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Topological Vacua, Spectra of Zero Modes and Dynamical Supersymmetry Breaking

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Abstract

Some aspects of supersymmetric gauge theories are discussed. A formula for the quantum spectra of the classical zero modes in supersymmetric gauge theories is derived. The topological vacua in higher dimensions are also discussed. The spectra and the discussion of the topological vacua are applied to the study of dynamical supersymmetry breaking. We argue that our previous argument that dynamical supersymmetry breaking does not occur in supersymmetric gauge theories free of both local (perturbative) and global (non-perturbative) gauge anomalies in higher dimensions is irrelevant to any possible vacuum-angle specification in the theories.

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The discoveries of Yang-Mills theory[1] and supersymmetry[2] led to the development of supersymmetric gauge theories[3] which have been one of the most interesting ideas in elementary particle physics. It is also known that the open superstring theory[4] can be approximated at low energy by a supersymmetric gauge theory. Supersymmetric gauge theories are of fundamental interest for unification theory[5] in particle physics beyond the standard model.

In the construction of realistic models based on supersymmetry, supersymmetry must be spontaneously broken since degenerate Bose-Fermi multiplets are not observed. Due to this fact, the study of the possibilities of spontaneous supersymmetry breaking is crucial in supersymmetric gauge theories.

It was noted by Witten[6-7] that generally there may be dynamical supersymmetry breaking by non-perturbative effects due to the non-zero energy of ground state created in the quantum dynamics. With extensive discussions, Witten has shown that dynamical supersymmetry breaking does not occur in certain interesting classes of theories. Witten's method has also been[8] applied to the cases of higher dimensions. Especially, we have shown that dynamical supersymmetry breaking does not occur in supersymmetric QED in higher dimensions, and argued that more generally there is no dynamical supersymmetry breaking in higher dimensions in any supersymmetric gauge theories free of gauge anomalies by

Witten's method of calculating the index[9] $Tr(-1)^F$.

In this paper, further discussions and results will be given intimately connecting to the study of dynamical supersymmetry breaking in gauge theories. By choosing a physical vacuum, We will construct the spectra of zero-momentum modes in the minimal supersymmetric gauge theories with an arbitrary compact connected simple gauge group G in a finite box with periodic boundary conditions. The spectra with the discussion of topological vacua which we will provide, are then applied to the study for the possibilities of dynamical supersymmetric breaking. As a complimentary to the previous discussions, we will argue that our previous argument generally for the anomaly-free theories in higher dimensions is independent of the any possible vacuum angle chosen in the theory. This is different from the case in four dimensions, where the theory has no dependence on the vacuum angle is assumed[7]. Note that the study of gauge theory in a finite box with an appropriate boundary condition is of interest generally for the understanding of other non-perturbative effects also, for example, see ref.9.

An essential feature in global supersymmetric theories is that the hamiltonian H is the sum of the squares of the supersymmetry charges which are hermitian. This implies that supersymmetry is spontaneously broken if and only if the ground state has energy greater than zero. For the analysis of zero energy states, Witten introduced an operator $(-1)^F$

for which the trace can be written as[7]

$$\mathrm{Tr}(-1)^F = n_B^{E=0} - n_F^{E=0} \quad , \tag{1}$$

where $n_B^{E=0}$ and $n_F^{E=0}$ are the numbers of zero-energy bosonic and fermionic states respectively. If $\mathrm{Tr}(-1)^F \neq 0$, then there exists at least one zero-energy ground state, the supersymmetry is not spontaneously broken. As shown by Witten, the trace in (1) can be regarded as the index[9] of an operator M, with supersymmetry charge Q written in the form of

$$Q = \begin{pmatrix} 0 & M^+ \\ M & 0 \end{pmatrix} \quad , \tag{2}$$

by splitting the Hilbert space of the theory into bosonic and fermionic subspaces. More generally, one can calculate the following trace

$$\mathrm{Tr}(-1)^F f(X) = \sum_{\lambda} f(\lambda) \mathrm{Tr}(-1)^F P_{\lambda} \quad , \tag{3}$$

where X is any operator commuting with the supersymmetry charges $[X, Q_\alpha]=0$, and thus commuting with the hamiltonian also. The P_λ denotes the projection from the Hilbert space onto its subspace with $X=\lambda$ for a definite eigenvalue. If $\mathrm{Tr}(-1)^F f(X)$ is non-zero for some choice of f(X), then supersymmetry is not spontaneously broken, thus there is no dynamical supersymmetry breaking. The usefulness of the formula(1) or (3) is due to the fact that the index of an operator is independent of the parameters of the theory if the different set of parameters can be reached to each other by continuous deformations, for example by conjugate transformations[7]. This implies that for those theories, the trace $Tr(-1)^F$ of $Tr(-1)^F f(X)$ can be calculated in a convenient limit, such as small volume, large bare mass, and weak coupling. Essentially, for this purpose one only needs to restrict to the minimal theory for the supersymmetric gauge theories[7], although additional fields may be present in general.

From the above discussion, one can easily see that the study of quantum spectra of classical zero-energy modes in a finite box is crucial to the determination of the traces $Tr(-1)^F f(X)$. Note that[11] there can be supersymmetric gauge theories only in the specific dimensions D=3, 4, 6, 10, and we will focus on the cases of D=4, 6, 10 dimensions. But we expect that our discussions are useful to the study of Yang-Mills system in general dimensions.

Consider the minimal supersymmetric gauge theory with the lagrangian density given by

$$\mathcal{L}=-\tfrac{1}{4}F^a{}_{\alpha\beta}F^{a\alpha\beta}+\tfrac{1}{2}\,\bar{\psi}^a\Gamma^\mu D_\mu\,\psi^a \quad , \tag{4}$$

where

$$F^a{}_{\alpha\beta}=\partial_\alpha A^a{}_\beta-\partial_\beta A^a{}_\alpha+gC_{abc}A_{b\alpha}A^c{}_\beta \quad , \tag{5}$$

$$D_\mu\,\psi^a=\partial_\mu\,\psi^a+gC_{abc}A^b{}_\mu\,\psi^c, \tag{6}$$

with gauge coupling constant g. The C_{abc} are the structure constants given by $[L_a,L_b]=iC_{abc}L_c$ with a basis $\{L_a=|a=1,2,\ldots \dim(L(G))\}$ for the Lie algebra L(G) of the gauge group G. We will assume here that G is a compact connected simple Lie group. In the case of an abelian G, see refs.7-8. Since A_μ and transform into each other under supersymmetry transformations, it is necessary that the spinor fields $\{\psi^a\}$ also form the adjoint representation of G under the gauge transformations. Furthermore, the spinors are Majorana in D=3, Majorana or Weyl in D=4, Weyl in D=6, Majorana and Weyl in D=10 respectively. The supersymmetry transformations that leave (4) invariant are

$$\delta A^a{}_\mu=\tfrac{1}{2}i[\bar{\varepsilon}\Gamma_\mu\psi^a-\bar{\psi}^a\Gamma_\mu\varepsilon] \ , \tag{7}$$

$$\delta\,\psi^a=\tfrac{1}{2}\Sigma_{\alpha\beta}F^{a\alpha\beta}\varepsilon \ , \tag{8}$$

$$\delta\,\bar{\psi}^a=-\tfrac{1}{2}\bar{\varepsilon}\Sigma_{\alpha\beta}F^{a\alpha\beta}, \tag{9}$$

with

$$\Sigma_{\alpha\beta}=\tfrac{1}{2}[\Gamma_\alpha,\Gamma_\beta] \ . \tag{10}$$

In the case of Majorana spinors in D=3, 4, or 10, (7) can be written in the simpler form

$$\delta A^a{}_\mu=i\bar{\varepsilon}\Gamma_\mu\,\psi^a \ . \tag{11}$$

In the case of D=6, it is necessary to keep both terms. The conserved supersymmetry current in the theory is

$$S_\mu=\Sigma_{\alpha\beta}F^{a\alpha\beta}\Gamma_\mu\,\psi^a\ . \tag{12}$$

Now consider the above supersymmetric gauge theory in a finite box of dimensions D (D=4, 6, 10) with periodic boundary conditions. We will choose the gauge $A_0=0$. Then, classically the gauge field modes corresponding to zero energy are those zero-momentum modes A_i which commute with each other[7]. Obviously, for these modes

$$F_{ij}=\partial_i A_j-\partial_j A_i-ig[A_i,A_j]=0\ . \tag{13}$$

Note here that a zero-momentum mode A_i is a constant in the Lie algebra L(G). For the minimal supersymmetric gauge theories in our discussions, the zero-momentum modes A_i are then constant matrices in the adjoint representation of the L(G).

The gauge field components A_i commuting with each other are contained in a Cartan subalgebra. Since the different Cartan subalgebra of a gauge group can be transformed into each other by a topologically trivial gauge transformation, in terms of a basis $\{H^s|s=1,2,\ldots r=\mathrm{rank}(G)\}$ of a Cartan subalgebra of L(G), we can write the zero-momentum modes of the gauge field as

$$A_i=\sum_{s=1}^{r}C_{is}H^s\qquad(i=1,2,\ldots D-1), \tag{14}$$

where C_{is} are constants. The Dirac field can be written as

$$\psi = \sum_a \psi^a L_a \; . \tag{15}$$

The zero modes of in the background field (14) are of the form

$$\psi = \sum_s^r \varepsilon^s H^s \; , \tag{16}$$

where ε^s are constant spinors.

Now what is crucial in our discussion is the spectrum for the zero-momentum modes after quantization. One may expect that the zero modes may remain as low-lying states even if they are lifted above zero energy after quantization. As we will see explicitly that this is true only in the weak coupling limit.

In order to quantize the zero modes (14) and (16), we need to determine the periodicity properties of the variables in the zero modes. In the following, we will show that the constants $\{C_{is}\}$ for the zero modes of the gauge field are variables modulo a class of vectors in the root space of the Lie algebra L(G).

In the special case of G=SU(2), as noted by Witten[7] the variable in the zero modes of the gauge field is a periodic variable with period $2\pi/gL$, where L is the length of the box. This can be easily seen explicitly, and can be used as a preliminary discussion of the general case. The zero-momentum modes of A_i can be written as

$$A_i = C_i L_3 \quad (i=1,2,\ldots D-1). \tag{17}$$

Obviously, the well defined and periodic gauge functions

$$U_i=\exp\{i2\pi/L\cdot x_iL_3\}$$
$$=\begin{pmatrix} \cos(2\pi/L\cdot x_i) & -\sin(2\pi/L\cdot x_i) & 0 \\ \sin(2\pi/L\cdot x_i) & \cos(2\pi/L\cdot x_i) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{18}$$

generate gauge transformations which shift C_1, C_2,...and C_{D-1} by $2\pi/gL$ for i=1, 2,...D-1 respectively. Where L_1, L_2, and L_3 in the adjoint representation of SU(2) can be written as

$$L_1=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad L_2=\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad L_3=\begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{19}$$

By the constraint of Gauss' law, the physical states are invariant under U_i (i=1, 2,...D-1), since the U_i are all topologically equivalent to the identity gauge function. In the Hilbert space with the constraint of Gauss's law, the C_i are periodic variables with period $2\pi/qL$.

Now consider generally for a compact connected simple gauge group G in the adjoint representation for the minimal supersymmetric gauge theories. Obviously, the gauge functions

$$U_i=\exp\{i4\pi x_i/L\cdot\sum_s^r \beta_s H^s\} \quad (i=1,\ 2,\ldots D-1), \tag{20}$$

generate gauge transformations which shift respectively C_{1s}, C_{2s},...and C_{D-1s} by $4\pi\beta_s/gL$ with constant β_s (s=1, 2,...,r).

However, in order to preserve the periodicity, only those U_i satisfying the condition

$$U_i\Big|_{x_i=L}=\exp\{i4\pi\sum_s ß_s H^s\}=1 \tag{21}$$

are well defined. The general solutions to eq.(21) are given by[12]

$$ß_{is}=\sum_{j=1}^{r} m^{(i)}{}_j(\alpha^j)_s/\langle\alpha^j,\alpha^j\rangle \quad s=1,\ 2,\ldots,r,\ \text{and}\ m^{(i)}{}_j\varepsilon Z. \tag{22}$$

Where α^j $(j=1,\ 2,\ldots,r)$ denote the simple roots of the Lie algebra L(G), and $\langle\alpha,\alpha'\rangle=\sum_{s=1}^{r}\alpha_s\alpha'^s$ denotes the inner product. As a matter of fact, up to a normalization, the expressions

$$\sum_{j=1}^{r} m^{(i)}{}_j\ \alpha^j/\langle\alpha^j,\alpha^j\rangle\ , \qquad m^{(i)}{}_j\varepsilon Z, \tag{23}$$

are weights of a dual group G^μ, (ref.12). G^μ is dual to G in the sense that $(G^\mu)^\mu=G$. Eq.(22) is of interest in the study of magnetic monoples, charge spectra of dyons associated with generalized magnetic monoples and the possibilities of quarks as dyons in a spontaneously broken gauge theory[12,13].

Now with $ß_s$ given by eq.(22), the gauge functions U_i in eq.(20) generate gauge transformations which shift C_{is} by

$$4\pi/gL\cdot\sum_{j=1}^{r} m^{(i)}{}_j(\alpha^j)_s/\langle\alpha^j,\alpha^j\rangle\ ,(i=1,\ 2,\ldots D-1). \tag{24}$$

Essentially, for the gauge functions

$$U_i=\exp\{i4\pi x_i/L\cdot\sum_{j,s=1}^{r} m^{(i)}{}_j(\alpha^j)_s H^s/\langle\alpha^j,\alpha^j\rangle \quad , \tag{25}$$

only the following ones are fundamental for any given i:

$$U_i(\alpha^j)=\exp\{i4\pi x_i/L\cdot\sum_s(\alpha^j)_s H^s/\langle\alpha^j,\alpha^j\rangle \quad (j=1,\ 2,\ldots r), \tag{26}$$

since the gauge transformations corresponding to the gauge functions (25) can be generated by those corresponding to (26) for the H^s in the Cartan subalgebra of L(G).

Since the U_i in (26) are all topologically equivalent to the identity gauge function, then the constraint of Gauss law requires that the physical states be invariant under the gauge transformations generated by U_i in (25) or (26). Therefore, by regarding each $\{C_{is}|s=1,\ 2,\ldots r\}$ as a vector in the r-dimensional root space of L(G), up to an overall normalization, the $\{C_{is}\}$ are only defined modulo the weights of $L(G^\mu)$ in the Hilbert space with the constraint of Gauss law.

We now need to quantize the degrees of freedom for the zero-momentum modes (14) and (16). For this purpose, one needs to choose a physical vacuum and expand around (14). The physical vacuum is expected to be a linear combination of the topological vacua with $F_{ij}=0$. In D=4 dimensions, the topological vacua are characterized by the homotopy group $\pi_3(G)=Z$. As is well known, the physical vacuum can be chosen as a Θ vacuum in this case. However, in higher dimensions, there can be either finite or infinite number of different topological vacua depending on the dimensions and the gauge group G. As an example relevant to the supersymmetric gauge theories in higher dimensions, consider the case of G=SU(2). The relevant homotopy groups[14] are given by

$$\pi_5(SU(2))=Z_2 \ , \tag{27}$$

$$\pi_9(SU(2))=Z_3 \ , \tag{28}$$

and there are only a finite number of different topological vacua in this case. Therefore, we expect that there can be only a finite number instanton solutions. In general, instanton tunnelling will physically connect the topological vacua. The gauge invariant physical vacuum can be constructed as a linear combination of the topological vacua. One can easily see that for Z_N topological vacua connected physically, up to an overall phase factor, the physical vacuum can be written as

$$|vac\rangle=\sum_{k=0}^{N-1}\exp\{-i2\pi k/N\}|k\rangle \ . \tag{29}$$

Under a topologically non-trivial gauge transformation with $|k\rangle\to|k+1\rangle$, we have

$$|vac\rangle\to\exp\{i2\pi/N\}|vac\rangle \ , \tag{30}$$

the $|vac\rangle$ is invariant up to a phase factor. It is straightforward to generalize the formula (29) to the case in which topological vacua correspond to a general finite-cyclic homotopy group. An interesting general feature in the case of finite number of physically connected topological vacua is that the gauge invariant physical vacuum is unique up to an overall phase factor. This is different from the case of an infinite many possibilities for the physical vacuum characterized by a vacuum angle.

From the above results, and the construction of the usual Θ vacuum, one can easily construct the gauge invariant physical vacuum when the topological vacua physically connected correspond generally to the direct sum of infinite cyclic and finite cyclic groups. For example, for the supersymmetric SO(10) gauge theories in D=10 dimensions, the relevant homotopy group is given by

$$\pi_9(SU(2))=Z + Z_2 \ . \tag{31}$$

Denote the topological vacua by $|n,\sigma\rangle$ with σ defined modulo 2. When the $|n,\sigma\rangle$ are physically connected, then up to an overall phase factor, the gauge invariant physical vacuum can be written as

$$|\Theta\rangle=\sum_{k=-\infty}^{\infty}\sum_{\sigma=0}^{1}\exp\{-in\Theta+i\pi\sigma\}|n,\sigma\rangle \ . \tag{32}$$

In any case, for our discussions of dynamical supersymmetry breaking, we only need to ensure the existence of at lease one zero-energy vacuum state, otherwise the supersymmetry is already spontaneously broken.

We now assume a zero-energy physical vacuum is chosen. It is then straightforward to quantize the degrees of freedom for zero-momentum modes (14) and (16) by expanding around (14). The relevant lagrangian for the zero-momentum modes can be written as

$$L=\int d^{D-1}x\ \tfrac{1}{2}\sum_{s=1}^{r}\Big(\sum_{i=1}^{D-1}(\dot{C}_{is})^2+\bar{\varepsilon}^s i\Gamma^0\dot{\varepsilon}^s\Big)$$

$$=\tfrac{1}{2}V\sum_{s=1}^{r}\Big(\sum_{i=1}^{D-1}(\dot{C}_{is})^2+\bar{\varepsilon}^s i\Gamma^0\dot{\varepsilon}^s\Big) \quad (D=4,\ 6,\ 10), \tag{33}$$

where $V=L^{D-1}$ is the volume of the box. After quantization, the corresponding hamiltonian is then of the form

$$H=\tfrac{1}{2}V^{-1}\sum_{s=1}^{r}\sum_{i=1}^{D-1}(\pi_{is})^2 \quad (D=4,\ 6,\ 10), \tag{34}$$

where $\pi_{is}=-i\partial/\partial C_{is}$ is the canonical momentum conjugate to C_{is}. The eigenfunctions of H in (34) with C_{is} as variables defined modulo the expressions in (24) can be easily constructed, by regarding each $C^{(i)}=\{C_{is}|s=1,\ 2,\ldots,r\}$ as a vector in the root space of L(G). The eigenfunctions are normalizable which, up to a normalization constant are given by

$$|\{m_{ij}\}\rangle=\exp\{igL\cdot\sum_{i=1}^{D-1}\sum_{j=1}^{r}m_{ij}\langle C^{(i)},\gamma^j\rangle\}\ ,\qquad m_{ij}\varepsilon Z. \tag{35}$$

where γ^j {s=1, 2,...,r} denote the fundamental weights[15] of the lie algebra L(G), satisfying the condition

$$2\langle\gamma^j,\alpha^{(k)}\rangle/\langle\alpha^{(k)},\alpha^k\rangle=\delta^{jk}\ ,\qquad j,\ k=1,\ 2,\ldots r, \tag{36}$$

corresponding to the simple root system $\{\alpha^i|\,i=1,\ 2,\ldots,r\}$. Obviously, when $C^{(i)}$ is shifted by $4\pi/gL\cdot\alpha^j/\langle\alpha^j,\alpha^j\rangle$ for some simple root α^j under a gauge transformation, $|\{m_{ij}\}\rangle$ remains unchanged. Therefore, the spectrum of the hamiltonian (34) is given by

$$E\{m_{ij}\}=g^2/2L^{D-3}\cdot\sum_{i=1}^{D-1}\sum_{s=1}^{r}\left(\sum_{j=1}^{r}m_{ij}(\gamma^j)^s\right)^2\qquad(m_{ij}\varepsilon Z). \tag{37}$$

This spectrum has a unique zero-energy ground state with a constant wave function. The states correspond to any given $\{m_{ij}\}$ in Eq.(37) in the weak coupling limit are all low-lying states as expected. Thus after quantization, the zero-momentum modes for the gauge field are excited with only one ground state remains as zero energy. For the physical ground state in the supersymmetric gauge theories, we must include zero-momentum fermions also by Fermi statistics. Moreover, physical states must satisfy the constraint of Gauss law.

The physical ground states can be constructed by noting the fact that the zero-momentum modes are contained in a Cartan subalgebra of the gauge group G. The Weyl group corresponding the Cartan subalgebra leave the subalgebra invariant, but will generally rearrange coefficients with respect to generators in it. This corresponds to the fact that Weyl group of G is isomorphic to the group generated by Weyl reflections with respect to the roots of the Lie algebra. Since the gauge transformations with gauge functions in a Weyl group are topologically equivalent to the identity gauge function, physical states then must be invariant under the Weyl group action by Gauss' law. Namely, Gauss' law requires that the physical states form a trivial representation of the Weyl group. Especially, the index $Tr(-1)^F$ should be calculated in the Hilbert space invariant under the Weyl group. Each fermion corresponding a given generator H^s in the Cartan subalgebra have two spin states. Witten[7] has shown that there are totally r+1 different zero-energy physical states, and $Tr(-1)^F=r+1\neq 0$ up to a possible sign ambiguity for the gauge group G of rank r. Therefore, we expect that there is no dynamical supersymmetry breaking in the relevant supersymmetric gauge theories. However, there is an essential difference between the case in D=4 dimensions and D=6, 10 dimensions which will be given below.

In D=4 dimensions, dynamical supersymmetry breaking does not occur in the Θ-independent gauge theories[7], since the conjugation transformation which continuously changes the gauge coupling constant is gauge invariant only when the theory is independent of the vacuum angle Θ. The operator

$$K=\frac{1}{2}\int dx^{D-1}\varepsilon_{ijk}(A_i{}^a\partial_j A_k{}^a-2/3C_{abc}A_i{}^aA_j{}^bA_k{}^c) \tag{38}$$

relevant to instantons in four dimensions can be used in the conjugation. The discussion of the continuous deformation in higher dimensions is more involved and will be given elsewhere. We found that the operator implementing the continuous transformation of the gauge coupling constant in the higher dimensions leaves the physical vacuum invariant due to the fact that the operator is invariant with respect to both topologically trivial and non-trivial gauge transformations. When the physical vacuum is constructed from only a finite number of physically connected topological vacua, the gauge invariant physical vacuum is unique up to an overall phase factor as we have seen. If the physical vacuum is constructed from an infinite number of physically connected topological vacua when the homotopy group $\pi_{D-1}(G)$ contains an infinite cyclic group, then any possible vacuum angle chosen in the theory will be invariant under the action of the operator implementing the conjugation transformation of the gauge coupling constant. This implies that our conclusion for the dynamical supersymmetry breaking in higher dimensions does not dependent on any possible vacuum angle chosen in the theory. This is different from the case of four dimensions, where the operator (38) is invariant only under local gauge transformations. In a Θ-dependent theory, under the action of the operator (38) on the physical states, the resulted states will not be invariant under a topologically non-trivial gauge transformation. Consequently, the gauge coupling constant can be continuously transformed into each other in four dimensions only in theories independent of the vacuum angle Θ^7.

In conclusion, we have obtained a formula for the quantum spectra of the classical zero modes for the study of gauge theory system in a finite box with periodic boundary conditions. We have also discussed about the topological vacua in gauge theories in higher dimensions. Our results obtained are then applied to the study of dynamical supersymmetry breaking in supersymmetric gauge theories. Especially, it is argued that our previous argument that dynamical supersymmetry breaking does not occur in supersymmetric gauge theories in higher dimensions free of both local and global gauge anomalies is irrelevant to any possible vacuum-angle specification in the theories.

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