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CHARMONIUM WITH IMPROVED WILSON FERMIONS I: A DETERMINATION OF THE STRONG COUPLING CONSTANT*

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I discuss the motivations and analysis of uncertainties of a recent determination of α_s from the 1P–1S splitting in Charmonium. The result is $\alpha_{\overline{MS}}(5\ \mathrm{GeV}) = 0.174 \pm 0.012$, or equivalently, $\Lambda^{(4)}_{\overline{MS}} = 160^{+47}_{-37}$ MeV.

The calculation of the masses of the proton and other light hadrons has occupied a prominent place in the efforts of lattice gauge theorists since the introduction of lattice gauge theory in 1974. In that year, the idea that nonperturbative calculations based on the fundamental QCD Lagrangian could quantitatively reproduce the hadron spectrum was an exciting hypothesis. Now, almost twenty years later, it is widely accepted that QCD is the correct theory of strong interactions, and even that lattice gauge theory will, at least eventually, produce real calculations in QCD. Light hadron spectrum calculations are now viewed as telling us more about the current state of lattice methods than about the physical world. The lattice calculations which give us new information about the world are the ones which aid in the extraction of the fundamental parameters of the standard model from experimental data, or which predict the behavior of QCD in extreme environments.

Heavy quark systems like the J/ψ and Υ systems received very little attention in the early days of lattice gauge theory since the very successful potential models provided a convincing phenomenology for these systems, which did not exist for the light hadrons. For precisely this reason, however, heavy quark systems can play a more important role than the light hadrons in illuminating the reliability and accuracy of present day lattice methods, and even, in the case of the strong coupling constant, in helping to determine the parameters of the standard model. The fact that these systems are well described by static potentials and wave functions can be used to bolster and make more precise estimates of corrections and uncertainties, even though potential model ideas play no role in the basic lattice calculations themselves. These points have often been made by Lepage in discussions of the nonrelativistic formulation of lattice fermions.[1] They apply equally well to calculations with Wilson fermions.

We at Fermilab have recently performed a study of the Charmonium system with Wilson fermions.[2,3] The study has several goals. One is the study of the phenomenology of Charmonium itself. Another is the careful study of systematic errors in a context where they can be very well understood, with the aim of guiding the analysis of systematic errors in calculations with lighter hadrons where the situation is murkier.

Yet another is the determination of the strong coupling constant. This is not the most crucial piece of standard model information that can be provided by lattice gauge theory, since

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it is known from a variety of short distance determinations to lie somewhere near the range $\alpha_{\overline{MS}}(5\ GeV) \approx 0.18 - 0.22$. It is, however, the one which can be obtained most accurately with present day lattice calculations.

The cleanest quantity in heavy quark systems from which to extract the strong coupling constant is the splitting between the spin averaged masses of the 1S and 1P states. This splitting is insensitive to errors in spin dependent interactions which are induced by the finite size of the lattice spacing. It is also known to be quite insensitive to any errors in the definition of the quark mass, since the 1P–1S splittings in the ψ and Υ systems are almost identical. We have calculated this splitting using standard Monte Carlo techniques at three lattice spacings: $\beta = 6.1$ on lattice volumes of 24^4 with 25 gauge configurations separated by 8000 pseudo-heat bath sweeps, $\beta = 5.9$ on volumes of 16^4 and $\beta = 5.7$ on volumes of $12^3 \times 24$, each with 25 configurations separated by 2000 pseudo-heat bath sweeps.

Figure 1 shows the Coulomb gauge wave function of the ψ meson. on the $\beta = 6.1$, 24^4 lattices. It has approximately the exponential shape associated with a Coulomb potential, with the softening at short distances expected from asymptotic freedom, and the faster fall-off at long distances arising from the linear potential. The effects of periodic boundary conditions are visible at $r \geq 12$. A short run on $\beta = 6.1$, 16^4 lattices showed a 1P–1S splitting 10–20% larger then on the 24^4 lattices. The assumption that finite volume effects are governed approximately by the wave function squared half way across the lattice, and the observation that the wave function is a factor of five smaller at $r = 12$ than at $r = 8$ leads to the estimate that finite volume errors in the splitting (and therefore in Λ) are less than around 1%, an estimate much smaller and much more accurate than could have been obtained

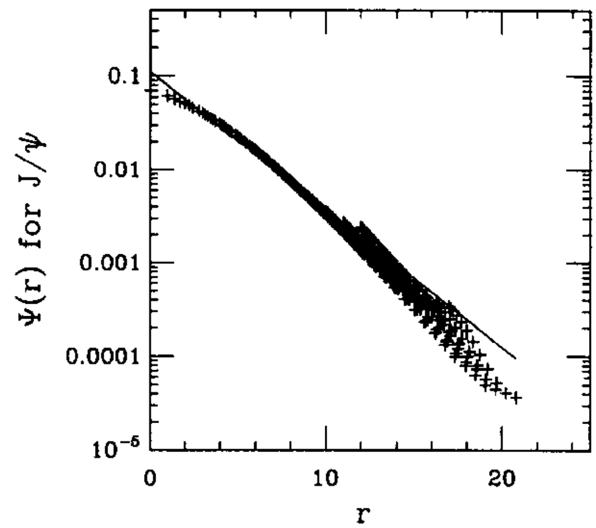

Fig. 1. The wave function of the ψ meson on the $\beta = 6.1$ lattices. The solid line shows the spreading function S(r), see [3].

from a simple comparison of the two volumes.

Similarly, a graph of Λ vs. a^2 shows relatively weak dependence on a^2. The wave functions can be used to evaluate the expectation values of the corrections to the lattice action to buttress or improve the theoretical prejudice in the extrapolation.

The most troublesome source of correction and uncertainty in the calculation arises from the omission of sea quarks. The dominant effect of this omission is a softening in the shape of the potential, resulting in a smaller value of the obtained bare coupling constant when the bare parameters are adjusted to obtain the correct physics at the charmonium momentum scale. The part of this effect arising at short distances may be reliably estimated with perturbation theory. The breakdown of perturbation theory at the medium distance scales of charmonium dynamics leads to the most significant source of uncertainty in the present calculation. It may be illuminated in the short run by the use of poten-

Source	Correction	Uncertainty
$g_0^2 \to g_{\overline{MS}}^2$ (One loop)	44%	-
$g_0^2 \to g_{\overline{MS}}^2$ (M. C.)	11%	-
$g^{(0)} \to g^{(4)}$	24%	6.6%
Statistics	-	2%
Finite lattice spacing	1%	1%
Finite volume	-	-

Table 1
Summary of corrections and uncertainties in the determination of $\alpha^{(4)}_{\overline{MS}}$. Note that all of the corrections have the same sign, raising the obtained value of α.

tial models instead of pure perturbation theory to estimate the correction. In the long run, it will of course be completely eliminated by the inclusion of sea quarks in the calculation.

We have thus arrived at a determination of a very interesting physical quantity from lattice gauge theory in which all known sources of systematic error are quantitatively estimated. Table 1 contains a summary of the corrections and uncertainties in the calculation which are detailed in reference [2]. Our result is

$$\alpha_{\overline{MS}}(5\ GeV) = 0.174 \pm 0.012. \quad (1)$$

We have not hesitated to use phenomenological information where necessary to make some of our corrections, much as experimentalists are sometimes forced to rely on theoretical prejudice in some data analysis. We are not yet as ambitious as Lüscher et al.[4] who attempt an entirely first principles determination of the running coupling constant in lattice gauge theory. It is clear though, that continued work can provide eventual removal of all phenomenological input from the calculation. In particular, the most important correction and uncertainty in the present calculation will be eliminated in a straightforward way by the inclusion of sea quarks in the calculation.

Although it is premature to make this claim to the physics community at large, our (perhaps prejudiced) opinion is that the lattice determination of α_s is already at least as reliable as the most widely quoted nonlattice result, the combined LEP determination: $\alpha_s(M_Z) = 0.119 \pm 0.006$. (Our result extrapolated to M_Z is $\alpha_s(M_Z) = 0.105 \pm 0.004$.) It seems likely that a straightforward, brute force inclusion of internal fermion loops over the next few years will reduce the uncertainties in the lattice calculation to a level which will be difficult for any standard short distance determination of α_s to approach.

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