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The Chromo-Electric Dipole Moment of the Heavy Quark and Purely Gluonic CP Violating Operators

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We study effects of induced CP non-conserving effective operators involving the gluon field strength as a result of integrating out the heavy quark field which carries a chromo-electric dipole moment. The induced gluonic operators of dimension 8 may be the dominant mechanism for generating the neutron electric dipole moment.

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Given a theory of CP violation, the mechanism for the violation typically involves heavy fields like the top quark in the Kobayashi-Maskawa model, the gluino/squarks in the minimal supersymmetric standard model, the right-handed gauge boson in the left-right models, or Higgs bosons in Higgs mediated models. At low energy (below the weak scale), the effect of these fields is carried by CP violating effective operators. One of the typical operators that can be significantly induced in most of the models is the chromo-electric dipole moment (CEDM) of the quarks. At even lower energies when these quarks are integrated out, many further interesting operators are in turn induced.

Weinberg[1] studied one of these interesting operators

$$O_6 = -\frac{g^3}{3} f^{abc} \tilde{G}^{a\mu\nu} G_{\lambda\mu}^b G_{\lambda\nu}^c \quad , \quad (1)$$

which can also be identified as the CEDM of the gluon itself[2], that provides a mechanism to generate the neutron electric dipole moment (NEDM). In Eq. (1), g is the gauge coupling; $G_{\mu\nu}^a$ is the gluon field strength; $\tilde{G}_{\mu\nu}^a = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{a\alpha\beta}$ is its dual with $\epsilon^{0123} = +1$; and f^{abc} is the totally antisymmetric tensor of $SU(3)$. Imposing the equation of motion for the sourceless gluon field, up to total derivatives, O_6 is the unique purely gluonic gauge invariant CP violating operator of dimension 6. It is also interesting to note that the dimension 4 topological term,

$$O_\theta = \tilde{G}_{\mu\nu}^a G^{a\mu\nu} \quad , \quad (2)$$

can provide an enormous contribution to the NEDM due to the nontrivial QCD θ vacuum. There are many ways to avoid this so-called strong CP problem. Therefore we will not address the issue associated with O_θ here[3]. Besides O_6 , it was also pointed out[1, 4] that certain dimension 8 purely gluonic operators can induce a NEDM. Earlier, Morozov[5] had investigated the renormalization group property of O_6 and the three independent CP violating gluonic operators of dimension 8:

$$\begin{aligned}
O_{8,1} &= g^4 \frac{1}{12} \bar{G}_{\mu\nu}^a G^{a\mu\nu} G_{\alpha\beta}^b G^{b\alpha\beta} \quad , \\
O_{8,2} &= g^4 \frac{1}{12} \bar{G}_{\mu\nu}^a G^{b\mu\nu} G_{\alpha\beta}^a G^{b\alpha\beta} \quad , \\
O_{8,3} &= g^4 \frac{1}{12} d^{abe} d^{ecd} \bar{G}_{\mu\nu}^a G^{b\mu\nu} G_{\alpha\beta}^c G^{d\alpha\beta} \quad .
\end{aligned} \tag{3}$$

Here d^{abc} is the totally symmetric tensor of $SU(3)$.

The induction of O_6 after integrating out the heavy quark with CEDM in the effective field theory has already been studied by several groups[6–9]. Here we investigate the effects caused by the induction of O_8 .

The higher dimensional operators are typically suppressed by additional powers of the heavy quark masses. However, unlike O_6 , which is suppressed by the QCD renormalization effect at low energy[5, 6], one of the dimension 8 operator[5] is QCD enhanced[10]. This enhancement can compensate the dimensional suppression which is not severe if the relevant scale is m_b (or m_c), but not M_W or higher.

Suppose a CEDM of a heavy quark Q with mass m is first induced at a high energy scale $\Lambda \gg m$ when particles with masses greater than Λ are integrated out from the low energy theory. At the renormalization scale $\mu = \bar{\Lambda}$, the effect of the heavy particles can be summarized by the following Lagrangian

$$L = -\frac{1}{2} \text{tr}_C G_{\mu\nu} G^{\mu\nu} + \bar{Q} (\not{P} - m - \frac{i}{2} g C \gamma_5 \sigma \cdot G) Q \quad , \tag{4}$$

where C is the CEDM of the heavy quark. Here $G_{\mu\nu} = G_{\mu\nu}^a T^a$ with T^a the gauge group generators in the representation of the fermion, $\text{tr}_C(T^a T^b) = \frac{1}{2} \delta^{ab}$; $\sigma \cdot G = \sigma_{\mu\nu} G^{\mu\nu}$ with $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$; and $\not{P} = \gamma_\mu P^\mu$ where $P_\mu = i\partial_\mu + g A_\mu$ with $A_\mu = A_\mu^a T^a$ the gauge connection. Assuming there is no other mass scales between Λ and m , one can evolve the theory from Λ to m via the standard renormalization group (RG) machinery. However, when μ passes below m , we change the effective theory to a new one with the heavy quark Q removed from the particle spectrum[11]. The new effective theory thus obtained involves an infinite tower of nonrenormalizable operators constructed

out of the field strength and its covariant derivative but with coefficients suppressed by inverse powers of the heavy quark mass. At the one loop level and to the first order in C , the relevant effective action is given by

$$\Delta S_{CP} = -gC \text{Tr}[\gamma_5 (P - m)^{-1} \frac{1}{2} \sigma \cdot G] \quad . \quad (5)$$

The symbol Tr denotes the functional trace including the space-time coordinates and other indices. The trace can be evaluated by the covariant derivative expansion method[12-14] The final expression for the off-shell effective action containing operators up to dimension 8 is

$$\begin{aligned} \Delta S_{CP} = & \frac{i}{32\pi^2} gC \text{tr} \int d^4x \gamma_5 \left\{ \frac{gm}{2} (\ln \Lambda_{UV}^2 + \text{finite}) (\sigma \cdot G)^2 \right. \\ & + \frac{1}{4m} \left[-\frac{g}{3} \sigma \cdot G D^2 \sigma \cdot G + \frac{g^2}{2} (\sigma \cdot G)^3 \right] \\ & + \frac{1}{24m^3} \left[g \left(\frac{1}{5} D_\mu D_\nu \sigma \cdot G D^\nu D^\mu \sigma \cdot G \right. \right. \\ & \left. \left. - g^2 G_{\mu\nu} G^{\mu\nu} (\sigma \cdot G)^2 \right) - \frac{3}{4} g^2 (D^2 \sigma \cdot G) (\sigma \cdot G)^2 \right. \\ & \left. \left. + \frac{1}{2} g^3 (\sigma \cdot G)^4 - \frac{i}{15} g^2 \sigma \cdot G D^\mu [D^\nu G_{\nu\mu}, \sigma \cdot G] \right] \right\} \quad . \quad (6) \end{aligned}$$

The symbol tr is the trace over the Dirac indices and the gauge group indices. The covariant derivative D_μ acting upon a matrix, $\mathcal{M} = \mathcal{M}^a T^a$, is defined as $D_\mu \mathcal{M} = \partial_\mu \mathcal{M} - ig[A_\mu, \mathcal{M}]$. The ultra-violet divergent term in Eq. (6) comes as no surprise because the induced operator, O_θ , is of dimension four. However it should be interpreted as the operator mixing between the quark CEDM and the topological term in the QCD renormalization group evolution. In fact, the coefficient of the divergence in Eq. (6) implies the anomalous dimension $\gamma_{Q\theta} = -2m$, which agrees with Morozov's result[5, 3]. Besides the operators involving only the field strength, there are also operators containing the covariant derivatives. For instance, in the case of dimension 6, the Weinberg operator $\text{tr} \gamma_5 (\sigma \cdot G)^3$ is accompanied with the operator $\text{tr} \gamma_5 \sigma \cdot G D^2 \sigma \cdot G$. However, by imposing the equation of motion for the sourceless external gauge field $D_\alpha G^{\alpha\beta} = 0$ and the Bianchi identity $D_\alpha G_{\beta\gamma} + D_\beta G_{\gamma\alpha} + D_\gamma G_{\alpha\beta} = 0$,

one can show, up to total derivatives, that all the latter operators can be expressed solely in terms of the field strength. For instance, the last term in Eq. (6) vanishes. Note that this result is true for the $SU(N)$ case which has four independent, on-shell, dimension 8, CP violating gluonic operators when $N > 3$. For $SU(3)$, one can further express all these operators in terms of the chosen basis O_θ , O_6 , and $O_{8,i}$ [15]. The effective action can be written as,

$$S_{\text{eff}} = S_{QCD} + S_{\text{light quarks}} + \int d^4x \left[C_6 O_6 + \sum_{i=1}^3 C_{8,i} O_{8,i} \right] , \quad (7)$$

$$C_6 = \frac{C}{32\pi^2 m}, \quad C_{8,1} = -\frac{C}{96\pi^2 m^3}, \quad C_{8,2} = 0, \quad C_{8,3} = -\frac{C}{64\pi^2 m^3}. \quad (8)$$

We have not included the topological term in Eq. (7). Its effects as well as its impact on the NEDM have been studied in Ref. [3].

It is actually straightforward to generalize Eq. (6) to arbitrary semi-simple Lie groups even though we have assumed there is a single gauge coupling constant g in the derivation. Such a general formula is very useful, for example when the heavy quark Q carries both the electric dipole moment and chromo-electric dipole moment. In this case, besides generating the gluonic operators, one has also the photonic operators and the mixed photon-gluonic operators. Some of these mixed operators have been considered in the literature[16]. The complete effective action involving not only the photon and the gluon fields strength but also their covariant derivatives will be presented including some technical details elsewhere[15].

The induction of O_6 from the the elimination of the quark CEDM in the effective field theory has been studied before in Refs. [6, 7]. Subsequently[10], we have briefly discussed the dimension 8 operators. Here we will give more details on their phenomenology. The most interesting case where a CEDM may be induced at high energy is clearly through the b quark. We thus need to study the models in which

an appreciable CEDM of b quark is generated at the weak scale or higher. This is not what happens in the standard Kobayashi-Maskawa model of CP violation. However, a significant CEDM for the b quark occurs in simple extensions of the standard model. Scenarios include charged Higgs exchange models, supersymmetric models, and left-right symmetric models, *etc.* The CEDM of the b quark occurs when the heavy particles, such as the top quark, the charged Higgs bosons, the gluinos and the bottom squarks, or the right-handed charged gauge bosons are integrated out at the high energy scale Λ . The effective Lagrangian has the general form:

$$L_{\text{eff}}(\Lambda) = \dots + C_b(\Lambda)O_b(\Lambda) + C_6(\Lambda)O_6(\Lambda) + \sum_{i=1}^3 C_{\mathbf{s},i}(\Lambda)O_{\mathbf{s},i}(\Lambda) \quad , \quad (9)$$

where $O_b(\Lambda) = -\frac{i}{2}g(\Lambda)\bar{b}i\gamma_5\sigma^{\mu\nu}G_{\mu\nu}b$ is the CEDM operator of the b quark defined at the scale Λ . The coefficient $C_b(\Lambda)$ depends on details of the models of CP violation. In the following, we summarize the results that can be found in the literature.

(i) In the minimal charged Higgs exchange model of CP violation[18] with three Higgs doublets ϕ_1, ϕ_2 and ϕ_3 , the first two are responsible for the masses of the t -like quarks and the b -like quarks respectively, while the last doublet is mainly responsible for the electroweak breaking. The mass eigenstates H_1^+ and H_2^+ together with the unphysical charged Goldstone boson H_3^+ are linear combinations of ϕ_1^+, ϕ_2^+ and ϕ_3^+ : $\phi_i^+ = \sum_{j=1}^3 U_{ij}H_j^+ (i = 1, 2, 3)$ where U_{ij} is complex in general. In this model, we obtain[7, 19]

$$C_b^H(\Lambda) = -\frac{m_b(\Lambda)}{16\pi^2} \sum_{i=1,2} \text{Im} \left(\frac{U_{1i}U_{2i}^*}{\langle \phi_1^0 \rangle \langle \phi_2^0 \rangle} \right) \frac{2\sigma_{ii}^H}{(1 - \sigma_{ii}^H)^2} \left(\sigma_{ii}^H - 3 - \frac{2\ln\sigma_{ii}^H}{1 - \sigma_{ii}^H} \right) \quad , \quad (10)$$

with $\sigma_{ii}^H = m_i^2/M_{H_i}^2$.

(ii) In minimal supersymmetric model based on $N = 1$ supergravity, we have[20]

$$C_b^{SUSY}(\Lambda) = \frac{g^2(\Lambda)m_b(\Lambda)}{8\pi^2} \frac{(Am_{3/2} + \tilde{\mu} \tan \alpha_H) \sin \phi}{m_\lambda^3} \sigma_b^2 \left(-2 + (1 + 2\sigma_b) \ln \frac{(1 + \sigma_b)}{\sigma_b} \right), \quad (11)$$

in which $\sigma_b = m_\lambda^2/(m_b^2 - m_\lambda^2)$ with m_λ and m_b the gluino and sbottom masses respectively. A is the Polonyi constant, $m_{3/2}$ is the gravitino mass, $\tilde{\mu}$ and $\tan \alpha_H$ are the mixing parameter and the ratio of the two VEVs of the two Higgs doublets respectively, and finally $\sin \phi$ is the CP violating phase arising from the complex gluino and/or squarks masses.

(iii) In left-right symmetric model, we obtain[7, 21]

$$C_b^{LR}(\Lambda) = \frac{m_t}{16\pi^2} \frac{e^2}{\sin^2 \theta_W} \sum_{i=1,2} \frac{\text{Im}(v_i a_i^*)}{2M_{W_i}^2} \frac{1}{(1 - \sigma_{ii}^W)^2} \left(1 + \frac{1}{4}\sigma_{ii}^W + \frac{1}{4}\sigma_{ii}^{W2} + \frac{3\sigma_{ii}^W \ln \sigma_{ii}^W}{2(1 - \sigma_{ii}^W)} \right). \quad (12)$$

Here $\sigma_{ii}^W = m_i^2/M_{W_i}^2$. v_i and a_i are defined in the charged currents $\bar{t}\gamma_\nu(v_i + a_i\gamma_5)b$ which couples to the mass eigenstates W_i of the charged gauge bosons $W_{L,R}$. They can related to the mixing angle ξ and the CP violation phase η as

$$\begin{aligned} v_1 &= \cos \xi + e^{i\eta} \sin \xi, & a_1 &= -\cos \xi + e^{i\eta} \sin \xi, \\ v_2 &= -\sin \xi e^{-i\eta} + \cos \xi, & a_2 &= e^{-i\eta} \sin \xi + \cos \xi. \end{aligned} \quad (13)$$

where ξ and η are defined by $W_1^+ = \cos \xi W_L^+ + e^{-i\eta} \sin \xi W_R^+$ and $W_2^+ = -e^{i\eta} \sin \xi W_L^+ + \cos \xi W_R^+$. Also, $M_{W_2} \gg M_{W_1} \simeq M_W$. For simplicity, we have used the condition $g_L = g_R = e/(\sqrt{8} \sin \theta_W)$. Note that, unlike in Eqs. (10,11) of previous models, m_b does not occur in Eq. (12).

To study the induction of the gluonic operators, we will assume that $C_6(\Lambda)$ and $C_8(\Lambda)$ are initially zero for the models mentioned above. Since O_b has lower dimension than O_6 and $O_{8,i}(i = 1, 2, 3)$, it will not induce these operators in the RG evolution. However, as one passes through the b quark threshold from above, finite O_6 and $O_{8,i}(i = 1, 2, 3)$ are induced with coefficients $C_6(\mu)$ and $C_{8,i}(\mu)$. These coefficients are

determined by matching physical amplitudes from below and above[7, 6, 8, 9] the b quark threshold with Eq. (8), where the CEDM $C_b \equiv C(m_b^+)$ is related to $C_b(\Lambda)$ by

$$C_b = C_b(\Lambda) \left(\frac{\alpha_s(m_b^+)}{\alpha_s(\Lambda)} \right)^{\frac{\gamma_b}{\beta_3}}; \quad \gamma_b = -2/3. \quad (14)$$

$\beta_n = (33-2n)/3$ is the QCD β -function coefficient with n active flavors of light quarks during the RG evolution. γ_b is the anomalous dimension of O_b . We assume 5 active flavors between the scales Λ and m_b . In order to obtain a more reliable estimation of the matrix element of the NEDM, we need to evolve the induced operators O_6 and O_8 from m_b down to the hadronic scale via the RG machinery. The RG equations are

$$\mu \frac{d}{d\mu} C_8 + \frac{\alpha_s}{2\pi} \gamma_8 C_8 = 0, \quad \gamma_8 = -18, \quad (15)$$

and

$$\mu \frac{d}{d\mu} C_8 + \frac{\alpha_s}{2\pi} C_8 \cdot \gamma = 0, \quad \gamma = \begin{pmatrix} 6 & -12 & 18 \\ 19 & -50 & 21 \\ 7 & -6 & -15 \end{pmatrix}, \quad (16)$$

in the matrix form. The eigenvalues $\gamma_i (i = 1, 2, 3)$ of this anomalous dimension matrix in Eq. (16) are 4.82, -42.98 , and -20.84 . The transformation matrix (S) which diagonalizes the matrix (γ) is obtained numerically

$$S = \begin{pmatrix} -0.912 & 0.190 & 0.275 \\ -0.394 & 1.038 & -0.224 \\ -0.203 & 0.175 & -0.560 \end{pmatrix}, \quad (17)$$

such that $S^{-1} \gamma S = \gamma_D$ and $(\gamma_D)_{ij} = \gamma_i \delta_{ij}$. In the new basis, $O'_8 = S^{-1} \cdot O_8$, $C'_8 = C_8 \cdot S$. One can see that the RG enhanced operator is mainly composed of $O_{8,1}$. Thus, at the hadronic scale, the Wilson's coefficients are

$$C_8(\mu) = \frac{C_b(\Lambda)}{32\pi^2 m_b} \left(\frac{\alpha_s(m_b)}{\alpha_s(\Lambda)} \right)^{\frac{\gamma_b}{\beta_3}} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{\frac{\gamma_8}{\beta_4}} \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right)^{\frac{\gamma_8}{\beta_3}}, \quad (18)$$

$$C'_{3,i}(\mu) = -\frac{2S_{1i} + 3S_{3i} C_b(\Lambda)}{192\pi^2} \frac{C_b(\Lambda)}{m_b^3} \left(\frac{\alpha_s(m_b)}{\alpha_s(\Lambda)} \right)^{\frac{\gamma_b}{\beta_3}} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{\frac{\gamma_i}{\beta_4}} \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right)^{\frac{\gamma_i}{\beta_3}} . \quad (19)$$

We estimate the size of the NEDM by using the naive dimensional analysis[17] accompanied with the unknown correction factors $\xi_6, \xi'_{8,i}$ which are naively of order about one,

$$\begin{aligned} D_N(O_6) &\simeq (eM_\chi/4\pi)g^3(\mu)C_6(\mu)\xi_6 , \\ D_N(O_8) &\simeq (eM_\chi^3/16\pi^2)g^4(\mu)C'_{8,i}(\mu)\xi'_{8,i} . \end{aligned} \quad (20)$$

Here $M_\chi = 4\pi F_\pi \simeq 1.19$ GeV is the chiral symmetry breaking scale. The strong coupling is set at $g(\mu) = 4\pi/\sqrt{6}$ as in Ref.[1]. It is interesting to compare the numerical ratio between $D_N(O_6)$ and $D_N(O_8)$,

$$\frac{D_N(O_8)}{D_N(O_6)} = -\frac{g(\mu) M_\chi^2}{4\pi m_b^2} \frac{2S_{1i} + 3S_{3i} \xi'_{8,i}}{6 \xi_6} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{\frac{\gamma_i - \gamma_6}{\beta_4}} \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right)^{\frac{\gamma_i - \gamma_6}{\beta_3}} . \quad (21)$$

We concentrate our attention to the case $i = 1$ for the reason of the QCD enhancement. Then,

$$D_N(O_8)/D_N(O_6) \simeq 3.6\xi'_{8,1}/\xi_6 . \quad (22)$$

The naive dimensional analysis is certainly not reliable since, when it is applied to operators of arbitrary normalization, we will obtain different predictions. Recently, Chemtob[22] used the QCD sum rule method to provide a more systematic estimate of the hadronic matrix elements of the operators O_6, O_8 , and O_8 . In this scheme if one assumes the nucleon pole dominance, the results are $\xi_6 = 0.07, \xi'_{8,1} = 0.08$, which correspond to smaller D_N compared to the dimensional estimates above. However, their ratio is still about 1, so the O_8 operators remain giving the dominant contribution to the NEDM. Using the current experimental bound[10] 10^{-25} e cm and the matrix elements of Chemtob, one can put a constraint on the CEDM of the b quark, *i.e.*

$$C_b \lesssim 0.6 G_F m_b / 16\pi^2 \quad . \quad (23)$$

If the chromo-electric dipole moment is given to the charm quark initially, the ratio in Eq. (22) will be even an order of magnitude larger because the quark mass suppression factor is less severe. In conclusion, the induced O_8 operators can place strong constraint on parameters of the CP violation.

Finally we note that, in the above models, the contribution to $O_{8,i}$ through the b quark CEDM in our approach corresponds to a RG improved two loop contribution from the viewpoint of the high energy Lagrangian. There are other two loop contributions to the $O_{8,i}$ in these models. However they are expected to be small as discussed in Ref.[10].

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