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Dirac neutrinos and SN 1987A

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Within the standard electroweak theory "wrong-helicity" neutrinos are produced in a nascent neutron star by "spin-flip" processes (at a rate proportional to m_ν^2), freely escape, and can lead to an excessively rapid cooling of the newly born neutron star. Previous work, based upon the neutrino-nucleon spin-flip scattering process alone, has shown that the observed cooling of the neutron star associated with SN 1987A excludes a Dirac-neutrino mass greater than ~ 20 keV for either ν_e, ν_μ, or ν_τ. We reexamine the emission of "wrong-helicity" Dirac neutrinos from SN 1987A and conclude that due to neutrino degeneracy and additional emission processes ($N+N \rightarrow N+N+\nu\bar{\nu}$, $\pi^- + p \rightarrow n + \nu\bar{\nu}$) the effect of a Dirac neutrino on the cooling of SN 1987A has been *underestimated*. While a precise Dirac-mass limit awaits the incorporation of our new rates into detailed numerical cooling models, we believe that the limit that follows from the cooling of SN 1987A is better, probably much better, than 10 keV. In particular, we believe that SN 1987A definitely excludes a 17-keV (purely) Dirac-mass neutrino that mixes with the electron neutrino at the 1% level.

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I. INTRODUCTION

According to the conventional theory [1], the initial cooling phase of a nascent neutron star lasts of the order of 10 sec, with thermal neutrino emission being the dominant cooling mechanism. This picture received dramatic confirmation when a neutrino burst of the order of 10 sec associated with SN 1987A was detected by both the Kamiokande II (KII) and Irvine-Michigan-Brookhaven (IMB) water Cherenkov detectors [2]. New, very weakly interacting particles, if they exist, can be produced in the core of a newly born, hot neutron star, can efficiently carry away energy, and can thereby accelerate the initial cooling process. The observable consequence of the emission of a new, very weakly interacting particle is the shortening of the duration of the neutrino burst associated with the early cooling phase. The potential shortening of the neutrino burst associated with SN 1987A has been used to severely constrain the properties of axions [3], "right-handed" neutrinos [4], and other hypothetical weakly interacting particles.

Our interest here is Dirac neutrinos; several authors have argued that a Dirac mass in the range of ~ 20 to ~ 300 keV for any of the three neutrino species is excluded by the KII and IMB observations of the initial cooling of the neutron star associated with SN 1987A [4]. The key to the argument involves the "wrong-helicity" states of a Dirac neutrino, ν_+ and $\bar{\nu}_-$, which are "sterile" for a massless Dirac neutrino, but are not quite sterile for a massive Dirac neutrino. The emission of "wrong-helicity" neutrinos can drastically alter the cooling of a nascent neutron star, because once produced they can freely escape and carry away a significant amount of the hot neutron star's thermal energy.

In the standard electroweak model, the $SU(2)_W \otimes U(1)_Y$ gauge theory, the weak-interaction states are the chirality states, "left" and "right:" only left-handed particles and right-handed antiparticles interact. For a massless neutrino, the chirality states and helicity states coincide: $\nu_L = \nu_-$ and $\nu_R = \nu_+$, $\bar{\nu}_L = \bar{\nu}_-$, and $\bar{\nu}_R = \bar{\nu}_+$, which implies that the "wrong-helicity" states ν_R and $\bar{\nu}_L$ do not interact (i.e., are sterile) and, therefore, in the context of the standard electroweak theory are irrelevant. Indeed, in the standard electroweak theory, wherein the neutrino is massless, the neutrino can be represented either as a four-component Dirac particle, with two sterile and irrelevant components, or as a two-component Majorana particle, whose components are ν_L and ν_R (for a Majorana neutrino $\nu = \bar{\nu}$).

The situation changes if the neutrino has mass: the chirality and helicity states no longer coincide. In the ultrarelativistic limit the projection of ν_- ($\bar{\nu}_+$) onto ν_L ($\bar{\nu}_R$) is of order unity, and so these helicity states have ordinary weak interactions (as in the massless case). On the other hand, the wrong-helicity states ν_+ ($\bar{\nu}_-$) have but a small, but nonzero, projection, of order $m_\nu / 2E_\nu$, on to the chirality states ν_L ($\bar{\nu}_R$), and are no longer sterile. Owing to this small projection onto the weak-interaction chirality states wrong-helicity neutrinos can be produced through ordinary weak interactions in "spin-flip" processes, e.g., $\nu_- + N \rightarrow \nu_+ + N$ or $\bar{\nu}_+ + N \rightarrow \bar{\nu}_- + N$. Of course it is also possible that the wrong-helicity states have other, new interactions (e.g., right-handed interactions). *We will not address that possibility here. We will assume that the massive Dirac neutrino only has the standard electroweak interactions, so that the wrong-helicity states have interactions only by virtue of their projections on to the chirality eigenstates ν_L and $\bar{\nu}_R$.*

(Once produced, wrong-helicity Dirac neutrinos can in principle interact with matter through their projections onto ν_L and $\bar{\nu}_R$; however for $m_\nu \lesssim 300$ keV, the mean free path for such interactions is large compared to the size of a neutron star. For $m_\nu \gtrsim 300$ keV the mean free path of a wrong-helicity neutrino in a neutron star becomes less than the radius of a neutron star, and thus wrong-helicity neutrinos should become trapped like their proper-helicity counterparts. Because the mean free path decreases as m_ν^{-2} the cooling effect of the wrong-helicity states diminishes with increasing mass; for a sufficiently large mass, the effect of wrong-helicity Dirac neutrinos on the cooling of a nascent neutron star will become "unobservable." The mass at which trapping is sufficient to make a Dirac species "supernova safe" certainly must be greater than 300 keV, and an accurate determination of this mass requires a careful treatment of wrong-helicity neutrino transport. This is a formidable task. For our purposes it suffices to say that the value of the "supernova safe" mass must certainly be greater than 300 keV, the mass where trapping sets in, and that for $m \lesssim 300$ keV wrong-helicity neutrinos once produced stream out.)

The most detailed study of the effect of a massive Dirac neutrino species on the cooling of SN 1987A is that of Gandhi and Burrows [5]. In numerical models of the early-cooling phase of SN 1987A they included the effect of wrong-helicity neutrinos produced by the spin-flip-scattering processes $\nu_- + N \to \nu_+ + N$ and $N + \bar{\nu}_+ \to N + \bar{\nu}_-$. For neutron-star models cooled by both proper- and wrong-helicity neutrinos they computed the flux of proper-helicity neutrinos and the response of the KII and IMB detectors to this flux. They concluded that the duration of the detected neutrino bursts exclude a Dirac mass greater than about 14 keV. In fact, their mass limit was extremely conservative; the effect of 14-keV Dirac neutrino was to reduce the burst duration expected to less than about 1 sec in either detector. Had they instead insisted that the neutrino burst duration expected be no shorter than half the duration of the actual burst, their limit would have been about 9 keV.

On the face of it their work seems to preclude a Dirac neutrino of mass 17 keV for example. Of course 17 keV is a very interesting mass since several β-decay experiments have found evidence for a 17-keV neutrino-mass eigenstates that mixes with the electron neutrino at the 1% level ($\sin^2\theta \simeq 0.01$) [6]. Moreover, the absence of neutrinoless double-β decay in several isotopes strongly suggests that the 17-keV mass eigenstate is of the Dirac type. Unfortunately, Gandhi and Burrows [5] recently discovered a simple factor of 4 error in the rate they used for the emission of wrong-helicity neutrinos, which has the effect of *doubling* their mass limit—raising their original limit to 28 keV (and the less conservative limit that one could derive from their results to about 18 keV). Our motivation for reexamining the emission of wrong-helicity neutrinos from SN 1987A hardly needs to be mentioned.

To summarize our results briefly, we find that due to a number of effects the volume emissivity (erg cm^{-3} sec^{-1}) of wrong-helicity neutrinos is at least as large as, and probably much larger than, that originally used by Gandhi and Burrows, implying that their original "conservative limit" of 14 keV stands. In particular the production of wrong-helicity neutrinos due to nucleon-nucleon, neutrino-pair bremsstrahlung is at least as important as spin-flip-scattering production. Since the cores of neutron stars are on the verge of pion condensation negative pions are likely to be present in great numbers. This being the case, the process $\pi^- + p \to n + \nu\bar{\nu}$ is likely to be even more important than the bremsstrahlung process (although less certain since the pion density depends critically upon the equation of state). Finally and most importantly, if there is a significant mixing between the massive Dirac neutrino and the electron neutrino (greater than few 0.1% for keV masses), then deep in the core of the neutron star the massive Dirac-neutrino species will, like the electron neutrino, be degenerate (with chemical potential $\mu_\nu \sim 200$ MeV), rather than nondegenerate as previously assumed; when this fact is taken into account the rate for wrong-helicity neutrino emission increases by an enormous factor, of the order of $(\mu_\nu/T)^4 \sim 10^3$.

Without recourse to a numerical study of the effect of a massive Dirac neutrino on the early cooling of a nascent neutron star it is premature to quote a definitive limit based upon the cooling of SN 1987A. However, it seems clear that when all of the additional effects discussed here are incorporated into detailed numerical models of the early phase of neutron-star cooling the Dirac-mass limit will be more stringent than 10 keV, and if the massive Dirac neutrino mixes with the electron neutrino at the 1% level, the limit will be closer to 1 keV.

II. SPIN-FLIP-SCATTERING PRODUCTION OF WRONG-HELICITY NEUTRINOS

A. Nondegenerate neutrinos

Positive-helicity neutrinos (and negative-helicity antineutrinos) are produced by the helicity-flip scattering processes $\nu_- + N \to \nu_+ + N$ and $\bar{\nu}_+ + N \to \bar{\nu}_- + N$, where N is a nucleon. The matrix-element squared for this process has been computed by Gaemers *et al.* [7]:

$$|\mathcal{M}_{\rm SF}|^2 = 8 G_F^2 m_\nu^2 m^2 [(c_V^2 + 3c_A^2) - (c_V^2 - c_A^2)\cos\theta] , \qquad (1)$$

where $|\mathcal{M}_{\rm SF}|^2$ has been summed over initial and final nucleon spins, θ is the angle between the incoming and outgoing neutrinos, m is the nucleon mass, m_ν is the Dirac-neutrino mass, $G_F \simeq 1.17 \times 10^{-5}$ GeV^2 is the Fermi constant, and $c_V(p) = (1 - 4\sin^2\theta_W)/2 \simeq 0$, $c_A(p) \simeq g_A/2$, $c_V(n) = -\frac{1}{2}$, $c_A(n) \simeq -g_A/2$, and $g_A = 1.26$. Unless stated otherwise we will work in units where $\hbar = k_B = c = 1$.

The volume emissivity (erg cm^{-3} sec^{-1}) of wrong-helicity neutrinos is given by

$$\dot{\varepsilon}_{\rm SF} = \int |\mathcal{M}_{\rm SF}|^2 (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2) \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3k_1}{2k_1(2\pi)^3} \frac{d^3k_2}{2k_2(2\pi)^3} f_1(1-f_2) f_\nu k_2 , \qquad (2)$$

where p_i is the four-momentum of the incoming ($i=1$)/outgoing ($i=2$) nucleon, k_i is the four momentum of the incoming ($i=1$)/outgoing ($i=2$) neutrino, $f_\nu=[\exp(k_1/T)+1]^{-1}$ is the phase-space distribution function of the incoming neutrino, and $f_i=[\exp(E_i/T-\mu_i/T)+1]^{-1}$ are the phase-space distribution functions of the nucleons. Note that we have allowed for nucleon degeneracy, but we have assumed that neutrinos are nondegenerate (according to the conventional model of a nascent neutrino star [1], this is a good assumption for ν_μ and ν_τ, but not for ν_e). We will return shortly to the important issue of neutrino degeneracy.

Making three reasonable assumptions this 12-dimensional integral can be reduced to a single integral. They are (1) nonrelativistic nucleons, (2) negligible three-momentum carried by the neutrinos (compared to the nucleons), and (3) incoming and outgoing neutrinos have the same energy (elastic scattering). The volume emissivity for the process $\nu_-+N\rightarrow\nu_++N$ can then be written as

$$\dot{\varepsilon}_{\rm SF}=\frac{G_F^2 m_\nu^2(c_V^2+3c_A^2)\rho_\nu}{4\pi}\left[\frac{\sqrt{2}(mT)^{3/2}}{\pi^2}\frac{\partial}{\partial y}\int_0^\infty\frac{\sqrt{u}\,du}{e^{u-y}+1}\right], \tag{3a}$$

where $y=(\mu-m)/T$ and $\rho_\nu=7\pi^2T^3/240$ is the energy density in thermal neutrinos. In the nondegenerate limit $y\ll-1$ the final term reduces to n_N. The emission of wrong-helicity antineutrinos is given by the same expression with $\rho_\nu\rightarrow\rho_{\bar\nu}=\rho_\nu$. Taking the nondegenerate limit and including the antineutrino process as well, the total volume emissivity becomes

$$\dot{\varepsilon}_{\rm SF}=1.2\times10^{32}m_{100}^2\rho_{14}T_{10}^4(0.9+0.2X_n)\,{\rm erg\,cm^{-3}\,sec^{-1}}, \tag{3b}$$

where m_{100} is the neutrino mass in units of 100 keV, ρ_{14} is the total mass density in units of 10^{14} g cm^{-3}, T_{10} is the temperature in units of 10 MeV, and X_n is the neutron fraction (the proton fraction $X_p=1-X_n$). For arbitrary nucleon degeneracy,

$$\dot{\varepsilon}_{\rm SF}=\frac{7G_F^2m_\nu^2m^{3/2}T^{11/2}}{2^{9/2}\times15\times\pi}[1.2I(y_p)+1.4I(y_n)] \tag{4a}$$

$$\simeq2.6\times10^{31}m_{100}^2(m/0.94\,{\rm GeV})^{3/2}T_{10}^{11/2}[1.2I(y_p)+1.4I(y_n)]\,{\rm erg\,cm^{-3}\,sec^{-1}}, \tag{4b}$$

where $I(y)\equiv(\partial/\partial y)\int_0^\infty\sqrt{u}\,du/[\exp(u-y)+1]$ and we have displayed explicitly the dependence upon the nucleon mass m because the effective nucleon mass in nuclear matter is expected to be reduced by a factor of the order of $\frac{1}{2}$. In the nondegenerate limit $\dot{\varepsilon}_{\rm SF}$ does not depend upon the value of the nucleon mass, cf. Eq. (3); in the degenerate limit $I(y)\propto y^{1/2}$ and $y\propto m^{-1}$, so that $\dot{\varepsilon}_{\rm SF}\propto m$. The conditions at the core of the neutron star are expected to be closer to nondegenerate than degenerate and $\dot{\varepsilon}_{\rm SF}$ should be insensitive to the effective value of the nucleon mass.

The following is a simple fit to $I(y)$ that is accurate to better than 12% (typically accurate to a few %):

$$I_{\rm fit}(y)^{-1}=\frac{2e^{-y}}{\sqrt{\pi}}+\frac{1}{\sqrt{1+|y|}}-\frac{1}{8(1+|y|)^{3/2}}.$$

Before going on to consider neutrino degeneracy we should compare our expression for $\dot{\varepsilon}_{\rm SF}$ with that used by Burrows and Gandhi [5]. After correcting the spin-flip cross section in their paper for their errant factor of 4, we find that our volume emissivity for ($X_n=\frac{1}{2}$) is a factor of about 1.9 larger than theirs. Most of the difference traces to one fact: We use $|c_A(n,p)|=g_A/2=0.63$ and they use 0.5. We believe that $g_A/2$ is the appropriate choice [8].

B. Neutrino degeneracy

At the core of a newly born neutron star, where the density is several times nuclear density and the temperature is of order 30 MeV, electrons are ultrarelativistic and highly degenerate with $\mu_e\simeq p_F(e)\simeq240(X_p\rho_{14})^{1/3}$ MeV (this follows from charge neutrality: $n_e=n_p=X_p\rho/m$). On the other hand, neutrons and protons are only semidegenerate, with $\mu_n,\mu_p\sim30$ MeV $\ll\mu_e$. From these facts it follows that electron neutrinos are also highly degenerate, as β equilibrium $(n+\nu_e\leftrightarrow p+e^-)$ enforces: $\mu_{\nu_e}=\mu_e+\mu_p-\mu_n\sim\mu_e\sim200$ MeV. In the absence of interactions that interconvert neutrinos of different flavor, μ and τ neutrinos are not degenerate ($\mu_\nu\ll T$). Since we know that the electron-neutrino mass is less than about 10 eV (more precisely, the mass of the dominant mass eigenstate associated with ν_e), only the production of wrong-helicity μ and τ neutrinos is of interest in setting a Dirac mass limit, justifying the previous assumption that the massive Dirac neutrino is nondegenerate.

Neutrino mixing can radically change the story. If there is mixing between ν_e and the massive Dirac neutrino species, then μ and/or τ neutrinos can become degenerate through "$\nu_e\leftrightarrow\nu_\mu$" and/or "$\nu_e\leftrightarrow\nu_\tau$" oscillations. For keV neutrino masses, the (matter) oscillation length ($l_{\rm osc}\sim4\pi/G_Fn_e\sim10^{-4}$ cm) is much shorter than the neutrino interaction length ($l_{\rm int}\sim4\pi/G_F^2n_eE^2\sim10^2$ cm), and thus it makes sense to work with neutrino-mass eigenstates rather than neutrino-flavor eigenstates. (The mass eigenstates are the eigenstates of a freely propagating neutrino, while the flavor eigenstates are the weak-interaction eigenstates.) We can express the flavor eigenstates in terms of the mass eigenstates (and vice versa); for simplicity consider only two mass ($i=1,2$) and flavor eigenstates ($i=e,\tau$):

$$|\nu_e\rangle=\cos\theta|\nu_1\rangle+\sin\theta|\nu_2\rangle ,$$
$$|\nu_\tau\rangle=-\sin\theta|\nu_1\rangle+\cos\theta|\nu_2\rangle ,$$

or

$$|\nu_1\rangle=\cos\theta|\nu_e\rangle-\sin\theta|\nu_\tau\rangle ,$$
$$|\nu_2\rangle=\sin\theta|\nu_e\rangle+\cos\theta|\nu_\tau\rangle .$$

Here θ is the "ν_e-ν_τ" mixing angle, which is expected to be small and for the masses of interest is constrained by experiment to be less than ~ 0.2. Thus mass eigenstate 1 is primarily ν_e with a small admixture of ν_τ, and vice versa for mass eigenstate 2.

The ordinary charged-current interactions that enforce β equilibrium will as before populate a degenerate sea of ν_1's, and the relationship $\mu_1=\mu_e+\mu_p-\mu_n\sim\mu_e$ will hold (μ_1 and μ_2 denote the chemical potentials of the two neutrino-mass eigenstates). To begin, assume that there are no ν_2, $\bar\nu_2$'s. Neutral-current interactions can rapidly create $\nu_2\bar\nu_2$ pairs, e.g., $\nu_1+\bar\nu_1\to\nu_2+\bar\nu_2$, thereby rapidly populating a *thermal* sea of $\nu_2\bar\nu_2$'s. Because neutral-current interactions are diagonal in the mass-eigenstate basis they cannot interconvert ν_1 to ν_2. Such interconversions are needed to populate a degenerate sea of ν_2's.

Charged-current interactions can interconvert ν_1 and ν_2: for example, $\nu_1+e^-\to\nu_2+e^-$ or $e^-+p\to n+\nu_2$. The amplitude for the first process is a factor of $\sin\theta\cos\theta$ times that of the charged-current diagram for $\nu_e+e^-\to\nu_e+e^-$; the amplitude for the second process is a factor of $\sin\theta$ times that for the familiar charged-current process $e^-+p\to n+\nu_e$. The process $e^-+p\to\nu_2+n$ is the more important process for populating a degenerate ν_2 sea because the process $\nu_1+e^-\to\nu_2+e^-$ has a smaller cross section and is Pauli suppressed due to the final-state electron. Thus we focus solely on the process $e^-+p\to\nu_2+n$.

The volume production rate of ν_2's (number per cm^3 per sec) by the process $e^-+p\to\nu_2+n$ is given by

$$\dot n_2=\int|\mathcal{M}|^2(2\pi)^4\delta^4(p_1+k_1-p_2-k_2)\frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3k_1}{2\omega_1(2\pi)^3}\frac{d^3k_2}{2\omega_2(2\pi)^3}f_ef_1 , \tag{5a}$$

where p_1,p_2 are four-momenta of the incoming proton and outgoing neutron, k_1,k_2 are the four-momenta of the incoming electron and outgoing ν_2, and f_e, f_1 are the phase-space distribution functions of electrons and protons. For simplicity the Pauli-blocking factor for the outgoing neutron has been neglected, a reasonable approximation since neutrons are only semidegenerate; until a degenerate sea of ν_2's builds up there is no Pauli-blocking factor for ν_2's.

To further simplify we will assume that (1) nucleons are nonrelativistic and (2) electrons are ultrarelativistic and degenerate. It then follows that the energy of the outgoing ν_2 is equal to that of the incoming e^-: $\omega_1=\omega_2$, and that the matrix-element squared (summed over initial and final spins) for $e^-+p\to\nu_2+n$ is

$$|\mathcal{M}|^2=32G_F^2m^2\omega_1^2\sin^2\theta_m[(1+3g_A^2)+(1-g_A^2)\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2] ,$$

where θ_m is the neutrino-mixing angle in matter. The matter mixing angle is related to the vacuum-mixing angle by [9]

$$\sin 2\theta_m\simeq\sin 2\theta\min[1,\Delta/V] ,$$
$$\Delta/V\simeq 0.05\left[\frac{\delta m^2}{10^8\,\mathrm{eV}^2}\right]\left[\frac{100\,\mathrm{MeV}}{E_\nu}\right]\left[\frac{3}{\rho_{14}}\right] ,$$
$$\Delta=\frac{\delta m^2}{2E_\nu} ,\quad V\simeq 3\rho_{14}\mathrm{eV} ,$$

where E_ν is the neutrino energy, $\delta m^2=m_2^2-m_1^2$, and V is the (weak-interaction) energy difference associated with the interaction of electron neutrinos and tau neutrinos with the background (neutrons, protons, electrons, positrons, and neutrinos in the core of the neutron star). (Note, by using the min$[1,\Delta/V]$ we have not allowed for the possibility of resonant conversion; it would only further enhance neutrino mixing as $\sin 2\theta_m\to 1$ in this case.)

It is now straightforward to evaluate Eq. (5a) for the production of ν_2's:

$$\dot n_2=\frac{(1+3g_A^2)G_F^2\sin^2\theta_m}{10\pi^3}n_p\mu_e^5 . \tag{5b}$$

If the production rate $\dot n_2$ is sufficiently large, then ν_2's will be brought into chemical equilibrium with ν_1's resulting in $\mu_2=\mu_1$. In chemical equilibrium the number density of ν_2's is given by $n_2^{\mathrm{eq}}\simeq\mu_2^3/6\pi^2\sim\mu_e^3/6\pi^2$. From this we can estimate the time required to populate a degenerate sea of ν_2's:

$$\tau\simeq\frac{n_2^{\mathrm{eq}}}{\dot n_2}=\frac{5\pi}{3(1+g_A^2)}\frac{\sin^{-2}\theta_m}{G_F^2\mu_e^2n_p}$$
$$\sim 10^{-5}\left[\frac{0.01}{\sin^2\theta}\right]\left[\frac{\rho_{14}}{X_p}\right]\left[\frac{10^8\,\mathrm{eV}^2}{\delta m^2}\right]^2\mathrm{sec} .$$

That is, the mixing of a tau neutrino of mass of order 10 keV with the electron neutrino at the 1% level is sufficient to very rapidly populate a degenerate sea of "τ neutrinos" in the core of a hot neutron star. (Rapid here means much less than the cooling time of the neutron star: $\tau\ll 1$ sec.) Note that neutrino oscillations mix flavors, but not helicity states, and so the degenerate sea of massive Dirac neutrinos filled by neutrino oscillations are proper-helicity neutrinos. The wrong-helicity neutrinos must still be produced by spin-flip interactions.

Degeneracy of μ (and/or τ) neutrinos will of course modify the "chemical composition" of the neutron star (by which we mean the abundance of n, p, e^-, ν_e, etc.). The chemical composition is determined by the chemical potentials for ν_1, ν_2, electrons, neutrons, and protons. In turn they are fixed by the various constraints at hand: β equilibrium, $\mu_1=\mu_2=\mu_p-\mu_n+\mu_e$; charge conservation, $n_e=n_p$; and the approximate conservation of total lepton-number conservation, $n_1-n_{\bar{1}}+n_e-n_{\bar{e}}+n_2-n_{\bar{2}}\simeq$const. (Lepton-number conservation is only approximate; in the conventional scenario electron neutrinos carry off lepton number, and here lepton number can be carried away both by electron neutrinos and wrong-helicity ν_2's. Thus, the total lepton number slowly decreases, which implies that μ_1 and μ_2 decrease with time.)

Qualitatively we can see what must happen if a massive Dirac neutrino species mixes sufficiently with the electron neutrino to become degenerate: Additional protons will have to decay to supply the neutrinos in the degenerate ν_2 sea; this will increase μ_n and decrease μ_p, μ_e, and μ_1. To be more quantitative consider the following simple model: degenerate electrons, nucleons, and N neutrino species; and $\mu_e=\mu_1=\cdots=\mu_N\gg\mu_n,\ \mu_p$. Denoting the initial lepton number per nucleon by Y_i and for simplicity assuming total lepton number is conserved, it follows that $\mu_e=\mu_1=\cdots=\mu_N=208$ MeV $[3/(2+N)]^{1/3}(Y_i\rho_{14})^{1/3}$ and $X_p=2Y_i/(2+N)$. If only the electron neutrino is degenerate, $N=1$, $[3/(2+N)]^{1/3}=1$, and $X_p=0.67Y_i$; if the electron neutrino and a heavy neutrino species are degenerate, $N=2$, $[3/(2+N)]^{1/3}\simeq0.91$, and $X_p=0.5Y_i$, so that the neutrino chemical potential is reduced by about 10% and the proton fraction is reduced by about 33%. (Note too, that if ν_μ's become degenerate, then μ's will be degenerate, and in the limit $m_\mu\ll\mu$, the $2+N$ factor becomes $4+N$.)

Where necessary for an estimate we will assume that $\mu_e\sim\mu_1\sim\mu_2\sim200$ MeV. However, a careful treatment of the cooling of a hot neutron star in the presence of a massive Dirac neutrino that mixes with the electron neutrino must take into account the evolution of the various chemical potentials and the fact that lepton number will decrease as electron neutrinos and wrong-helicity ν_2's escape the neutron star.

If the massive Dirac neutrino species is degenerate, which seems likely for keV masses and mixing angles of order 1%, then our calculation of $\dot{\varepsilon}_{SF}$ must be revised: The neutrino distribution function must be changed to $f_\nu=[\exp(E_\nu/T-\mu_\nu/T)+1]^{-1}$. (No blocking factor need be added for the final-state neutrino since it is a wrong-helicity neutrino.) Making the same approximations as before, the form of Eq. (3a) for $\dot{\varepsilon}_{SF}$ is unchanged; however, the energy density of the massive Dirac neutrino ($=\rho_\nu$) is now given by

$$\rho_\nu=\frac{T^4}{2\pi^2}\int_0^\infty\frac{u^3du}{e^{u-y}+1}\to\frac{\mu_\nu^4}{8\pi^2}\quad(\text{for } \mu_\nu\gg T)\ ,$$

where $y=\mu_\nu/T$. In the highly degenerate limit, the energy density of neutrinos is much larger, of order μ^4 rather than of order T^4, there are more neutrinos and they have higher energies, and the volume emissivity is increased relative to the previous result, cf. Eq. (3b), by a factor of

$$\frac{15}{7\pi^4}\frac{\mu^4}{T^4}\simeq4\times10^3(\mu/200\,\text{MeV})^4/T_{10}^4\ .$$

In the highly degenerate limit the process involving antineutrinos is severely suppressed because $\mu_{\bar{\nu}}=-\mu_\nu$, which means that

$$\rho_{\bar{\nu}}=\frac{T^4}{2\pi^2}\int_0^\infty\frac{u^3du}{e^{u+y}+1}\ll\rho_\nu\ .$$

In contrast with the nondegenerate case where the emission of wrong-helicity neutrinos and antineutrinos is identical (since $\rho_\nu=\rho_{\bar{\nu}}$), in the degenerate case only wrong-helicity neutrinos are emitted.

Bringing everything together, when we allow for the degeneracy of the massive Dirac neutrino, the emissivity due to spin-flip neutrino-nucleon scattering is given by

$$\dot{\varepsilon}_{SF}=\frac{G_F^2m_\nu^2m^{3/2}T^{11/2}}{2^{5/2}\pi^5}[1.2I(y_p)+1.4I(y_n)]\left[\frac{1}{4}\left[\frac{\mu}{T}\right]^4+\frac{\pi^2}{2}\left[\frac{\mu}{T}\right]^2+\frac{7\pi^4}{60}\right]\tag{6a}$$

$$\to9.2\times10^{34}m_{100}^2(m/0.94\,\text{GeV})^{3/2}(\mu_\nu/200\,\text{MeV})^4T_{10}^{3/2}[1.2I(y_p)+1.4I(y_n)]\,\text{erg cm}^{-3}\,\text{sec}^{-1}\quad(\text{for } \mu_\nu\gg T)\ .\tag{6b}$$

Note that neutrino degeneracy always increases $\dot{\varepsilon}_{SF}$ because $(\rho_\nu+\rho_{\bar{\nu}})$ is minimized for $\mu_\nu=0$.

Provided the massive Dirac neutrino is degenerate, the volume emissivity of wrong-helicity neutrinos is increased by a whopping factor of $\sim10^4$, which should naively improve the mass limit by a factor of 100. However, for a vacuum-mixing angle of 1%, the time scale for populating the degenerate sea becomes longer than $\sim10^{-1}$ sec for a mass of less than ~1 keV; in this mass regime there is not enough time to populate a degenerate sea of massive Dirac neutrinos and the nondegenerate formula for $\dot{\varepsilon}_{SF}$ applies. The SN 1987A mass limit clearly depends upon the mixing angle, and for 1% mixing it should be around 1 keV.

III. BREMSSTRAHLUNG-PAIR PRODUCTION OF WRONG-HELICITY NEUTRINOS

Wrong-helicity neutrinos can be produced through another spin-flip process: nucleon-nucleon, neutrino-pair bremsstrahlung, $N+N\to N+N+\nu\bar{\nu}$, where $\nu\bar{\nu}=\nu_-\bar{\nu}_-$ or $\nu_+\bar{\nu}_+$ and N is a nucleon. The rate for this process can be found by using the matrix-element squared calculated by Friman and Maxwell [10] and the phase-space volume calculated by Brinkmann and Turner [11].

The volume emissivity for $N+N\to N+N+\nu_{+}+\bar{\nu}_{+}$ is given by

$$\dot{\varepsilon}_{\rm brem}=\int S|\mathcal{M}|^2(2\pi)^4\delta^4(p_1+p_2-p_3-p_4-q_1-q_2)$$
$$\times\frac{d^3p_1}{2E_1(2\pi)^3}\cdots\frac{d^3p_4}{2E_3(2\pi)^3}\frac{d^3q_1}{2\omega_1(2\pi)^3}\frac{d^3q_2}{2\omega_2(2\pi)^3}f_1f_2(1-f_3)(1-f_4)\omega_1\,, \qquad (7)$$

where p_i are the four-momenta of the nucleons, $f_i=[\exp(E_i/T-\mu_i/T)+1]^{-1}$ are the nucleon phase-space distribution functions, $q_{1,2}$ are the four-momenta of the neutrino and/or antineutrino, $\omega_{1,2}$ are the energies of the neutrino and/or antineutrino, ω_1 is the energy of the wrong-helicity neutrino, S is the symmetry factor (a factor of $1/2!$ for any pair of identical particles in the initial or final state), and the matrix-element squared is to be summed over initial and final nucleon spins. To begin we will assume that neutrinos are nondegenerate.

Friman and Maxwell [10] have calculated the matrix element for the ordinary (no spin flip) pair-production bremsstrahlung process in the one-pion-exchange approximation; their matrix element can be used to obtain the matrix element for the spin-flip process by multiplying by $m_\nu/2\omega_1$. Doing so and pulling out the only factor in the matrix element that depends upon the neutrino energies the desired matrix element can be written as

$$|\mathcal{M}|^2=|\mathcal{M}'_{\rm FM}|^2\left[\frac{m_\nu}{2\omega_1}\right]^2\left[\frac{\omega_1\omega_2}{\omega^2}\right], \qquad (8)$$

where $\omega=\omega_1+\omega_2$ is the total energy carried off by the neutrino and antineutrino.

There are actually three different bremsstrahlung processes: neutron-neutron, proton-proton, and neutron-proton. The matrix elements for the first two are the same. Calculating the matrix-element squared is a tedious process involving the square of the sum of eight different diagrams (four direct and four exchange). Friman and Maxwell [10] have computed the square of the sum of the direct diagrams and the square of the sum of the exchange diagrams, but not the interference term. From previous experience with nucleon-nucleon, axion bremsstrahlung [11], for which the matrix element has a similar structure, we know that in the nondegenerate limit the interference term is very small, while in the degenerate limit the contribution of the interference term *increases* $|\mathcal{M}|^2$ by about 50% (over the incoherent sum of the direct and exchange terms). Based on this we will ignore the interference terms (thereby likely *underestimating* the matrix element in the degenerate limit). The two matrix-elements squared (and summed over nucleon spins) are

$$|\mathcal{M}'_{\rm FM}(nn,pp)|^2=2\times2^{10}G_F^2g_A^2f^4\left[\frac{m}{m_\pi}\right]^4,$$

$$|\mathcal{M}'_{\rm FM}(np)|^2=3\times2^{10}G_F^2g_A^2f^4\left[\frac{m}{m_\pi}\right]^4,$$

where $G_F=1.17\times10^{-5}$ GeV^{-2} is the Fermi constant, $g_A\simeq1.26$ is the axial-vector coupling constant, $f\sim1.1$ is the pion-nucleon coupling, m is the nucleon mass, and $m_\pi\simeq0.135$ GeV is the pion mass. Note that (1) we have already factored out the neutrino-energy dependence from the matrix-element squared and (2) as presented, the Friman-Maxwell matrix-element squared must be multiplied by a factor of $(2m)^4$ because of their nucleon-spinor normalization convention.

Now we discuss the phase-space volume integration. Brinkmann and Turner [11] have evaluated the five-particle phase-space volume element for nucleon-nucleon, axion bremsstrahlung for arbitrary nucleon degeneracy with the following assumptions: (1) nonrelativistic nucleons; (2) negligible axion three-momentum (compared to that of the nucleons); and (3) constant matrix element. As we will see very shortly the six-particle phase-space integral needed here can be reduced to the very same phase-space integral. For the neutrino bremsstrahlung case the analogous assumptions are (1) nonrelativistic nucleons and (2) negligible neutrino three-momenta. In the axion case the integral over the axion's momentum in the expression for $\dot{\varepsilon}_a$ that is analogous to Eq. (6) can be reduced to a single integral over the axion's energy:

$$\int\frac{d^3q_a}{2E_a(2\pi)^3}E_a=\frac{1}{4\pi^2}\int E_a^2dE_a\,.$$

In the present circumstance the integral over the momenta of the neutrino pair can be reduced to an integral over the sum of their energies:

$$\int\frac{d^3q_1}{2\omega_1(2\pi)^3}\frac{d^3q_2}{2\omega_2(2\pi)^3}\frac{\omega_1\omega_2}{\omega^2}\left[\frac{m_\nu}{2\omega_1}\right]^2\omega_1=\left[\frac{1}{4\pi^2}\right]^2\frac{m_\nu^2}{48}\int\omega^2d\omega\,.$$

Thus, by simply multiplying the results of Brinkmann and Turner [9] for the phase-space integrals over $p_1,\ldots,p_4$ and E_a by a factor of $m_\nu^2/192\pi^2$ we can obtain the phase-space integration needed here.

The axion phase-space integral can be expressed as

$$\int\frac{d^3p_1}{2E_1(2\pi)^3}\cdots\frac{d^3p_4}{2E_4(2\pi)^3}\frac{E_a^2}{4\pi^2}dE_a(2\pi)^4\delta^4(p_1+p_2-p_3-p_4-q_a)f_1f_2(1-f_3)(1-f_4)$$
$$=\begin{cases}m^{1/2}T^{13/2}\exp(y_1+y_2)/140\pi^{13/2} & \text{(nondegenerate limit)}\,,\\ m^{1/2}T^{13/2}I(y_1,y_2) & \text{(in general)}\,,\end{cases} \qquad (9)$$

where $y_i \equiv (\mu_i - m)/T$, and $I(y_1,y_2)$ is a (different) dimensionless function that must be evaluated numerically (see below). In the nondegenerate limit,

$$n_i = 2\left[\frac{mT}{2\pi}\right]^{3/2} e^{y_i} ,$$

where n_i is the number density of species i (i = neutron or proton).

To begin, consider the nondegenerate limit, a reasonable approximation to the conditions that pertain [11]. In this circumstance the volume emissivity can be expressed as

$$\dot{\varepsilon}_{\rm brem} = \frac{16 g_A^2 f^4}{105\pi^{11/2}} \frac{G_F^2 m_\nu^2 m^{3/2} T^{7/2}}{m_\pi^4} f(X_n) n_N^2 , \tag{10a}$$

$$\dot{\varepsilon}_{\rm brem} \simeq 1.5\times 10^{31} f(X_n) \rho_{14}^2 T_{10}^{7/2} m_{100}^2 \ {\rm erg\,cm^{-3}\,sec^{-1}} , \tag{10b}$$

where n_N is the total nucleon density, X_n is again the neutron fraction, the function $f(X_n) = 0.5 + 2X_n(1-X_n)$ varies between 0.5 (for $X_n = 0,1$) and 1.0 ($X_n = \frac{1}{2}$), and we have included a factor of 2 to account for both the process where the neutrino has the wrong helicity and the one where the antineutrino has the wrong helicity. We can compare this energy-loss rate to that from neutrino-neutron spin-flip scattering (taking the nondegenerate limit for both):

$$\frac{\dot{\varepsilon}_{\rm brem}}{\dot{\varepsilon}_{\rm SF}} \simeq 0.14 \frac{\rho_{14}}{T_{10}^{1/2}} \frac{f(X_n)}{0.9+0.2X_n} . \tag{11}$$

At the core of the newly born neutron star where most of the emission of wrong-helicity neutrinos occurs $\rho_{14} \sim 4-10$ and $T_{10} \sim 3-10$, and thus the bremsstrahlung process should be of comparable importance.

In the general case the volume emissivity is

$$\dot{\varepsilon}_{\rm brem} = \frac{160 f^4 g_A^2}{15\pi^2} \frac{G_F^2 m_\nu^2 m^{9/2} T^{13/2}}{m_\pi^4} \{0.5[I(y_1,y_1)+I(y_2,y_2)] + 3I(y_1,y_2)\} \tag{12a}$$

$$\simeq 2.4\times 10^{35} (m/0.94\,{\rm GeV})^{1/2} m_{100}^2 T_{10}^{13/2} \{0.5[I(y_1,y_1)+I(y_2,y_2)] + 3I(y_1,y_2)\} {\rm erg\,cm^{-3}\,sec^{-1}} , \tag{12b}$$

where we have displayed explicitly the kinematical dependence upon the nucleon mass (i.e., we have not pulled out the m^4 factor associated with the pion-nucleon coupling, m^4/m_π^4). On the basis of the nonlinear σ model it has been argued that the ratio of the nucleon mass to the pion mass should not change significantly with density [12]. In the nondegenerate limit, $\dot{\varepsilon}_{\rm brem}$ varies as $m^{-5/2}$ and would increase by a significant factor if the effective nucleon mass is half its vacuum value. In the degenerate limit $\dot{\varepsilon}_{\rm brem}$ is independent of the effective nucleon mass.

Since the nucleons in a newly born, hot neutron star are closer to being nondegenerate than degenerate, $\dot{\varepsilon}_{\rm brem} \propto m^{-5/2}$ will increase by a factor of ~ 6 if the effective nucleon mass is half its vacuum value, while $\dot{\varepsilon}_{\rm SF} \propto m^0$ does not change. Thus, if the effective nucleon mass is substantially smaller than its vacuum value, the numerical factor in Eq. (11) is closer to unity, implying that the bremsstrahlung process dominates the spin-flip scattering process.

Brinkmann and Turner [11] give a simple fit to $I(y_1,y_2)$ that is accurate to better than 25% for all values of y_1 and y_2:

$$I_{\rm fit}(y_1,y_2)^{-1} = 2.39\times 10^5 (e^{-y_1-y_2} + 0.25e^{-y_1} + 0.25e^{-y_2}) + 1.73\times 10^4 (1+|\bar{y}|)^{-1/2} + 6.92\times 10^4 (1+|\bar{y}|)^{-3/2} + 1.73\times 10^4 (1+|\bar{y}|)^{-5/2} , \tag{13}$$

where $\bar{y} \equiv (y_1+y_2)/2$.

If one is interested in producing helicity-flipped electron neutrinos, the URCA process can be very important ($n+p \to n+n+e^+ + \nu_+$ and $p+p \to n+p+e^+ + \nu_+$). (Note the process where an electron rather than a positron is produced in the final state is highly suppressed because of electron degeneracy: $\mu_e \sim 200$ MeV.) The matrix element for this process is four times larger than that for the neutron-neutron or proton-proton process. However, we are interested in the production of wrong-handed μ and τ neutrinos.

Finally, in the nondegenerate limit Grifols and Masso [13] have also calculated the volume emissivity due to the bremsstrahlung process. Our result in this limit is larger than theirs by about a factor of approximately 5. The difference traces to several factors. First, they did not take into account the exchange diagrams (about a factor of 2); second, they did not account for both wrong-helicity neutrino and antineutrino emission; and third, they did not take into account the *pp* bremsstrahlung process, which is important since during the early cooling phases $X_n \sim X_p \sim \frac{1}{2}$.

A. Neutrino degeneracy

In computing the rate for the bremsstrahlung process we have assumed that proper-helicity neutrinos are nondegenerate. In light of our discussion of the important effect of neutrino degeneracy upon the spin-flip-scattering process we should reexamine that assumption.

The effect here is far less pronounced. If the neutrino seas are degenerate, then there will be a significant blocking factor that suppresses the emission a proper-helicity neutrino, but not a proper-helicity antineutrino. In the nondegenerate limit the two processes, $N+N \to N+N+\nu_+ + \bar{\nu}_+$ and $N+N \to N+N+\nu_- + \bar{\nu}_-$, contribute equally to $\dot{\epsilon}_{\rm brem}$; in the highly degenerate limit only the first of these will contribute, the second being suppressed by the degenerate sea of ν_-'s. The net effect is a reduction of the volume emissivity by a factor of 2. However, in this limit the spin-flip-scattering process is enhanced so much that the bremsstrahlung process becomes subdominant and unimportant.

B. Pion-nucleon, neutrino-pair production

The conditions at the core of a neutron star are very close to those where a pion condensate should form. Because of this, the abundance of negative pions may well be comparable or greater than that of nucleons [14]. Needless to say, the abundance of pions in the core depends critically upon the equation of state. If the pion abundance is large, then the process $\pi^- + p \to n + \nu_+ + \bar{\nu}_+$ may also be important—in fact it may be dominant [15].

The matrix element squared for this process is

$$|\mathcal{M}|^2 = \frac{32 G_F^2 f^2 m^2 m_\nu^2 \omega_2}{m_\pi^2 \omega_1}[g_A^2(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}_1)(1-\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}_2)+0.5(1-0.5\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}_1-0.5\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}_2)]\ ,$$

where we have assumed that the nucleons are nonrelativistic, that the pions are relativistic, and summed over initial and final nucleon spins. The three-momenta of the pion, wrong-helicity neutrino, and proper-helicity antineutrino are $\mathbf{k}$, $\mathbf{q}_1$, and $\mathbf{q}_2$, respectively, and the energies of these particles are $\omega(=\omega_1+\omega_2)$, ω_1, and ω_2, respectively. The volume emissivity for this process is given by

$$\dot{\epsilon}_{\pi N} = 2\int \frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3k}{2\omega(2\pi)^3}\frac{d^3q_1}{2\omega_1(2\pi)^3}\frac{d^3q_2}{2\omega_2(2\pi)^3}(2\pi)^4|\mathcal{M}|^2\delta^4(p_1+k-p_2-q_1-q_2)f_1 f_\pi \omega_1$$

$$= \frac{5(g_A^2+0.5)f^2}{\pi^3}\frac{G_F^2 m_\nu^2 (m/m_\pi)^2}{m^2} T^3 X_p n_\pi n_N \tag{14a}$$

$$\simeq 2.1\times 10^{33} m_{100}^2 (m/0.94\,{\rm GeV})^{-2}(n_\pi/n_N) X_p \rho_{14}^2 T_{10}^3\ {\rm erg\,cm^{-3}\,sec^{-1}}\ , \tag{14b}$$

where we have assumed that nucleons are nondegenerate, that the pion phase-space distribution function $f_\pi = \exp(\alpha-\omega/T)$, and included a factor of 2 to account for both the process where the neutrino has the wrong helicity and the one where the antineutrino has the wrong helicity. Note that $\dot{\epsilon}_{\pi N} \propto m^{-2}$, so that it is a factor of 4 larger if the effective nucleon mass is half its vacuum value.

Now compare this production process to the bremsstrahlung process; taking the nondegenerate limit for $\dot{\epsilon}_{\rm brem}$, cf. Eq. (10b), we find

$$\frac{\dot{\epsilon}_{\pi N}}{\dot{\epsilon}_{\rm brem}} \simeq 140\left[\frac{n_\pi}{n_N}\right]\left[\frac{X_p}{f(X_p)}\right]T_{10}^{-1/2}\ . \tag{15}$$

Thus, if the number density of negative pions is comparable to that of nucleons, pion-nucleon, neutrino-pair production is even more important than the bremsstrahlung process.

C. Axions

In passing we note that it has been assumed that the dominant axion emission process for a hot young neutron is nucleon-nucleon, axion bremsstrahlung, $N+N \to N+N+a$ [3]. If negative pions are very abundant in the core of a hot neutron star, then pion-axion conversion, $\pi^- + p \to n + a$, the analogue of the process just discussed for neutrinos, can be the dominant axion emission process. We have computed the axion emissivity due to this process, making the same assumptions as above, and we find

$$\dot{\epsilon}_a = \frac{30 f^2 \bar{g}_{aN}^2 T^3}{\pi m^2 m_\pi^2} X_p n_\pi n_N \tag{16a}$$

$$\simeq 4.9\times 10^{49} X_p\left[\frac{n_\pi}{n_N}\right]\bar{g}_{aN}^2 \rho_{14}^2 T_{10}^3\ {\rm erg\,cm^{-3}\,sec^{-1}}\ , \tag{16b}$$

where $\bar{g}_{aN} \simeq 8\times 10^{-13}(m_a/10^{-5}\ {\rm eV})$ is a combination of axion-proton and axion-neutron couplings. The ratio of this process to the usual axion bremsstrahlung process is about $50(n_\pi/n_N)T_{10}^{-1/2}$; if the abundance of negative pions is comparable to nucleons this process will be the dominant one. If this is the case, then the upper limit to the axion mass derived from SN 1987A improves by almost an order of magnitude: from about 10^{-3} eV to almost 10^{-4} eV [16,17]. This result has important consequences: If the production of relic axions via axionic-string decay is the dominant process, then $\Omega_a \sim 1$ is achieved for $m_a \simeq 10^{-3}$ eV and $\Omega_a \gtrsim 1$ for $m_a \lesssim 10^{-3}$ eV. Improving the SN 1987 A limit to the axion mass to 10^{-4} eV would preclude the possibility that 10^{-3} eV axions provide the closure density and solve the dark matter problem, implying that the only axion solution to the dark-matter problem is 10^{-6} eV-10^{-4} eV axions produced by the misalignment mechanism [18].

IV. DISCUSSION

Within the context of the standard electroweak theory the "wrong-helicity" states of a Dirac neutrino can be produced by spin-flip processes. We have computed the emission of wrong-helicity neutrinos from a nascent neutron star due to the spin-flip neutrino-nucleon scattering and nucleon-nucleon, neutrino-pair bremsstrahlung, both for arbitrary nucleon degeneracy and nondegenerate neutrinos. The two processes are found to be of comparable importance at the core of a newly born hot neutron star. Relative to the volume emissivity used by Gandhi and Burrows [5] (corrected for their errant factor of 4) the total volume emissivity we calculate is larger by about a factor of 4, which should restore their original, very conservative mass limit of 14 keV.

We have also calculated the production of wrong-helicity neutrinos (and antineutrinos) due to the process $\pi^- + p \rightarrow n + \nu\bar{\nu}$, and find that if the number density of negative pions is comparable to that of nucleons (as is expected since the core of a neutron star is on the verge of pion condensation), this process dominates both spin-flip scatterings and bremsstrahlung by a large factor. Taking this process into account improves the Dirac-mass limit to of the order of a few keV. However, because the pion abundance is very sensitive to the equation of state, it is difficult to argue convincingly that such a bound is rigorous.

The most important effect that we have discussed is neutrino degeneracy. Electron neutrinos are certainly degenerate at the core of a neutron star (with chemical potential $\mu_e \sim 200$ MeV) [1]; a Dirac neutrino of mass greater than about 1 keV that mixes with the electron neutrino at the 1% level or larger will also be degenerate. [In general, $\sin^2\theta$ need only be greater than about 10^{-2} $(\mathrm{keV}/m_\nu)^4$ to ensure that the massive neutrino becomes degenerate.] The degeneracy of the massive Dirac species has the effect of increasing the emission of wrong-helicity neutrinos due to the spin-flip-scattering process by an enormous factor, of the order of $(\mu/T)^4 \sim 10^3$ relative to the nondegenerate rates previously used, which should improve the mass limit to around 1 keV. (Of course, the massive Dirac neutrino need not mix with the electron neutrino at all.)

A precise Dirac-neutrino mass limit based upon SN 1987A awaits incorporation of the effects discussed here into detailed numerical cooling models, work which is currently in progress [17]. However, it seems clear that the limit obtained will be more stringent than 10 keV, and if the massive Dirac neutrino mixes with the electron neutrino at the 1% level or more probably as stringent as 1 keV. Thus the early cooling of the neutron star associated with SN 1987A precludes a "pure" Dirac neutrino of mass 17 keV that mixes with the electron neutrino at the 1% level. This has important consequences for particle-physics models that attempt to incorporate a 17-keV neutrino; in particular it makes a strong case that if the 17-keV exists, it must be a "pseudo Dirac" neutrino [19].

Note added. The possibility that the rates for the bremsstrahlung process should be reduced by a modest factor due to a collective effect akin to the Landau-Pomeranchuk-Migdal effect has been raised very recently by G. G. Raffelt and D. Seckel, Phys. Rev. Lett. **67**, 2605 (1991).

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