\theta > 45°$. The ratio of the imaginary and real part of impedance is:

$$\frac{\mathrm{Im}Z}{\mathrm{Re}Z} = \tan\theta \tag{12}$$

For large θ, one gets large $\mathrm{Im}Z$ and small $\mathrm{Re}Z$. For instance, when $\theta = 84.3°$, the ratio is 10. A special type of cavity, the biconical structure, is of particular interest for this application. This is because its fundamental and higher order modes (HOM) cover a unique spectrum: f_0, $3f_0$, $5f_0$, $7f_0$, *etc.* The existence of a beam pipe attached to this structure will certainly skew the Slater plot slightly but not too much. Table 1 shows the URMEL simulation results of a biconical resonator (17 cm in radius) connected with a beam pipe (2 cm in radius). It is seen the Slater perturbation is small. This special spectrum of resonance modes in this beam-excited resonator indicates that, while the beam induced reactive voltage will offset the potential well (the slope of the rf voltage) and thereby the bunch length, the peak voltage can remain flat due to superposition of the HOMs. (All the HOMs are on the "right" frequencies.)

Table 1. Modes of a Biconical Resonator with Beam Pipe

Mode	Freqency (MHz)	R/Q (Ω)	Q
TM_{01}	487.48 ($1 \times f_0$)	131.8	23788
TM_{02}	1459.6 ($2.994 \times f_0$)	37.04	34204
TM_{03}	2422.9 ($4.970 \times f_0$)	15.53	43231
TM_{04}	3369.6 ($6.912 \times f_0$)	6.159	52383

B. Transverse impedance

The control of transverse impedance is based on the observation of the so-called negative transverse impedance.[3]-[4] For certain type of structure, *e.g.,* the CERN SPS adaptor, which has a circular cavity connected with rectangular beam pipes on both ends, the first peak of the transverse wakefield in one plane is negative. This means focusing in that direction, *i.e.,* any deflection of the beam from the axis will see a kick to force it back to the axis. Simulations and measurements both show the horizontal and vertical transverse impedance in the SPS are of opposite signs. Thus, one may design special vacuum ports as local impedance insert to control the total transverse machine impedance.

Negative transverse impedance means energy gain of a beam. From the viewpoint of energy conservation, one plane's gain must be another plane's loss. In other words, there would be more positive transverse impedance in the other plane. That plane should be chosen as a less critical one if both planes are not equivalent (which is usually the case in most machines). Furthermore, it will be interesting to see what would be the combined effect if a beam sees alternate focusing and defocusing transverse wakefield kicks.

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