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Superstring in Two Dimensional Black Hole

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abstract

We construct superstring theory in two dimensional black hole background based on supersymmetric $SU(1,1)/U(1)$ gauged Wess-Zumino-Witten model.

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Recently it was shown that the $SU(1,1)/U(1)$ gauged Wess-Zumino-Witten (GWZW) model describes strings in a two dimensional black hole.^[1] The string propagation and Hawking radiation in this black hole were discussed in Ref.2. When the level k of $SU(1,1)$ current algebra equals to $9/4$, this model can be regarded as a two dimensional gravity coupled with $c = 1$ superconformal matter. We expect that this model could be one of toy models which provide a clue to solve the dynamics of more “realistic” string models.

The supersymmetric extension of this model appeared^[3] as an exact solution of ten-dimensional superstring theory corresponding to black fivebranes.^[4] In this paper, we will consider the supersymmetric extension based on $SU(1,1)/U(1)$ supersymmetric GWZW (SGWZW) model. It has been shown^[5] that supersymmetric $SU(1,1)/U(1)$ coset model has $N = 2$ supersymmetry due to Kazama-Suzuki^[6] mechanism and this model is equivalent to $N = 2$ superconformal models proposed by Dixon, Lykken and Peskin.^[7] The central charge c of this system is given by,

$$c = \frac{3k}{k-2} . \tag{1}$$

Here k is the level of $SU(1,1)$ current algebra. When $k = \frac{5}{2}$, the central charge c equals to 15 and this conformal field theory describes a critical string theory. The $N = 1$ supergravity coupled with $c = \frac{3}{2}\hat{c} = \frac{3}{2}$ superconformal matter would be described by this critical theory. Furthermore the $N = 2$ superconformal symmetry of this model suggests that pure $N = 2$ supergravity would be also described by this model. Therefore the pure $N = 2$ supergravity in two dimensions would be equivalent to the $N = 1$ supergravity coupled with $c = \frac{3}{2}\hat{c} = \frac{3}{2}$ superconformal matter. Due to $N = 2$ superconformal symmetry, superstring theories in the two dimensional black hole background can be constructed by imposing GSO projection. It is also expected that this superstring theory would be equivalent to the matrix models which have space-time supersymmetry^[8,9] and topological superstring theories based on $N = 2$ superconformal topological field theories.^[10]

The action $S^{\text{WZW}}(G)$ of $N = 1$ supersymmetric Wess-Zumino-Witten model is given by,^[11]

$$S^{\text{WZW}}(G) = \frac{k}{2\pi} \int d^2z d^2\theta \text{tr} G^{-1} D G G^{-1} \bar{D} G - \frac{k}{2\pi} \int dt d^2z d^2\theta [\text{tr} G^{-1} D G G^{-1} \bar{D} G G^{-1} \partial_t G + (D \leftrightarrow \bar{D} \text{term})] . \quad (2)$$

Here we define covariant derivatives D and \bar{D} by using holomorphic and anti-holomorphic Grassmann coordinates θ and $\bar{\theta}$

$$D \equiv \frac{\partial}{\partial \theta} - \theta \frac{\partial}{\partial z} , \quad \bar{D} \equiv \frac{\partial}{\partial \bar{\theta}} - \bar{\theta} \frac{\partial}{\partial \bar{z}} . \quad (3)$$

The matrix superfield G , which is an element of a group \mathcal{G} , is given by

$$G = \exp(i \sum_a T^a \Phi^a) . \quad (4)$$

Here Φ^a is a superfield $\Phi^a = \phi^a + \theta \psi^a + \bar{\theta} \bar{\psi}^a + \theta \bar{\theta} f^a$ and T^a is a generator of the algebra corresponding to \mathcal{G} . The action (2) satisfies Polyakov-Wiegmann type formula:

$$S^{\text{WZW}}(GH) = S^{\text{WZW}}(G) + S^{\text{WZW}}(H) + \frac{k}{\pi} \int d^2z d^2\theta \text{tr} G^{-1} D G \bar{D} H H^{-1} . \quad (5)$$

Here H is also an element of \mathcal{G} . This formula guarantees that the system described by the action (2) has super Kac-Moody symmetry and $N = 1$ superconformal symmetry.

If \mathcal{F} is a $U(1)$ subgroup of \mathcal{G} , we can gauge the global symmetry under the following axial $U(1)$ transformation, which is given by an element F of \mathcal{F} ,

$$G \rightarrow F G F . \quad (6)$$

The action of supersymmetric \mathcal{G}/\mathcal{F} gauged Wess-Zumino-Witten is given by,

$$\begin{aligned} S^{\mathcal{G}/\mathcal{F}}(G, A) &= S^{\text{WZW}}(F_L G F_R) - S^{\text{WZW}}(F_L^{-1} F^R) \\ &= S^{\text{WZW}}(G) \\ &\quad + \frac{k}{2\pi} \int d^2 z d^2 \theta \text{tr}(A \bar{A} + A G \bar{A} G^{-1} + G^{-1} D G \bar{A} + A \bar{D} G G^{-1}). \end{aligned} \quad (7)$$

Here F_L and F_R are elements of \mathcal{F} and gauge fields A and \bar{A} are defined by,

$$A = F_L^{-1} D F_L, \quad \bar{A} = F_R^{-1} \bar{D} F_R. \quad (8)$$

The action (7) is invariant under the following $U(1)$ gauge transformation

$$\begin{aligned} G &\rightarrow F G F, \quad A \rightarrow A + F^{-1} D F, \quad \bar{A} \rightarrow \bar{A} + F^{-1} \bar{D} F. \\ &(F_{L,R} \rightarrow F_{L,R} F) \end{aligned} \quad (9)$$

A supersymmetric extension of string theory in a two dimensional black hole background is given by setting $\mathcal{G} = SU(1,1)$ in the action (7). We start with considering $SU(1,1)$ SWZW model. By parametrizing G by,

$$G = \exp\left(\frac{i}{2} \Phi_L \sigma_2\right) \exp\left(\frac{1}{2} R \sigma_1\right) \exp\left(\frac{i}{2} \Phi_R \sigma_2\right), \quad (10)$$

with σ_i the Pauli matrices, we obtain the action $S^{SU(1,1)}$ of $SU(1,1)$ SWZW model

$$\begin{aligned} S^{SU(1,1)} &= \frac{k}{2\pi} \int d^2 z d^2 \theta \left[-\frac{1}{2} D \Phi_L \bar{D} \Phi_L - \frac{1}{2} D \Phi_R \bar{D} \Phi_R \right. \\ &\quad \left. - \cosh R D \Phi_L \bar{D} \Phi_R + \frac{1}{2} D R \bar{D} R \right]. \end{aligned} \quad (11)$$

The holomorphic (anti-holomorphic) conserved currents J_i (\bar{J}_i) of this system are given by,

$$2kG^{-1}DG = J_1\sigma_1 + iJ_2\sigma_2 + J_3\sigma_3, \quad 2kG^{-1}\bar{D}G = \bar{J}_1\sigma_1 + i\bar{J}_2\sigma_2 + \bar{J}_3\sigma_3, \quad (12)$$

$$J_i = j_i + \theta \bar{J}_i + \dots, \quad \bar{J}_i = \bar{j}_i + \bar{\theta} \bar{J}_i + \dots. \quad (13)$$

Here \dots express the terms which vanish by using the equations of motion. If we

define new currents \hat{J}_i and $\bar{\hat{J}}_i$ by the following equation

$$\hat{J}_i = \tilde{J}_i - \frac{1}{2k} \epsilon_{ilm} j_l j_m, \quad \bar{\hat{J}}_i = \bar{\tilde{J}}_i - \frac{1}{2k} \epsilon_{ilm} \bar{j}_l \bar{j}_m, \quad (14)$$

These currents \hat{J}_i and $\bar{\hat{J}}_i$ do not depend on the fermion currents j_i and \bar{j}_i .

By expanding superfields $\Phi_{L,R}$ and R into components,

$$\begin{aligned} \Phi_{L,R} &= \phi_{L,R} + \theta \psi_{L,R} + \bar{\theta} \bar{\psi}_{L,R} + \theta \bar{\theta} f_{L,R}, \\ \frac{R}{2} &= s + \theta \eta + \bar{\theta} \bar{\eta} + \theta \bar{\theta} g, \end{aligned} \quad (15)$$

we can rewrite the $SU(1,1)$ SWZW action $S^{SU(1,1)}$ in Eq.(11) by a sum of non-supersymmetric $SU(1,1)$ WZW action $\tilde{S}^{SU(1,1)}$ and free fermion actions:

$$S^{SU(1,1)} = \tilde{S}^{SU(1,1)} + \frac{1}{4k\pi} \int d^2 z [j_+ \bar{\partial} j_- - j_- \bar{\partial} j_+ + \bar{j}_+ \partial \bar{j}_- - \bar{j}_- \partial \bar{j}_+], \quad (16)$$

$$\begin{aligned} \tilde{S}^{SU(1,1)} &= \frac{k}{2\pi} \int d^2 z [-\frac{1}{2} (\partial \phi_L \bar{\partial} \phi_L + \partial \phi_R \bar{\partial} \phi_R) \\ &\quad - \cosh(2s) \partial \phi_L \bar{\partial} \phi_R + 2\partial s \bar{\partial} s]. \end{aligned} \quad (17)$$

Here j_{\pm} and \bar{j}_{\pm} are defined by

$$j_{\pm} \equiv j_1 \pm i j_3, \quad \bar{j}_{\pm} \equiv \bar{j}_1 \pm i \bar{j}_3. \quad (18)$$

The conserved currents corresponding to the non-supersymmetric $SU(1,1)$ WZW action $\tilde{S}^{SU(1,1)}$ (17) are given by \hat{J}_i and $\bar{\hat{J}}_i$ in Eq.(14).

Fermionic currents j_{\pm} and \bar{j}_{\pm} can be written as

$$\begin{aligned} j_{\pm} &= \frac{k}{2} \exp(\mp i \phi_R) (\eta \pm \frac{i}{2} \sinh(2s) \psi_L), \\ \bar{j}_{\pm} &= \frac{k}{2} \exp(\mp i \phi_L) (\bar{\eta} \pm \frac{i}{2} \sinh(2s) \bar{\psi}_R). \end{aligned} \quad (19)$$

Note that there appear bosonic factors $\exp(\mp i \phi_R)$ and $\exp(\mp i \phi_L)$. Due to these factors, the boundary conditions of j_{\pm} and \bar{j}_{\pm} are twisted although fermions η ,

$\bar{\eta}$, ψ_L and $\bar{\psi}_R$, which is identified later with space-time fermionic coordinates, are periodic or anti-periodic. Therefore the eigenvalues of the zero modes of fermion number currents K and \bar{K} ,

$$K = \frac{1}{4k}(j_+j_- - j_-j_+) , \quad \bar{K} = \frac{1}{4k}(\bar{j}_+\bar{j}_- - \bar{j}_-\bar{j}_+) , \quad (20)$$

which satisfy the following operator product expansions

$$K(z)j_{\pm}(w) \sim \pm \frac{1}{z-w}j_{\pm} , \quad \bar{K}(\bar{z})\bar{j}_{\pm}(\bar{w}) \sim \pm \frac{1}{\bar{z}-\bar{w}}\bar{j}_{\pm} , \quad (21)$$

are not quantized.

We now gauge the $U(1)$ symmetry in the action (11) by following Eq. (7). We consider the case that the $U(1)$ symmetry is generated by σ_2 . Since the $U(1)$ symmetry is compact, the resulting conformal field theory describes the Euclidean black hole. The theory of the Lorentzian black hole can be obtained by replacing σ_2 by σ_3 or simply by analytic continuing $\Phi_{L,R} \rightarrow i\Phi_{L,R}$.

By the parametrization (10) the $SU(1,1)/U(1)$ gauged SWZW action takes the form

$$S^{SU(1,1)/U(1)} = S^{SU(1,1)} + \frac{k}{2\pi} \int d^2z d^2\theta [4(1 + \cosh R)A\bar{A} + 2iA(\bar{D}\Phi_L + \cosh R \bar{D}\Phi_R) + 2i(D\Phi_L + \cosh R D\Phi_R)\bar{A}] . \quad (22)$$

Here $S^{SU(1,1)}$ is $SU(1,1)$ SWZW action in Eq.(11). By using the following redefinitions,

$$\begin{aligned} \Phi &\equiv \Phi_L - \Phi_R , \\ A' &\equiv A + \frac{i \cosh R D\Phi_L + D\Phi_R}{1 + \cosh R} , \\ \bar{A}' &\equiv \bar{A} + \frac{i \cosh R \bar{D}\Phi_R + \bar{D}\Phi_L}{1 + \cosh R} , \end{aligned} \quad (23)$$

the action (22) can be rewritten as follows,

$$S^{SU(1,1)/U(1)} = \frac{k}{2\pi} \int d^2z d^2\theta \left[\frac{1}{2} \tanh^2 \frac{R}{2} D\Phi \bar{D}\Phi + \frac{1}{2} DR \bar{D}R + 4(1 + \cosh R) A' \bar{A}' \right]. \quad (24)$$

The action (22) and (24) are invariant under the following infinitesimal gauge transformation corresponding to Eq.(9),

$$\delta\Phi_L = \delta\Phi_R = \Lambda, \quad \delta A = -\frac{i}{2} D\Lambda, \quad \delta\bar{A} = -\frac{i}{2} \bar{D}\Lambda. \quad (25)$$

We fix this gauge symmetry by imposing the gauge condition

$$\Phi_L = -\Phi_R = \bar{\Phi}. \quad (26)$$

By integrating gauge fields A and \bar{A} in the action (22) or (24),^{*} and by integrating auxiliary fields f and g in Eq.(26), we obtain the following action,

$$S^{(1)} = \frac{k}{\pi} \int d^2z \left[\tanh^2 s (\partial\phi\bar{\partial}\phi - \partial\bar{\psi}\bar{\psi} + \psi\bar{\partial}\psi) - 2 \frac{\sinh s}{\cosh^3 s} (\eta\psi\bar{\partial}\phi + \bar{\eta}\bar{\psi}\partial\phi) + 4 \tanh^2 \eta\bar{\eta}\psi\bar{\psi} + \partial s \bar{\partial} s - \partial\bar{\eta}\bar{\eta} + \eta\bar{\partial}\eta \right]. \quad (27)$$

Here we write superfields $\bar{\Phi}$ and R in terms of components:

$$\begin{aligned} \bar{\Phi} &= \phi + \theta\psi + \bar{\theta}\bar{\psi} + \theta\bar{\theta}f, \\ \frac{R}{2} &= s + \theta\eta + \bar{\theta}\bar{\eta} + \theta\bar{\theta}g \end{aligned} \quad (28)$$

This system has $N = 1$ supersymmetry since the starting action (22) and gauge condition (25) are manifestly supersymmetric. In fact, this action is nothing but the action of (1,1) supersymmetric σ model^[13] in two dimensional black hole background.

* The integration of the gauge fields induces the dilaton term in the action but we now neglect this term. The gauge fixed action which is correct at the quantum level is given later in this paper.

The $N = 1$ supersymmetry in the action (27) is extended to $N = 2$ supersymmetry since this action is invariant under the following holomorphic (anti-holomorphic) $U(1)$ symmetry:

$$\begin{aligned}\delta\psi &= -\frac{u(z)}{\tanh s}\eta \\ \delta\eta &= u(z)\tanh s\psi ,\end{aligned}\tag{29}$$

$$\begin{aligned}\delta\bar{\psi} &= -\frac{\bar{u}(\bar{z})}{\tanh s}\bar{\eta} \\ \delta\bar{\eta} &= \bar{u}(\bar{z})\tanh s\bar{\psi} .\end{aligned}\tag{30}$$

Here $u(z)$ ($\bar{u}(\bar{z})$) is a holomorphic (anti-holomorphic) parameters of the transformation. The transformations (29) and (30) tell that the currents of this $U(1)$ symmetry can be regarded as fermion number currents with respect to space-time fermion coordinates, η , ψ , $\bar{\eta}$ and $\bar{\psi}$. By commuting this $U(1)$ symmetry transformation with the original $N = 1$ supersymmetry transformation, we obtain another supersymmetry transformation and we find that the action has $N = 2$ supersymmetry. On the other hand, in case of the Lorentzian black hole, the obtained algebra is not exactly $N = 2$ superconformal algebra.[†] Usual $N = 2$ superconformal algebra is given by

$$\begin{aligned}\{G_n^+, G_m^-\} &= 4L_{n+m} + 2(m-n)J_{n+m} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} , \\ [J_n, G_m^\pm] &= \pm G_{n+m} , \quad \text{etc.}\end{aligned}\tag{31}$$

and the hermiticities of the operators are assigned by

$$(G_n^+)^\dagger = G_{-n}^- , \quad J_n^\dagger = J_{-n} .\tag{32}$$

The algebra which appears in the Lorentzian case is identical with Eq.(31), but

† Note that any Lorentzian manifold is not Kähler.

the assignment of the hermiticities is different from Eq.(32):

$$(G_n^+)^\dagger = G_{-n}^+ , \quad (G_n^-)^\dagger = G_{-n}^- , \quad J_n^\dagger = -J_{-n} . \quad (33)$$

This is not so surprizing since this algebra also appears in flat two dimensional Lorentzian space-time which is a subspace of flat ten dimensional space-time in usual Neveu-Schwarz-Ramond model. Even in Lorentzian case, we have a $U(1)$ current and superstring theories can be constructed by imposing GSO projection.

In order to consider the spectrum of this theory, we choose the following gauge condition instead of Eq. (25) ,

$$\bar{D}A - D\bar{A} = 0 . \quad (34)$$

This gauge conditon allows us to parametrize the gauge fields A and \bar{A} as

$$A = D\Pi , \quad \bar{A} = -\bar{D}\Pi . \quad (35)$$

By shifting the fields $\Phi_{L,R}$,

$$\Phi_L \rightarrow \Phi_L + 2i\Pi , \quad \Phi_R \rightarrow \Phi_R - 2i\Pi , \quad (36)$$

the gauge fixed action $S^{(2)}$ is given by a sum of $SU(1,1)$ SWZW action $S^{SU(1,1)}$ in Eq. (11), free field action S^Π and (free) ghost action S^{FP} .

$$\begin{aligned} S^{(2)} &= S^{SU(1,1)} + S^\Pi + S^{\text{FP}} , \\ S^\Pi &= -\frac{4k}{\pi} \int d^2z d^2\theta D\Pi\bar{D}\Pi , \\ S^{\text{FP}} &= \frac{k}{2\pi} \int d^2z d^2\theta B D\bar{D}C . \end{aligned} \quad (37)$$

Here B and C are anti-ghost and ghost superfields.

The BRS charge Q_B which defines the physical states is given by

$$Q_B = \oint dz C(D\Pi - \frac{i}{4k}J_2) + \oint d\bar{z}\bar{C}(\bar{D}\Pi + \frac{i}{4k}\bar{J}_2) . \quad (38)$$

This BRS charge gives constraints on the physical states,

$$D\Pi - \frac{i}{2}J_2 = \bar{D}\Pi + \frac{i}{2}\bar{J}_2 = 0 , \quad (39)$$

which tell that B, C, Π and J_2 (or \bar{J}_2) make so-called ‘‘quartet’’ structure^[13] similar to the structure which appeared in the quantization of Neveu-Schwarz-Ramond model based on BRS symmetry.^[14,15]

The action which describes superstring theory in the two dimensional black hole is simply given by a sum of $SU(1,1)$ WZW action (17), free fermion actions (16) and the actions of free superfield and free ghost and anti-ghost superfields (37). Furthermore the constraints (39) imposed by the BRS charge (38) can be easily solved with respect to free superfields Π . Therefore if we can find the spectrum of the bosonic string in the two dimensional black hole,^[1,2] we can also find the spectrum of this string theory.

The $U(1)$ current, which corresponds to the transformations (29) and (30) are given by,^[8]

$$J = \frac{-2i}{k-2}\hat{J}_2 + \frac{k}{k-2}K , \quad \bar{J} = \frac{-2i}{k-2}\bar{\hat{J}}_2 + \frac{k}{k-2}\bar{K} . \quad (40)$$

Here \hat{J}_2 and $\bar{\hat{J}}_2$ are defined by Eq.(14) and fermion number currents K and \bar{K} are defined by Eq.(20). These $U(1)$ currents commute with the BRS charge (38) and we can impose GSO projection consistently. Note that GSO projection does not give any constraint on the representations of $SU(1,1)$ current algebra since the eigenvalues of the zero modes in the currents K and \bar{K} are not quantized although those in J and \bar{J} are quantized.

Recently the string model based on $\frac{SU(1,1) \times U(1)}{U(1)}$ coset model was discussed.^[16,17] This model describes the strings in two^[16] or three dimensional^[17] charged black

holes. By adjusting the radius of the $U(1)$ boson, we will obtain $N = 2$ superconformal theory with $c > 3$ ^[7] in the same way as $N = 2$ minimal model was constructed from $\frac{SU(2) \times U(1)}{U(1)}$ ^[18]. The obtained model should be equivalent to the model discussed here.

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