



Light domain walls, massive neutrinos and the large scale structure of the Universe

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Abstract

We consider domain walls generated through a cosmological phase transition, which interact non-gravitationally with light neutrinos ($m_\nu \sim \mathcal{O}(1)eV$). At a redshift $z \geq 10^4$ the network grows rapidly and is virtually decoupled from the matter. As the friction with the matter becomes dominant, a comoving network scale close to that of the comoving horizon scale at $z \sim 10^4$ gets frozen in. During the later phases, the walls produce matter wakes of thickness $d \sim 10h^{-1}Mpc$, that may become seeds for the formation of the large scale structure ($30 - 130h^{-1}Mpc$) observed in the Universe.



The discoveries of supergalactic structures at a scale comparable to $100h^{-1}Mpc$ and of an intergalactic medium already ionized at $z \sim 5$ prompted new interest in models of galaxy formation alternative to the standard cold dark matter (CDM) scenario.^{1,2} As is well known, the CDM model finds it difficult to generate the very large clustering scales we observe, given the constraints from the microwave background. This model also predicts that most of the galaxy formation should take place at a redshift $z \leq 2$, and can hardly explain the production of big quantities of ionizing radiation at early times. An alternative class of models is based on the fact that cosmological phase transitions could give rise to highly concentrated pockets of energy, that may become seeds for baryonic infall and lead to galaxy and structure formation earlier than what the standard model postulates.

Cosmological phase transitions can result from the spontaneous breaking of a symmetry associated with “Higgs-like” scalar fields. The symmetry breaking is usually considered to take place at a temperature close to the electroweak or the grand-unification (GUT) energy scales. The phase transitions can give rise to topological defects. Depending on the choice of the scalar fields one may, in particular, generate 1-d or 2-d objects like “strings” or “domain walls”, formed of a “false vacuum” phase. As many studies show, these solitons can be associated with galactic and supergalactic scales and generate fluctuations in the distribution of matter. Typically, the defects move at relativistic speeds and create matter fluctuations by their gravitational action.³

In the first model dealing with cosmological domain walls, Zel’dovich et al.(1974)⁴ considered a phase transition taking place at the GUT scale. The main results of the paper were discouraging. The walls produced were far too heavy, giving rise to unphysically large distortions of the microwave background radiation. The idea was therefore abandoned until a possible mechanism was found to form domain walls at

a much lower transition temperature.⁵ It was found⁶ that the surface energy density σ of the walls is compatible with the constraints of the microwave background, by requiring $\sigma < 10MeV^3$, but still gravitationally significant, if $\sigma \sim 10MeV^3$. Numerical simulations^{7,8} showed in detail that the typical scale associated with the network is always close to that of the horizon, provided that the walls freely stretch under their surface tension.

In a recent publication,⁹ I discussed a variation of the previous domain wall models that could possibly generate both the large scale structure and give rise to a very early galaxy formation (at $z \sim 10 - 30$). A simple non-gravitational Lagrangian coupling was introduced between the scalar field associated with the walls and a component of the dark matter. We assumed that the domain walls gave rise to a symmetric potential barrier for the dark matter, resulting in elastic scattering.¹⁰ As a simplifying assumption, the barrier was taken to be high enough to reflect elastically all the incident particles, regardless of their impact energy. The result of such an interaction is to keep the velocity of the walls to very non relativistic values (typically $v \sim 10^{-2} - 10^{-3}c$). Therefore, the network just stretches with the universal expansion, the comoving scale of the system being frozen. Summarizing the results, for this to occur, $\Omega_{walls}/\Omega_{DM} \simeq 10^{-3}$. The walls, by pushing ahead the dark matter, create a void region behind them and a high density wake in front of them, whose thickness can be of order $d \sim 10h^{-1}Mpc$.

In the present paper, we modify the assumptions of the previous work. We consider the effects of a finite barrier height E_o for the particle reflection.¹⁰ We assume that particles hitting the domain wall with a relative kinetic energy $E \leq E_o$ are scattered, and for $E \geq E_o$ they pass through the barrier. As we show in ref.10, this is always a good approximation, in the limit $E_o \ll m$, where m is the mass of the dark matter particles. We are going to limit our calculations to such a limit. In considering the

dark matter as a fermion gas in thermal equilibrium, we will find a link between the comoving scale of the network \bar{r} and the mass m of the particles.

We begin by deriving the equation of motion of the walls, under the assumption that they move through a homogeneous medium formed by a fermion gas in thermal equilibrium. The action of the dark matter gives rise to a pressure P_f on the domain walls. Deriving P_f is the first step of our calculation.

If we define as the x -axis the normal to the moving surface, the momentum exchange between wall and incident particles is $\Delta p \simeq 2m\gamma(v-v_x)$ (notice that, for the interaction to occur, $(v-v_x) \ll 1$, since $E_o \ll m$), where v_x is the normal component of the particle thermal speed in the background frame and $\gamma \equiv (1-v^2)^{-1/2} \simeq (1-v_x^2)^{-1/2}$. The hit rate is given by $n(v-v_x)$, where n is the particle number density. It is convenient to introduce the dimensionless variables $y \equiv m\gamma v/T$ and $y_x \equiv p_x/T$ (where p_x is the x -momentum of the particles in the rest frame and T is the temperature).

We now introduce the thermal distribution $f(y_x)$ of the particles in the momentum component p_x (averaging over the other directions).⁹ This function is defined so that $\int_{-\infty}^{\infty} f(y_x) d(y_x) = 1$. Since the particles are fermions,

$$f(y_x) = \frac{1}{3\zeta(3)} \int_0^{\infty} \frac{y_{\perp} dy_{\perp}}{\exp\sqrt{y_x^2 + y_{\perp}^2} + 1} \quad (1)$$

where $y_{\perp} \equiv p_{\perp}/T$ (where $p_{\perp}^2 \equiv p^2 - p_x^2$ and p is the particle momentum). Recalling that the reflections only occur if $(v-v_x) \ll 1$, the pressure P_f is

$$P_f(y) = 2\gamma T^2 n/m \left[\int_{y-y_o}^y (y-y_x)^2 f(y_x) dy_x - \int_y^{y+y_o} (y-y_x)^2 f(y_x) dy_x \right], \quad (2)$$

where y_o is a limiting value simply related to E_o . In fact, in the rest frame of the wall, the maximum value of the incident particle momentum giving rise to reflection is approximately $p_o = \sqrt{2mE_o}$ (since $E_o \ll m$). By boosting such value back to the universal rest frame we find $y_o = \gamma\sqrt{2mE_o}/T$ to a good approximation.

The first integral in eq.(2) is the momentum exchanged per unit time and area between the wall and the particles in front of it, while the second integral refers to particles hitting the wall from the back. P_f can be rewritten, by changing the variable of integration, as

$$P_f = 2\gamma T^2 n/m \left[\int_0^{y_o} y_1^2 f(y - y_1) dy_1 - \int_0^{y_o} y_1^2 f(y + y_1) dy_1 \right], \quad (3)$$

by introducing $y_1 \equiv y - y_x$ in the first and $y_1 \equiv y_x - y$ in the second integral of eq.(2).

The range $\gamma\sqrt{2mE_o} \gg T$ ($y_o \gg 1$) corresponds to the high barrier limit, that we already calculated in ref.9. In this limit, P_f can be written as follows:

$$P_f = 12mnv^2 \quad mv \gg T, E_o > mv^2/2 \quad (4)$$

$$= \frac{3}{\pi} v T^4 \quad mv \ll T, E_o > mv^2/2 \quad (5)$$

$$= 0 \quad mv \gg T, E_o < mv^2/2, \quad (6)$$

where v is the physical speed of the wall through the medium.

We now want to study the limit $\gamma\sqrt{2mE_o} \ll T$ ($y_o \ll 1$), since we are interested in the behavior of the network at high z . It will turn out that the friction is ineffective at very early times, so that the network initially evolves only subject to its surface tension. Expanding f in power series around y , if $\gamma\sqrt{2mE_o}/T \ll 1$ we get

$$P_f = -4\gamma n T^2 f'(y)/m \left[\int_0^{y_o} y_1^3 dy_1 \right]. \quad (7)$$

The function $f'(y)$ is given by:

$$f'(y) = -\frac{1}{3\zeta(3)} \int_0^\infty \frac{\exp\sqrt{y^2 + y_1^2} y y_1 dy_1}{\sqrt{y^2 + y_1^2} [\exp\sqrt{y^2 + y_1^2} + 1]^2} = -\frac{1}{3\zeta(3)} \frac{y}{e^y + 1}. \quad (8)$$

Substituting eq.(8) into eq.(7) gives

$$P_f = -4\gamma n T^2 / m f'(y) \int_0^{y_o} y_1^3 dy_1 = \frac{2m^2 E_o^2 v}{\pi^2} \frac{\gamma^4}{e^y + 1}. \quad (9)$$

For $y > 1$ (recall that $y \equiv mv\gamma/T$), the pressure gets exponentially small. Physically, when $v \gg \bar{v}_{thermal} \gg (2E_o/m)^{1/2}$, the walls behave like virtually decoupled from the matter (see eq.9).

In the limit $y \ll 1$, the T -independence of eq.(9) comes from the fact that the number of particles reflected by the walls is constant during the expansion, while the momentum exchanged is $\Delta p_o \sim (E_o/m)^{1/2}$.

The equation of motion of the domain walls, describing an infinitesimal wall segment of curvature $1/\tilde{R}$ (in physical coordinates), is given by

$$\gamma^2 \dot{v} + 3 \frac{\dot{a}}{a} v + \frac{P_f}{\sigma \gamma} = -\frac{1}{\tilde{R}}, \quad (10)$$

where σ is the surface energy density of the kinks. The derivatives are taken respect to the universal time t . This equation is valid if the wall thickness $\Delta \ll \tilde{R}$, which becomes valid soon after the phase transition. Eq.(10) is just the relativistic generalization of Newton's second law divided by $\gamma\sigma$, where σ is the energy density of the walls. The damping term $3(\dot{a}/a)v$ derives from the universal expansion, while the r.h.s. is the tension that drives the motion of the walls.

It is useful to recall the results of the limit $P_f \rightarrow 0$, that has been already studied in several works.^{4,7,8} It was shown that the network reaches a scaling regime of growth, the scale of the system being given by $\bar{R} = 2\beta t^\alpha/3$, with $\beta = const. \sim \mathcal{O}(1)$, and $\alpha \simeq 1$. A very small percentage of the network reaches highly relativistic speeds, and the average physical speed is constant.

During the early stage of the wall evolution, there could be a phase during which $T \gg \sqrt{2mE_o}$ is satisfied. In this regime, $P_f \leq m^2 E_o^2 v / \pi^2$ for most of the network ($\gamma \sim 1$). The cosmological damping term, instead, is $3\dot{a}v/a \sim t^{-1}v$. Consequently, if the transition temperature $T_c \gg \sqrt{2mE_o}$ and $t_c^{-1} > m^2 E_o^2 / \pi^2$ (where t_c is the age

of the Universe at the phase transition), the initial phase of the network evolution is frictionless and rapid. Under additional conditions that we will discuss, the comoving scale of the network freezes in at the value it had at the time $t \sim (m^2 E_o^2 / \pi^2)^{-1}$.

Solving the general form of the equation of motion in the regime $T \gg \sqrt{2mE_o}$ is a difficult task, since the expression for P_f is quite involved. There is, however, a wide range of parameters such that eq.(10) can be simplified. Given the assumption that $\gamma m v < T$ we can write the approximate expression:

$$\gamma^2 \dot{v} + 3 \frac{\dot{a}}{a} v + \frac{m^2 E_o^2 v}{\pi^2 \sigma} = -\frac{1}{R}, \quad (11)$$

This equation can be studied analytically, provided that virtually all of the network obeys it. The linear dependence of P_f in v ensures a selfsimilar evolution throughout the process.

With the onset of the friction, the walls slow down, until they stop evolving and just stretch conformally with the expansion. If P_f becomes dominant at $z \sim 10^4$ or so for all of the network, the freeze in gives a comoving scale of the order $10 - 10^2 Mpc$, comparable to the horizon scale at that time.

Since the network evolves self-similarly, we can perform an averaging procedure to transform eq.(11) into an equation describing the evolution of the average "interwall" distance. Such a procedure is analogous to the one used when dealing with domain walls in condensed matter physics.¹¹

Multiplying eq.(11) by v and averaging over the surface of the network we get

$$\left(\frac{3\dot{a}}{a} + \frac{m^2 E_o^2}{\pi^2 \sigma} \right) \langle v^2 \rangle + \frac{1}{a} \langle \gamma^2 \dot{v} \cdot v \rangle = \frac{1}{a^2} \left\langle \frac{v}{R} \right\rangle.$$

During the period in which P_f is negligible, $\langle \gamma^2 \dot{v} \cdot v \rangle \simeq \langle \dot{v} \cdot v \rangle = d(\langle v^2 \rangle)/dt = 0$. When P_f dominates, since $\dot{v}/v \sim -\dot{a}/a$, we get $\langle \dot{v} \cdot v \rangle \simeq -\dot{a}/a \langle v^2 \rangle$. This term can be always safely neglected.

It is convenient to express the parameters in comoving coordinates ($\tilde{R} \equiv a\tilde{r}$ and $v \equiv a\dot{\tilde{r}}$). From simple geometry, it follows¹¹ that $\langle v/\tilde{R} \rangle = \langle \dot{\tilde{r}}/\tilde{r} \rangle = \tilde{r}^{-2}d(\tilde{r}^2)/dt$, where \tilde{r} is defined as the average comoving "interwall" distance, related to the average curvature of the walls by the relation $\beta/\tilde{r} \equiv \langle 1/\tilde{r}^2 \rangle^{1/2}$. Therefore we get,

$$\frac{d\tilde{r}^2}{dt} \left[3\frac{\dot{a}}{a} + \frac{m^2 E_o^2}{\sigma} \right] = \frac{\beta^2}{a^2}. \quad (12)$$

where $\beta = 3$ if there is one wall per horizon during the uncoupled phase. In what follows we will suppose that the dark matter is constituted by only one particle species. For an open Universe, the solution to eq.(12) can be written in the approximate form

$$\tilde{r} = 5h^{-1}\beta\sqrt{\frac{1}{K}}\ln^{1/2}(1 + Kt_{5f}/2)Mpc, \quad (13)$$

where we introduced $K \equiv (m/10eV)^2(E_o/10^{-4}eV)^2(\sigma/1MeV^3)^{-1}$ for convenience, since typically $K \sim 1$ (see fig.(1)). The reason for the choice of the normalization of K respect to E_o and σ will be clear soon. The quantity t_{5f} is the time of equivalence of matter and radiation: $t_{5f} \equiv (t_{equiv}/t(z = 10^5))$. After equivalence, the evolution of the network virtually stops. The values of the parameters involved indicate that the largest scales are obtained for small m . This is consistent with an open Universe, if the only forms of matter are neutrinos and baryons.

The assumptions we have made in calculating \tilde{r} are selfconsistent if the network remains always confined to a range of speeds where $P_f \sim v$. This is approximately true if the network remains confined to the region where P_f is not exponentially suppressed.¹² Assuming that the distribution of the curvatures is roughly Gaussian, all but a negligible fraction (1%) of the wall sections have a comoving curvature $\tilde{r}^{-1} < 2.5\tilde{r}^{-1}$, and the condition above translates to a bound on K , namely $K \leq 1.5(m/10eV)^2$. For $K > 1.5(m/10eV)^2$ our analytical approach is invalid, since part of the network is decoupled and part is strongly coupled. In this complicated regime, a numerical simulation would be needed to determine the evolution of the network.

Considerations about the later evolution of the network can help us in further constraining the parameters. Up to this point we have calculated the evolution of the walls at $T > (mE_o)^{1/2}$. As soon as the Universe cools down below $T \simeq (mE_o)^{1/2}$, eq.(4) holds. P_f decreases rapidly, due to its steep T dependence, and v starts increasing. Shortly, v becomes bigger than the thermal velocity of the neutrinos and the walls are in the regime described by eq.(5). During this phase, as shown preliminarily in ref.9, the walls deplete the volume they sweep of dark matter, generating a matter wake in front of them. The speed of the walls increases at a slower rate: $v \sim a$.

The details of the evolution through these different regimes are being investigated numerically.¹³ Some order of magnitude estimate is already possible. The walls decouple again as soon as $v > (2E_o/m)^{1/2}$. By that time (define it as t_d), the comoving distance the walls have swept is roughly $d/t_u \simeq v_d t_d / a_d = v_d = (2mE_o)^{1/2}$ (where t_u is the present age of the Universe, $v_d \equiv v(t_d)$ and $a_d \equiv a(t_d)$). If we consider $m = 1 - 10eV$ and $d = 5 - 20h^{-1}Mpc$, we get $E_o = 10^{-5.5} - 10^{-3.5}eV$. From the constraint on K , we also obtain $\sigma < (E_o/10^{-4}eV)/1.5MeV^3$ for our analytical approach to hold. The wall “re-decoupling” typically takes place at $a_d \sim \mathcal{O}(10^{-1})$. Since the network rapidly reaches relativistic speeds, we expect one or few walls to be within our present horizon. However, the wakes generated at $t < t_d$ should have gravitationally collapsed, in that involving also the baryon component. This collapse may be responsible for the formation of a first generation of protogalaxies at a z higher than the values predicted by the CDM model.

The distortions of the cosmic microwave background radiation (CMBR) generated by walls and by the dark component of the wakes are beyond the present capability of detection. We give a fairly detailed analysis of the problem in ref.(9), and therefore we limit the present discussion to a brief summary of the main results.

The biggest source of distortion associated with walls and neutrinos is the grav-

itational potential of the matter wakes generated by the wall motion. Because of the Reese-Sciama effect,¹⁴ these wakes give rise to a microwave background temperature distortion $\Delta T/T \sim G\rho d^3 \bar{R}^{-1/2} t_u^{-1/2} \sim 10^{-7}$. An additional effect is due to the domain walls themselves, through the same mechanism. Since today there should be roughly one stretched and relativistic infinite wall within our horizon (plus collapsing wall bubbles), we expect $\Delta T/T \sim G\sigma t_u \sim 10^{-6}$, for $\sigma \sim 1 \text{ Mev}^3$, as shown in previous work.⁶ Both the effects are lower than the current limit $\Delta T/T \simeq 10^{-5}$. The effects on the CMBR due to the baryons have not yet been treated. We expect that a major contribution to the distortions is due to the Zeldovich-Sunyaev effect,¹⁵ which is the scattering of low energy photons by free and hot electrons. This kind of CMBR distortion is typical of any model dealing with early formation of protogalactic objects.

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Figure caption

The comoving scale of the network \bar{r} in $h^{-1}Mpc$ units, as a function of a . The friction begins to slow down the network at $a \simeq 10^{-5}$. The curves refer to different values of the parameter K and the neutrino mass m . For each value of m we take the limiting value $K = 1.5(m/10eV)^2$. We also take $\beta = 3$ (see eq.(13)).

