Fermi National Accelerator Laboratory

FERMILAB-PUB-91/96-T
PURD-TH-91-01
April, 1991

Dynamical Symmetry Breaking and the Top Quark Mass in the Minimal Supersymmetric Standard Model

M. Carena†, T. E. Clark , C. E. M. Wagner†

Dept. of Physics, Purdue University, West Lafayette, IN 47907

W. A. Bardeen

Fermi National Accelerator Laboratory, Batavia, IL 60510

K. Sasaki

Dept. of Physics, Yokohama National University, Yokohama 240, Japan

Abstract

The minimal supersymmetric extension of the standard model is studied, when the electroweak symmetry breakdown relies on the formation of condensates involving the third generation of quarks and their supersymmetric partners. The top quark mass is obtained as a function of the compositeness scale Λ and the soft supersymmetry breaking scale, Δ_S. The dependence of the top quark mass on the ratio of the Higgs vacuum expectation values, as well as a function of the lightest Higgs mass, is analyzed. We show that, when $\Lambda \simeq 10^{16}$ GeV, the characteristic top quark mass in this model is $140GeV \leq m_t \leq 195GeV$, a prediction that is only slightly dependent on the value of Δ_S.

† Address after September 1, 1991 : Max Planck Institut, Föhringer Ring 6, D-8000 München 40, Germany.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

1 Introduction

The dynamical breakdown of symmetries[1] has played an important role in the physical understanding of the low energy behavior of quantum chromodynamics, and has been proposed as an alternative to the Higgs mechanism to explain the breakdown of the electroweak symmetry. Based on the present experimental lower bound on the top mass, it has been realized that the top quark may be sufficiently heavy as to induce the formation of a condensate, which catalyzes the electroweak symmetry breakdown at low energies [2]-[5]. A possible physical realization of this mechanism is given by the gauged Nambu-Jona-Lasinio model in which the dynamical fermions have effective four Fermi interactions at the compositeness scale Λ. The four Fermi coupling constant acquires a critical value, which separates the region in coupling constant space in which the chiral $SU(2)_L \times U(1)$ symmetry is broken from the one in which this symmetry is preserved. When the four Fermi coupling is tuned to its critical value, a composite, scalar Higgs multiplet appears at low energies as a new dynamical degree of freedom.

For values of $\Lambda >> M_Z$, this model can be viewed as a particular limiting case of the standard model. In fact, the top quark and Higgs masses obtained at low energies are those consistent with the renormalization group trajectories associated with the "triviality" bounds on these quantities[4]. On these trajectories, the renormalized Yukawa and scalar self couplings diverge at the compositeness scale Λ. Hence, the top quark mass values obtained within this model can be understood as upper bounds for a given effective cutoff scale Λ, at which, quite generally, new physics should appear. For a compositeness scale $\Lambda \simeq 10^{16}$ GeV, the top quark mass turns out to be $m_t \simeq 230 GeV$, a value that could be too large to be consistent with the experimental constraints coming from the ρ parameter measurement.

The gauged Nambu-Jona-Lasinio model described above requires an unnatural fine tuning of the four Fermi coupling constant in order to give sensible physical results. In fact, this is not a special feature of this model. In the standard model, due to the quadratic divergence of the scalar mass, a fine tuning of the Higgs mass parameter at a high energy scale Λ is required in order to get the proper electroweak symmetry breaking scale. Supersymmetry provides a possible solution to this problem. In a supersymmetric extension of the standard model the quadratic divergences disappear, and hence no fine tuning of the four Fermi coupling constants, or in general of the Higgs mass parameters, is required [6]. As has been pointed out in Ref. [7], a potential problem for the supersymmetric composite-Higgs model is that a large four Fermi coupling, of the order of the inverse of the supersymmetry breaking scale squared, could induce unitarity violations at scales which are large compared to the supersymmetry breaking scale but still small compared to the compositeness scale Λ. A definitive answer to this question may necessarily come from a complete nonperturbative analysis of the supersymmetric model. However, the bubble sum approximation provides a solution to this problem in the direct four fermion channels, while supersymmetry may yet protect the model in other multiparticle channels. In other words, from the renormalization group point of view, at energies lower than the compositeness scale this model is nothing but a particular limiting case of the minimal supersymmetric standard model, hence no unitarity violations will appear in the low energy theory.

The minimal supersymmetric extension of the composite-Higgs model was first studied in Ref.[6], in a simplified version in which only one of the scalar Higgs doublets acquires a vacuum expectation value, and hence, the bottom quark remains massless. This supersymmetric composite-Higgs model is obtained, in analogy to the standard one, for those values of the low energy parameters which are consistent with the renormalization group trajectories associated with the triviality bounds on the top quark mass. As is already well known, if the top Yukawa coupling becomes large at a high energy scale Λ, the top quark mass is governed by an infrared quasi-fixed point [8], which determines the low energy top Yukawa coupling as a function of the low energy QCD gauge coupling constant. In the supersymmetric model, the infrared quasi-fixed point value of the top Yukawa coupling is lower than in the case of the standard model. Consequently, the top quark mass values obtained in the supersymmetric composite-Higgs model studied in Ref.[6] are lower than those obtained, for the same compositeness scale, in its standard version.

In this article, we shall analyze the minimal supersymmetric extension of the standard model with dynamical symmetry breaking in further detail. We will study the effects of considering nontrivial vacuum expectation values for the two scalar Higgs doublets, which give masses to the upper and lower quark and lepton isospin components, respectively. Since both neutral Higgs particles give contributions to the Z^o mass, their individual vacuum expectation values are lower than that of the standard model Higgs particle. Thus, the top quark mass values can be significantly lower than those obtained in Ref.[6]. In addition, we will show that, the value of the lightest neutral Higgs mass, within the supersymmetric top-condensate model, strongly depends on the value of the supersymmetry breaking scale. This may seem surprising, since the lightest Higgs scalar mass obtained using the tree level supersymmetric potential with soft supersymmetry breaking terms is only weakly dependent on the supersymmetry breaking scale. However, for the large top Yukawa couplings consistent with the supersymmetric infrared fixed point, large radiative corrections to the tree level predictions are induced[9]. An important consequence of these radiative effects is to invalidate previous phenomenological constraints on the ratio R of the Higgs vacuum expectation values, and hence, as will be discussed in section 2, lower values for the top quark mass are allowed within the model under study. Moreover, we will analyze the modifications to the top quark mass predictions induced by the appearance of a finite bottom quark Yukawa coupling. In section 3, we will compute the complete Higgs mass spectrum as a function of the radiatively corrected quartic scalar couplings and the explicit scalar mass terms. The top quark mass value will be computed, as a function of the lightest Higgs mass for different values of the compositeness scale Λ and the tree level CP-odd Higgs mass, and for a supersymmetry breaking scale of the order of 1 TeV.

2 Renormalization Group Flow of the Low Energy Parameters.

To describe the dynamics responsible for the top quark multiplet condensation, we shall consider an $SU(3) \times SU(2) \times U(1)$ invariant gauged supersymmetric Nambu-Jona-Lasinio model[6], [10]-[12], with explicit soft supersymmetry breaking terms. In this simplified model, we shall first ignore all quark and lepton Yukawa couplings besides the one associated with the top quark, since they are unessential for the qualitative description of the phenomena under study. These additional interactions will be introduced below, while analyzing the complete supersymmetric standard model.

Written in terms of the two composite chiral Higgs superfields H_1 and H_2, the action of the gauged Nambu-Jona-Lasinio model at the scale Λ takes the form

$$
\begin{aligned}
\Gamma_\Lambda &= \Gamma_{YM} + \int dV \left[\bar{Q}e^{2V_Q}Q + T^C e^{-2V_T}\bar{T}^C + B^C e^{-2V_B}\bar{B}^C\right](1-\Delta^2\theta^2\bar{\theta}^2) \\
&+ \int dV \bar{H}_1 e^{2V_{H_1}} H_1(1-M_H^2\theta^2\bar{\theta}^2) - \int dS \epsilon_{ij}\left(m_0 H_1^i H_2^j(1+B_0\theta^2) - g_{T_0} H_2^j Q^i T^C(1+A_0\theta^2)\right) \\
&- \int d\bar{S}\epsilon_{ij}\left(m_0 \bar{H}_1^i \bar{H}_2^j(1+B_0\bar{\theta}^2) - g_{T_0}\bar{T}^C\bar{Q}^i\bar{H}_2^j(1+A_0\bar{\theta}^2)\right), \qquad (1)
\end{aligned}
$$

where $Q = \binom{T}{B}$ is the SU(2) doublet of top and bottom quark chiral superfield multiplets, T^C (B^C) is the SU(2) singlet charge conjugate top (bottom) quark chiral multiplet, and we have denoted the superspace integration measures $dV = d^4x d\theta^2 d\bar{\theta}^2$, $dS = d^4x d\theta^2$ and $d\bar{S} = d^4x d\bar{\theta}^2$ [13]. An equivalent form of the above action, only in terms of the fundamental quark chiral superfields, can be obtained by integrating out the static composite chiral superfields, or equivalently, by substituting in Eq.(1) the fields H_1 and H_2 in terms of their Euler-Lagrange equations. Γ_{YM} includes the usual supersymmetric gauge field kinetic and the supersymmetry breaking gaugino mass terms, while the quark and Higgs multiplets interact with the SU(3)×SU(2)×U(1) gauge fields via

$$
\begin{aligned}
V_Q &= g_3 G^a \frac{1}{2}\lambda^a + g_2 W^i \frac{1}{2}\sigma^i + \frac{1}{6}g_1 Y, \qquad V_T = g_3 G^a \frac{1}{2}\lambda^a + \frac{2}{3}g_1 Y \\
V_B &= g_3 G^a \frac{1}{2}\lambda^a - \frac{1}{3}g_1 Y, \qquad V_{H_1} = \frac{g_2}{2}W^i\sigma^i - \frac{1}{2}g_1 Y. \qquad (2)
\end{aligned}
$$

Generalizing the model of Ref.[6], we have included two soft supersymmetry breaking terms A_0 and B_0 which are proportional to the scalar trilinear and bilinear terms appearing in the superpotential. The gauged Nambu-Jona-Lasinio model depends only on $\delta = A_0 - B_0$, as can be easily verified by integrating out the chiral Higgs superfields. As will be discussed in section 3, the inclusion of the δ induced terms in the low energy theory is essential in order to obtain nontrivial vacuum expectation values for both neutral scalar Higgs particles without inducing an unacceptably light axion[14]. As in Ref.[6], Δ^2 and M_H^2 provide explicit soft supersymmetry breaking scalar mass terms.

It follows from the H_2 Euler-Lagrange equations that the scalar component of the H_1 chiral superfield and the quark superfields are related by

$$m_0 H_1 = g_{T_0}\tilde{Q}\tilde{T}^C, \qquad (3)$$

where we have denoted by $\tilde{Q}$ ($\tilde{T}^C$) the scalar component of the chiral quark superfield Q (T^C). Since the soft supersymmetry breaking terms are thought to arise from an underlying supergravity [15], it is reasonable to assume that the higher dimension composite fields H_1 feel twice the breaking strength as do the individual $\tilde{Q}$ or $\tilde{T}^C$ fields. It is straightforward to prove that this is achieved when the H_1-explicit supersymmetry breaking mass term is given by $M_H^2 = 2\Delta^2 + \delta^2$.

In the presence of a condensate of top quark superfields, a dynamical mass for the top quark is generated. Its value may be determined in a self consistent way by using the Schwinger-Dyson equations in the bubble approximation. The Schwinger-Dyson equations for the top quark mass are depicted in Figure 1. Note that, generalizing the expression for the superfield propagators derived in Ref.[16], there is a left-right scalar quark propagator induced by the inclusion of the soft supersymmetry breaking term δ. For a nontrivial solution of the Schwinger-Dyson equations, the gap equation

$$G^{-1} = \frac{N_C \Delta^2}{16\pi^2} \left[\left(1 + \frac{2m_t^2 + \delta^2 \alpha}{2\Delta^2}\right) \ln\left(\frac{\Lambda^4}{\left(m_t^2 + \Delta^2\right)^2 - m_{QQ^C}^4}\right) - \frac{2m_t^2}{\Delta^2} \ln\left(\frac{\Lambda^2}{m_t^2}\right)\right]. \tag{4}$$

must be fulfilled, where $G = g_{T_0}^2/m_0^2$ and $\alpha = m_{QQ^C}^2/(\delta m_t)$ is given by

$$\alpha^{-1} = 1 + \frac{G M_H^2 N_C}{32\pi^2} \ln\left(\frac{\Lambda^4}{\left(m_t^2 + \Delta^2\right)^2 - m_{QQ^C}^4}\right). \tag{5}$$

The logarithmic term in Eq.(5) comes from the interactions induced by the explicit scalar supersymmetry breaking mass term associated with the scalar field H_1. The gap equation takes a simpler form in the case in which this explicit mass term vanishes. In general, however, the critical value of the four Fermi coupling G may be obtained by solving the above two coupled equations. The logarithmic dependence on the compositeness scale Λ is a direct consequence of the supersymmetric nonrenormalization theorems [17]. The usual quadratic dependence on Λ, appearing in the standard top-condensate models has been replaced by a mild quadratic dependence on the supersymmetry breaking scales Δ and δ. Thus, as we have already mentioned, no fine tuning is necessary in this model.

In the scaling region, in which the four Fermi coupling constant is close to its critical value, a gauge invariant kinetic term for H_2 is induced at low energies. In the large N_C limit, it may be obtained by computing the contributions depicted in Fig. 2, and is given by [6]

$$Z_{H_2} \int dV\, H_2 e^{2V_{H_2}} \bar{H}_2 (1 + A_0\theta^2 + A_0\bar{\theta}^2 + (2\Delta^2 + A_0^2)\theta^2\bar{\theta}^2), \tag{6}$$

where Z_{H_2} is the H_2 wavefunction renormalization constant, which at a normalization scale μ is given by

$$Z_{H_2} = \frac{g_{T_0}^2 N_C}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \tag{7}$$

and $V_{H_2} = -V_{H_1}$. When μ approaches Λ, Z_{H_2} tends to zero and H_2 has no independent dynamics. For energies much lower than the compositeness scale Λ, instead, H_2 appears as an independent dynamical degree of freedom. Rescaling the field $H_2 \to H_2(1 - A_0\theta^2)/\sqrt{Z_{H_2}}$, so that it has a canonically normalized kinetic term, the low energy model is given by

$$\begin{aligned}
\Gamma_Z \quad &= \Gamma_{YM} + \int dV \left[\bar{Q} e^{2V_Q} Q + T^C e^{-2V_T} \bar{T}^C + B^C e^{-2V_B} \bar{B}^C\right](1 - \Delta^2\theta^2\bar{\theta}^2) \\
+ \quad &\int dV\, \bar{H}_1 e^{2V_{H_1}} H_1 (1 - M_H^2 \theta^2\bar{\theta}^2) - \int dS \epsilon_{ij} \left(m H_1^i H_2^j (1 + \delta\theta^2) - h_t H_2^j Q^i T^C\right) \\
- \quad &\int d\bar{S} \epsilon_{ij} \left(m \bar{H}_1^i \bar{H}_2^j (1 + \delta\bar{\theta}^2) - h_t \bar{T}^C \bar{Q}^i \bar{H}_2^j\right) + \int dV\, \bar{H}_2 e^{2V_{H_2}} H_2 (1 + 2\Delta^2\theta^2\bar{\theta}^2).
\end{aligned} \tag{8}$$

where we have defined the renormalized mass, $m = m_0/\sqrt{Z_{H_2}}$, and Yukawa coupling, $h_t = g_{T_0}/\sqrt{Z_{H_2}}$. Observe that, since m_0 and g_{T_0} have finite values, these renormalized couplings diverge at the scale Λ. Once H_2 is canonically normalized, the effective supersymmetry breaking terms proportional to the bilinear and trilinear terms of the superpotential are $B = \delta$ and $A = 0$, respectively. The negative value of the induced mass parameter for the scalar Higgs H_2 may generate the electroweak symmetry breakdown in the low energy effective theory, even for the case B=0 [15]. However, as mentioned above, a nonvanishing B is necessary in order to induce a nontrivial vacuum expectation value for the scalar Higgs H_1, and therefore give masses to the bottom quarks and leptons of the theory.

The corrections induced by the inclusion of the gauge couplings may be obtained by going beyond the bubble sum approximation. The qualitative features associated with the composite-Higgs scenario, however, are properly described by the leading order in $1/N_C$ results presented above. Instead of computing gauge fields corrections and higher order in $1/N_C$ effects, in order to obtain physical information at low energies it proves convenient to work with the full renormalization group equations of the supersymmetric standard model [4],[6]. The effect of the interactions ignored in the above discussion can be obtained by analyzing the modifications to the renormalization group trajectories consistent with the compositeness condition, $Z_{H_2}(\mu = \Lambda) = 0$, which are induced by their inclusion in the low energy theory. Concomitantly, when studying the scalar sector of the theory, we will not restrict ourselves to the particular assignments for the scalar mass parameter which were obtained in Eq.(8). It is important to remark, that although the cancellation of the supersymmetry breaking term $A(\mu)$ at all scales is only a property of the bubble sum approximation, the relation $A(\mu)|_{\mu\to\Lambda} = 0$ is a prediction of the model.

The top quark mass value is given by $m_t = h_t(m_t)v_2$, where v_i is the vacuum expectation value of the scalar Higgs H_i. The low energy value of the top quark Yukawa coupling can be obtained by using the renormalization group flows in which h_t becomes large at the compositeness scale Λ. The relevant renormalization group equations in the minimal supersymmetric model are given by[19]

$$\begin{aligned}
\frac{d\alpha_3}{dt} &= 3\frac{\alpha_3^2}{4\pi} \\
\frac{d\alpha_2}{dt} &= -\frac{\alpha_2^2}{4\pi} \\
\frac{d\alpha_1}{dt} &= -11\frac{\alpha_1^2}{4\pi} \\
\frac{dY_t}{dt} &= Y_t\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{9}\tilde{\alpha}_1 - 6Y_t - Y_b\right) \\
\frac{dY_b}{dt} &= Y_b\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{9}\tilde{\alpha}_1 - 6Y_b - Y_t\right)
\end{aligned} \tag{9}$$

where $\alpha_i = g_i^2/4\pi$, $\tilde{\alpha}_i = \alpha_i/4\pi$, $Y_b = (h_b/4\pi)^2$, $Y_t = (h_t/4\pi)^2$ and $t = \log(\Lambda/\mu)^2$. The solution to these equations provides the renormalization group flow for energy scales $\Delta_S \leq \mu \leq \Lambda$. In general, if supersymmetry is broken at an energy scale Δ_S larger than the electroweak scale, the low energy effective theory is equivalent to the Standard Model with one or two light Higgs doublets, depending on the value of the mass parameters appearing in the scalar potential. Hence, at scales below Δ_S, the proper renormalization group flow is described by the solutions to the standard model renormalization group equations, which are given by[8]

$$\begin{aligned}
\frac{d\alpha_3}{dt} &= 7\frac{\alpha_3^2}{4\pi} \\
\frac{d\alpha_2}{dt} &= \beta_2\frac{\alpha_2^2}{4\pi} \\
\frac{d\alpha_1}{dt} &= -\beta_1\frac{\alpha_1^2}{4\pi}
\end{aligned}$$

$$
\begin{aligned}
\frac{dY_t}{dt} &= Y_t\left(\frac{24}{3}\tilde{\alpha}_3 + \frac{9}{4}\tilde{\alpha}_2 + \frac{17}{12}\tilde{\alpha}_1 - \frac{9}{2}Y_t - \frac{\alpha_b}{2}Y_b\right) \\
\frac{dY_b}{dt} &= Y_b\left(\frac{24}{3}\tilde{\alpha}_3 + \frac{9}{4}\tilde{\alpha}_2 + \frac{5}{12}\tilde{\alpha}_1 - \frac{9}{2}Y_b - \frac{\alpha_t}{2}Y_t\right)
\end{aligned} \tag{10}
$$

where $\beta_2 = 3(19/6)$, $\beta_1 = 7(41/6)$, $\alpha_b = 1(3)$, $\alpha_t = 1(3)$ if there are two (one) light scalar Higgs doublets. In the case in which only one light scalar Higgs doublet ϕ remains at low energies, it will be given by a combination of the original Higgs doublets H_1 and H_2 [9]

$$
\phi = H_1\cos(\theta_M) + i\tau_2 H_2^* \sin(\theta_M) \tag{11}
$$

where θ_M is the mixing angle. As we will show in the next section, when there is only one light Higgs doublet below Δ_S the mixing angle $\tan(\theta_M) = R$, where $R = v_2/v_1$ is the ratio of the Higgs vacuum expectation values. Hence $<\phi>^T = (v, 0)$, where $v = \sqrt{v_1^2 + v_2^2}$. The top and bottom Yukawa couplings appearing in the renormalization group equation (10) are the effective couplings of the doublet ϕ with the top and bottom quarks. These are related to the supersymmetric Yukawa couplings at the scale Δ_S by $h_b^{eff} = h_b\cos(\theta_M)$ and $h_t^{eff} = h_t\sin(\theta_M)$.

For a compositeness scale $\Lambda >> M_Z$ and a supersymmetry breaking scale which is of the order of the weak scale, an estimate of the top quark mass may be obtained by using the supersymmetric infrared quasi-fixed point, $h_t(M_Z) \simeq \sqrt{\frac{8}{9}}g_3(M_Z)$. This yields a top quark mass approximately equal to $m_t \simeq 196 GeV R/\sqrt{1+R^2}$. Consequently, if the ratio $R \simeq 1$, the top quark mass can be significantly lower than the values obtained in the model of Ref.[6].

A more accurate estimate of the top quark mass can be obtained by numerical integration of the renormalization group equations, Eqs.(9)-(10). In Fig.3, we show the results obtained for the top quark mass as a function of the ratio R for three different values of the compositeness scale Λ and a supersymmetry breaking scale $\Delta_S = 1 TeV$. In the numerical work, we impose the compositeness condition on the top quark Yukawa coupling $Y_t(\Lambda)^{-1} = 0$. The boundary conditions for the gauge couplings are chosen to be $\alpha_3(M_Z) = 0.115$, $\alpha_2(M_Z) = 0.0336$ and $\alpha_1(M_Z) = 0.0102$, which are consistent with present experimental constraints[18]. The low energy bottom Yukawa coupling was fixed by requiring the bottom mass to be consistent with its experimental value $m_b \simeq 5 GeV$. The values of the gauge and Yukawa couplings at a given energy scale μ are obtained by integrating the renormalization group equations, asking for continuity at the supersymmetry breaking scale Δ_S. The perturbative one loop renormalization group equations may not be reliably used to determine the evolution of the Yukawa couplings at energy scales μ close to the compositeness scale Λ. However, as we have already mentioned in the introduction, the action of the infrared quasi-fixed point makes the top quark mass predictions very insensitive to the precise high value of the top quark Yukawa coupling at the scale Λ. A slight variation, of less than 1% (2%), of the top quark mass value is obtained by setting $Y_t(\Lambda) = 0.1$, for a compositeness scale $\Lambda \geq 10^{16}$ GeV ($\Lambda \geq 10^{10}$ GeV).

As it is apparent from Figure 3, the minimal value of the top quark mass is obtained for the lowest value of R. In general, the top quark mass values are insensitive to whether only one or two light Higgs doublets appear in the spectrum. However, for low values of R, the top quark mass predictions obtained if there are two light Higgs doublets are slightly lower than those obtained for the one light Higgs doublet case. As we will show in the next section, in the one light Higgs doublet case the ratio R is bounded to be larger than one. In the two light Higgs doublets case, although for characteristic values of the low energy parameters $R \geq 1$, R could be slightly lower than one.

The top quark mass has the same qualitative behavior for the different values of Λ and Δ_S. Since $v_2(R) = v_2(R=1)R\sqrt{2/(1+R^2)}$, the top quark mass is expected to increase with R, tending to a constant value for large values of R. Such behavior is actually observed for low and intermediate values of R. However, since $m_b = h_b v_1$, the bottom Yukawa coupling depends on R as follows

$$h_b = h_b(R=1)\sqrt{(1+R^2)/2}. \tag{12}$$

Thus, for larger values of R, the bottom Yukawa coupling becomes larger and the infrared quasi-fixed point is reached for lower values of the top Yukawa coupling. In addition, since v_2 varies only slightly with R in the large R regime, the top quark mass decreases with R, a behavior that is clearly seen in Fig.3. If R becomes too large ($R \geq 36$ for $\Lambda \simeq 10^{16} GeV$), h_b becomes larger than the top Yukawa coupling. In our computations we have set an upper bound on R by requiring the top Yukawa coupling to be larger than h_b.

In Figure 4, the top quark mass as a function of the ratio R is depicted, for three different values of the supersymmetry breaking scale Δ_S. An important result of these computations is that, if the compositeness scale $\Lambda \simeq 10^{16} GeV (10^{10} GeV)$, and $\Delta_S \simeq 1 TeV$, the characteristic top quark mass is 140 GeV $< m_t <$ 195 GeV, (160 GeV $< m_t <$ 220 GeV). Furthermore, the top quark mass results obtained for $\Delta_S = 100$ GeV are very similar to the ones obtained for $\Delta_S = 1$ TeV. For Δ_S as large as 10 TeV, the top quark mass is shifted slightly towards larger values. Therefore, as may be seen from Figure 4, the low energy predictions for the top quark mass obtained in our analysis are stable under variations of Δ_S.

3 Scalar Higgs spectrum and the top quark mass

In recent years, there has been considerable effort to derive low energy supersymmetry from supergravity or superstring inspired models [19],[20]. In particular, it has been realized that there is a finite set of soft supersymmetry breaking parameters which can be included in the theory without spoiling the cancellation of quadratic divergences. These parameters are the gaugino masses, an explicit mass parameter for the scalar components of the chiral superfields, a scalar trilinear coupling A proportional to the Yukawa dependent part of the superpotential, and a bilinear coupling B proportional to the Higgs-bilinear term in the superpotential (see section 2). The values of these parameters must be adjusted at a high energy scale Λ in order to obtain a sensible low energy spectrum. In addition, the value of the mass parameter m appearing in the superpotential should be fixed in order to define a particular low energy model. There is a wide range of possible values for m and the soft supersymmetry breaking parameters at $\mu = \Lambda$, that lead to low energy predictions which are in agreement with both experimental and theoretical constraints[21]. The values of the mass parameters appearing in the scalar potential depend on the particular choice of the supersymmetry breaking scheme. The compositeness condition imposes the cancellation of the trilinear coupling $A(\Lambda)$, together with a relation between the supersymmetry breaking mass terms for the squarks and the Higgs field H_2 at the scale Λ. However, it imposes no constraints on the gaugino masses, the bilinear term B, the low energy value of the supersymmetric Higgs mass m and the overall scale of the Higgs and squarks supersymmetry breaking mass terms. Therefore, it proves convenient to get a general picture, independent of the boundary conditions at the compositeness scale Λ. For any sensible set of boundary conditions, the supersymmetry breaking scale Δ_S cannot be much larger than 1 TeV without requiring a fine tuning, which would spoil one of the relevant properties of the supersymmetric models. Hence, we will be particularly interested in studying models with $\Delta_S \leq 10TeV$.

At the supersymmetry breaking scale Δ_S, the low energy Higgs potential reads

$$V_{eff} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 \left(H_1^T i\tau_2 H_2 + h.c.\right) \\ +\frac{1}{8}\left(g_2^2 + g_1^2\right)\left(H_2^\dagger H_2 - H_1^\dagger H_1\right)^2 + \frac{1}{2} g_2^2 \left|H_2^\dagger H_1\right|^2 \tag{13}$$

where we have defined $m_3^2 = Bm$, and m_1^2 and m_2^2 are generic mass parameters. An interesting aspect of the minimal model under consideration is that it predicts that the lightest tree level Higgs mass is bounded to be below the Z^0 mass. However, for large values of the top and bottom Yukawa couplings, large radiative corrections to the tree level potential are induced[9]. At energies below the supersymmetry breaking scale the potential is given by the general expression

$$V_{eff} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 - m_3^2 \left(H_1^T i\tau_2 H_2 + h.c.\right) \\ +\frac{\lambda_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{\lambda_2}{2}\left(H_2^\dagger H_2\right)^2 + \lambda_3\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4\left|H_2^\dagger i\tau_2 H_1^*\right|^2 \tag{14}$$

where we have chosen the same notation as in ref.[8]. Observe that the appearence of other quartic couplings than the ones given in Eq.(14) is protected by either discrete symmetries ($H_2 \to -H_2$, $H_1 \to -H_1$) or a global PQ symmetry. In particular a quartic term $\lambda_5[(H_2^T i\tau_2 H_1)^2 + h.c.]$ breaks the PQ symmetry. The PQ symmetry is only broken by the mass term m_3^2 in the scalar potential, Eqs. (13) and (14). Hence, λ_5 will not receive any leading logarithmic contribution and will not appear in our renormalization group analysis. The minimization conditions for the above potential give

$$\sin(2\theta) = \frac{2m_3^2}{(m_1^2 + m_2^2) + \lambda_1 v_1^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4)v^2}, \tag{15}$$

while

$$R^2 = \frac{m_1^2 + \lambda_1 v_1^2}{m_2^2 + \lambda_2 v_2^2} = \frac{m_1^2 + \lambda_2 v^2 + (\lambda_1 - \lambda_2)v_1^2}{m_2^2 + \lambda_2 v^2} \tag{16}$$

where we have defined $\tan\theta = R$.

The value of the quartic couplings at a low energy scale μ may be computed through the renormalization group flows obtained by solving the corresponding renormalization group equations[8]

$$\begin{aligned}
16\pi^2 \frac{d\lambda_1}{dt} &= -6\left[\lambda_1^2 + (4\pi)^2\lambda_1(Y_b - \tilde{\alpha}_1/4 - 3\tilde{\alpha}_2/4) + (4\pi)^4(\tilde{\alpha}_1^2/16 + \tilde{\alpha}_1\tilde{\alpha}_2/8\right. \\
&+ \left. 3\tilde{\alpha}_2^2/16 - Y_b^2)\right] - 2\lambda_3^2 - 2\lambda_3\lambda_4 - \lambda_4^2 \\
16\pi^2 \frac{d\lambda_2}{dt} &= -6\left[\lambda_2^2 + (4\pi)^2\lambda_2(Y_t - \tilde{\alpha}_1/4 - 3\tilde{\alpha}_2/4) + (4\pi)^4(\tilde{\alpha}_1^2/16\right. \\
&+ \left. \tilde{\alpha}_1\tilde{\alpha}_2/8 + 3\tilde{\alpha}_2^2/16 - Y_t^2)\right] - 2\lambda_3^2 - 2\lambda_3\lambda_4 - \lambda_4^2 \\
-32\pi^2 \frac{d\lambda_3}{dt} &= (\lambda_2 + \lambda_1)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + \lambda_3(4\pi)^2(-3\tilde{\alpha}_1 - 9\tilde{\alpha}_2 \\
&+ 6Y_t + 6Y_b) + (4\pi)^4(9\tilde{\alpha}_2^2/4 + 3\tilde{\alpha}_1^2/4 - 3\tilde{\alpha}_1\tilde{\alpha}_2/2 + 12Y_tY_b) \\
-32\pi^2 \frac{d\lambda_4}{dt} &= \lambda_4(2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 4\lambda_4) + 3\lambda_4(4\pi)^2(-3\tilde{\alpha}_2 \\
&- \tilde{\alpha}_1 + 2Y_t + 2Y_b) + 3(4\pi)^4(\tilde{\alpha}_1\tilde{\alpha}_2 + 4Y_tY_b)
\end{aligned} \tag{17}$$

Contrary to what happens in the standard model, the tree level Higgs masses do not depend on the compositeness scale Λ. In the minimal supersymmetric model, this is a result of the quartic Higgs couplings being proportional to the electroweak gauge couplings, whose low energy values are fixed by experimental constraints. In fact, from eqs.(13)-(14), it follows that at the supersymmetry breaking scale the quartic couplings must fulfill the boundary conditions

$$\begin{aligned}
\lambda_1(\Delta_S) = \lambda_2(\Delta_S) &= \frac{g_1^2 + g_2^2}{4} \\
\lambda_3(\Delta_S) = \frac{g_2^2 - g_1^2}{4}, &\quad \lambda_4(\Delta_S) = -\frac{g_2^2}{2},
\end{aligned} \tag{18}$$

where we have used the relation

$$\left(H_2^\dagger H_2\right)\left(H_1^\dagger H_1\right) = \left|H_2^\dagger H_1\right|^2 + \left|H_2^\dagger i\tau_2 H_1^*\right|^2 . \tag{19}$$

It is easy to prove that, for $\Delta_S \leq 10TeV$, the stability conditions $\lambda_1 > 0$, $\lambda_2 > 0$,

$$\begin{aligned}
\sqrt{\lambda_1\lambda_2} &> -\lambda_3 + |\lambda_4| \qquad if\ \lambda_4 < 0 \\
\sqrt{\lambda_1\lambda_2} &> -\lambda_3 \qquad if\ \lambda_4 > 0
\end{aligned} \tag{20}$$

are always verified within this model. Hence, the Higgs scalar potential is stable.

In the absence of radiative corrections, M^2, defined as $m_1^2 + m_2^2 = 2M^2$, must be greater than $|m_3^2|$ to assure the stability of the potential along the direction $|H_1| = |H_2|$. As we have shown above, radiative corrections stabilize the potential even when this relation is not fulfilled. In the following, we will only assume that the mass parameter $M^2 > 0$. If M is of the order of the weak scale, two light Higgs doublets appear in the low energy spectrum. There are two neutral CP-even scalar states with masses given by

$$\begin{aligned} m_{H,h}^2 &= \frac{1}{2}\Big[2M^2 + 3\lambda_2 v_2^2 + 3\lambda_1 v_1^2 + (\lambda_3 + \lambda_4)v^2 \\ &\pm \sqrt{(-m_A^2 \cos(2\theta) + 2\lambda_1 v_1^2 - 2\lambda_2 v_2^2)^2 + (m_A^2 - 2(\lambda_3 + \lambda_4)v^2)^2 \sin^2(2\theta)}\Big] \end{aligned} \tag{21}$$

where m_A is the mass of the neutral CP-odd scalar state

$$m_A^2 = 2M^2 + \lambda_2 v_2^2 + \lambda_1 v_1^2 + (\lambda_3 + \lambda_4)v^2. \tag{22}$$

Observe that a vanishing value of m_3^2 would imply an unacceptable massless axion in the physical spectrum of the theory. Finally,

$$m_{ch}^2 = m_A^2 - \lambda_4 v^2 \tag{23}$$

is the squared mass of the charged Higgs eigenstate. The scalar masses may be computed by using the expressions given above, where the quartic couplings are evaluated at the renormalization scale $\mu^2 = m_i^2$, with m_i the corresponding scalar mass.

It is straightforward to prove that the mass matrices of the CP-odd and charged Higgs states are diagonalized with an $H_1 - H_2$ mixing angle, which is always given by $-\theta$. In addition, whenever the mass parameter $M >> M_Z$, the mixing angle for the neutral CP-even states is approximately given by θ and hence, the light CP-even state, together with the Goldstone modes, form a Higgs doublet ϕ, whose expression is given by Eq.(11) with a mixing angle $\theta_M = \theta$.

For a given compositeness scale Λ and a supersymmetry breaking scale Δ_S the top quark mass is only a function of R, while the Higgs spectrum depends on R and on the value of the mass parameter M. This functional relation is depicted in Fig. 5 for a supersymmetry breaking scale varying in the range $\Delta_S = 1TeV - 10TeV$, for compostiteness scales $\Lambda = 10^{16}$ GeV and $\Lambda = 10^{10}$ GeV and for three different values of the mass parameter M, and where we have assumed that the ratio $R \geq 1$. An interesting result of these computations is that whenever the mass parameter $M > 0$, and for ratios $R \geq 1$, the lightest Higgs mass is enhanced by radiative corrections to values that are beyond the present experimental constraint $m_h > 38$ GeV [22]. Hence, if the supersymmetry breaking scale Δ_S is of the order of 1 TeV, the top quark mass within the supersymmetric top-condensate model may be as low as $m_t \simeq 140 GeV$ without any fine tuning of the low energy parameters.

As we have discussed above, when $M >> M_Z$ only one light neutral Higgs particle ϕ^0 remains in the physical spectrum of the theory. The other three components of the doublet ϕ, Eq. (11), are the Goldstone bosons, which are eaten by the electroweak gauge bosons through the usual Higgs mechanism. An alternative way to compute the mass of ϕ^0 is by assuming from the beginning that at scales lower than the supersymmetry breaking scale one recovers the standard model with only one light Higgs doublet. Considering the effective potential for ϕ to be

$$V(\phi) = m_\phi^2 \phi^\dagger \phi + \frac{\lambda}{2}(\phi^\dagger \phi)^2, \tag{24}$$

the ϕ^0 mass is given by $m_{\phi^0}^2 = 2\lambda v^2$. The value of the quartic coupling λ at low energies may be obtained by solving the corresponding renormalization group equation

$$\begin{aligned} 16\pi^2 \frac{d\lambda}{dt} &= -6\left[\lambda^2 + (4\pi)^2\lambda(Y_t^{eff} + Y_b^{eff} - \tilde{\alpha}_1/4 - 3\tilde{\alpha}_2/4) + (4\pi)^4(\tilde{\alpha}_1^2/16\right. \\ &+ \left. \tilde{\alpha}_1\tilde{\alpha}_2/8 + 3\tilde{\alpha}_2^2/16 - (Y_t^{eff})^2 - (Y_b^{eff})^2)\right], \end{aligned} \tag{25}$$

with the boundary condition $\lambda(\Delta_S) = \frac{g_1^2+g_1^2}{4}\cos^2(2\theta)$, where $Y_{b,t}^{eff} = (h_{b,t}^{eff}/4\pi)^2$. The values obtained for the Higgs mass coincide remarkably well with the ones obtained by the procedure above, when the mass $M >> M_Z$, and are depicted with a dashed line in Figure 5.

In the light Higgs mass, m_h, computations we have assumed $R > 1$. A lower bound on R may be obtained by analyzing the renormalization group flow of the mass parameters appearing in the scalar Higgs potential. In all the supersymmetry breaking schemes studied so far, the mass parameters m_1^2 and m_2^2 acquire equal values at large energy scales. From the renormalization group equations for these parameters [19], it follows that if $m_1^2(\Lambda) = m_2^2(\Lambda)$ then $m_1^2(\mu) > m_2^2(\mu)$ for any energy scale $\mu < \Lambda$. Observe that this also applies to our $1/N_C$ study of section 2, where in the limit $\mu \to \Lambda$, $m_1^2(\mu)/m_2^2(\mu) \to 1$, while at low energies $m_1^2 - m_2^2 = M_H^2 + 2\Delta^2 > 0$. Using the expression for the CP-odd scalar mass m_A, Eq.(22), we can rewrite the minimization condition, Eq.(16), as

$$R^2 = \frac{m_A^2 + (\lambda_1 - \lambda_3 - \lambda_4)v^2 + M_{12}^2}{m_A^2 + (\lambda_2 - \lambda_3 - \lambda_4)v^2 - M_{12}^2}. \tag{26}$$

where $M_{12}^2 = m_1^2 - m_2^2$. Observe that the right hand side must be greater than zero in order to obtain a proper electroweak symmetry breakdown.

As it is apparent from Figure 3 and 4, a lower bound on R induces a lower bound on the top quark mass within this model. From Eq.(26), it follows that in the one light Higgs doublet case, i.e. $m_A^2 >> M_Z^2$, the ratio R is bounded to be $R \geq 1$. In the two light Higgs doublets case, the result depends on the value of the CP-odd mass and the squared mass difference M_{12}^2. For low values of R, $\lambda_1 \simeq -(\lambda_3 + \lambda_4) \simeq 0.13$, while λ_2 varies from $\lambda_2 \simeq 0.4$ for $\Lambda = 10^{16}$ GeV and $\Delta_S = 1TeV$ up to $\lambda_2 \simeq 0.8$ for $\Lambda = 10^{10} GeV$ and $\Delta_S = 10TeV$. Assuming a scalar CP-odd state with mass consistent with its present experimental bound, $m_A \geq 42$ GeV[22], and a small mass difference ($M_{12} \simeq 0$), it is easy to obtain a lower bound on R while setting m_A to be equal to its experimental lower bound. The lower bound on R decreases when the compositeness (supersymmetry breaking) scale decreases (increases). However, since under these conditions the top Yukawa coupling is increased, the lower bound on m_t is only slightly modified. We obtained that, for a supersymmetry breaking scale varying in the range $\Delta_S = 1 - 10$ TeV, and a compositeness scale $\Lambda = 10^{10} - 10^{16}$ GeV, the top quark mass $m_t > 120$ GeV. The lower bound increases rapidily for larger values of M_{12}. If M_{12} is assumed to be $M_{12} = \mathcal{O}(M_Z)$, the lower bound on R is generally obtained for the case of a heavy scalar CP-odd state, that is to say $R \geq 1$ and therefore, for a supersymmetry breaking scale $\Delta_S = 1$ TeV, the top quark mass $m_t \geq 140 GeV$. The same bound on R, $R \geq 1$, applies if, as suggested by our $1/N_C$ study of section 2, the mass difference M_{12} is of the order of the supersymmetry breaking scale.

Observe that the bound on m_t may not be relaxed by setting the supersymmetry breaking scale $\Delta_S = \mathcal{O}(M_Z)$. If the characteristic mass of the supersymmetric partners were of the order of 100 GeV, the quartic Higgs couplings would be approximately described by their supersymmetric expressions. Hence, the bounds $R > 1$ and $m_h < M_Z|\cos(2\theta)|$ would apply in this case. Moreover, from the present experimental bound on the lightest CP-even neutral Higgs mass, $m_h > 38 GeV$ [22], it follows that the ratio $R > 1.6$ and hence $m_t > 165$ GeV for $\Lambda = 10^{16}$ GeV.

A heavy CP-odd spectrum may be obtained only by enhancing the value of the mass parameter M. This is clearly shown in Figure 6.a (6.b), where we depicted the results for the CP-odd (charged) Higgs mass as a function of the mass parameter M for $\Lambda = 10^{16}$ GeV , $\Delta_S = 1$ TeV and for different values of the top quark mass. Note that, even if the mass $M \simeq 0$, the CP-odd mass acquires phenomenologically acceptable values.

4 Conclusions

In this article we have analyzed the values of the top quark mass compatible with a dynamical breakdown of the electroweak symmetry induced by condensates of the third generation of quark multiplets, in a minimal supersymmetric extension of the standard model. We have argued that these values may be reinterpreted as being the triviality bounds on the top quark mass, for a given cutoff energy scale provided by the compositeness scale Λ. We have shown, in a way independent of the supersymmetry breaking scheme, that, for a given supersymmetry breaking scale, the values of the top quark mass within this model only depend on the ratio R of Higgs vacuum expectation values. In addition, we have shown that, for $\Delta_S = \mathcal{O}(1TeV)$, no additional constraint is obtained from the present experimental bound on the lightest Higgs mass $m_h > 38GeV$. For the compositeness scale $\Lambda \simeq 10^{16}GeV$ and the supersymmetry breaking scale $\Delta_S = 1TeV$, the characteristic top quark mass is predicted to be in the range $140GeV \leq m_t \leq 195GeV$. This prediction has only a slight dependence on the exact value of the supersymmetry breaking scale.

Finally, when trying to give masses to all observed quarks and leptons, additional four Fermi couplings to the ones considered in this article have to be included in the supersymmetric model. Since the relevant four Fermi coupling constants are not suppressed by powers of Λ, these additional interactions may induce flavor changing neutral currents at the supersymmetry breaking scale. These processes, if they are not supressed as in the non-supersymmetric model, may provide a means to experimentally test this model. As well, there is a correlation between the Higgs spectrum and top quark masses that provides a signature for the supersymmetric top-condensate model.

ACKNOWLEDGEMENTS

K.S., C.W. and M.C. would like to thank the members of the theory group for discussions and hospitality extended to them while visiting Fermilab, where portions of this work were done. K.S. would also like to thank Professor Muta for hospitality extended to him at Hiroshima University. T.C. thanks S. Love for interesting comments. W. A. B. was partially supported by a grant from the Ministry of Education of Japan. This work was partially supported by the U. S. Department of Energy.

References

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122(1961)345.

[2] Y. Nambu, *Bootstrap Symmetry Breaking in Electroweak Unification*, Enrico Fermi Institute Preprint 89-08(1989); *BCS Mechanism, Quasi- Supersymmetry, and Fermion Mass Matrix* Talk presented at the Kasimirz Conference, EFI 88-39 (July 1988).

[3] V. Miransky, M. Tanabashi and K. Yamawaki, Mod. Phys. Lett. A4(1989)1043, Phys. Lett. B221(1989)177.

[4] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D41 (1990) 1647.

[5] W.J. Marciano Phys. Rev. Lett.62(1989)2793; Brookhaven National Laboratory Preprint, *Dynamical Symmetry Breaking and the Top Quark Mass*, September 1989.

[6] W. A. Bardeen, T. E. Clark and S. T. Love, Phys. Lett. B 237 (1990) 235.

[7] C. Hill, *Dynamical Symmetry Breaking of the Electroweak Interactions and the Renormalization Group*, FERMI-CONF-90/170-T.

[8] C.T. Hill Phys. Rev.D24(1981)691.
C. Hill, C. N. Leung and S. Rao, Nucl. Phys. B262(1985) 517.

[9] Y. Okada, M. Yamaguchi and T. Yanagida, Tohoku U. preprints TU-360 (1990) and TU-363 (1990),
J. Ellis, G. Ridolfi and F. Zwirner, CERN-TH 5936 (1990),
H. Haber and R. Hempfling, preprint SCIPP-90/142.

[10] W.A. Bardeen, C.N. Leung and S.T. Love, Phys. Rev. Lett.56 (1986)1230, Nucl. Phys.B273 (1986)649, Nucl. Phys.B323(1989)493.

[11] W. Buchmüller and S.T. Love, Nucl. Phys. B204(1982)213.

[12] W. Buchmüller and U. Ellwanger Nucl. Phys. B245(1984)237.

[13] J. Wess and J. Bagger, Supersymmetry and Supergravity, Princeton U. Press, Princeton, 1983.

[14] K. Inoue, A. Kakuto and S. Takeshita, Prog. Theor. Phys. 67 (1982) 1889; ibid 68 (1982) 927;
H. P. Nilles, Phys. Rep 110 (1984) 1;
K. Sasaki, Phys. Lett. 199B (1987) 395;
H. Okada and K. Sasaki, Phys. Rev D40 (1989) 3743.

[15] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Phys. Lett. B79 (1978)231, Nucl. Phys.B147 (1979)105;
E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. B116 (1982)231, Nucl. Phys. B212 (1983)413;
S.K. Soni and H.A. Weldon, Phys. Lett. 126B(1983)215;
L. Alvarez-Gaumé, J. Polchinski and M.B. Wise Nucl. Phys. B221(1983) 495.

[16] T.E. Clark and S.T. Love, Nucl. Phys. B310(1988)371.

[17] F. Fujikawa and W. Lang, Nucl. Phys.B88 (1975)61.

[18] P. Langacker and M. Luo, Penn. U. preprint UPR-0466T, February 1991.

[19] J. Bagger, S. Dimopoulos and E. Massó, Phys. Rev. Lett.55(1985) 920;
L. E. Ibañez, C. Lopez and C. Muñoz, Nucl. Phys. B256 (1985) 299;
B. Gato, J. Leon, J. Perez-Mercader and M. Quiros, Nucl.Phys. B253 (1985) 285;
N. K. Falck, Z. Phys. C30 (1986) 247.

[20] C. Kounnas, A. Lahanas, D. Nanopoulos, M. Quiros, Nucl. Phys. B236 (1984) 438
J. P. Derendinger, C. A. Savoy, Nucl. Phys. B237 (1984) 307.

[21] See for example, S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, preprint MPI-PAE/TH 43/90, July, 1990, and references therein.

[22] G. Bonneaud, Talk given at the Conference Pascos91, Northeastern U., March 1991.

FIGURE CAPTIONS.

Fig. 1. The Schwinger-Dyson equation for the top quark chiral superfield self energy in the bubble approximation, where the coupling $G(\theta,\bar{\theta}) = G(1 - 2\Delta^2\theta^2\bar{\theta}^2 + \delta\bar{\theta}^2 + \delta\theta^2)$.

Fig. 2. The Feymann diagram contributing to the induced kinetic term for the Higgs chiral superfield H_2 in leading order in $1/N_C$, where the coupling $g_T(\theta) = g_{T_0}(1 + \delta\theta^2)$.

Fig. 3. Top quark mass as a function of the ratio R, for a supersymmetry breaking scale Δ_S= 1 TeV and three different values of the compositeness scale Λ, for the case of one light Higgs doublet (dashed line) and two light Higgs doublets(solid line).

Fig. 4. Top quark mass as a function of the ratio R, for three different values of the supersymmetry breaking scale, for the case of two Higgs doublets and a compositeness scale a) $\Lambda = 10^{10}$ GeV and b) $\Lambda = 10^{16}$ GeV.

Fig. 5. Top quark mass as a function of the lightest Higgs mass m_h, for three different values of the mass parameter M (solid lines), and the same functional relation for the case of one light Higgs doublet (dashed line), for 1. $\Lambda = 10^{10}$ GeV and 2. $\Lambda = 10^{16}$ GeV, and a supersymmetry breaking scale a) $\Delta_S = 1$ TeV and b) $\Delta_S = 10$ TeV.

Fig. 6. a) the CP-odd Higgs mass, m_A, and b) the charged Higgs mass, m_{ch}, as a function of the mass parameter M for a supersymmetry breaking scale $\Delta_S = 1$ TeV, a compositeness scale $\Lambda = 10^{16}$ GeV, and different values of the top quark mass, m_t.

Figure 1

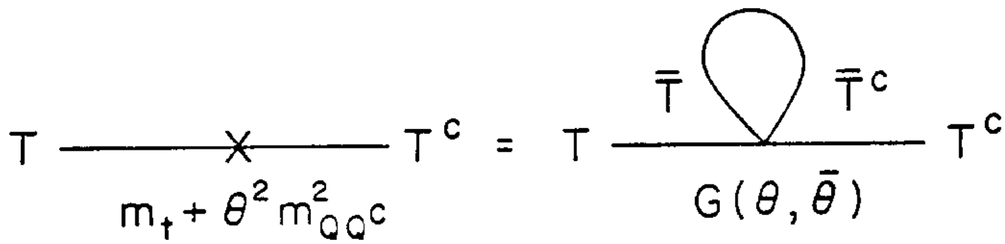


Figure 2

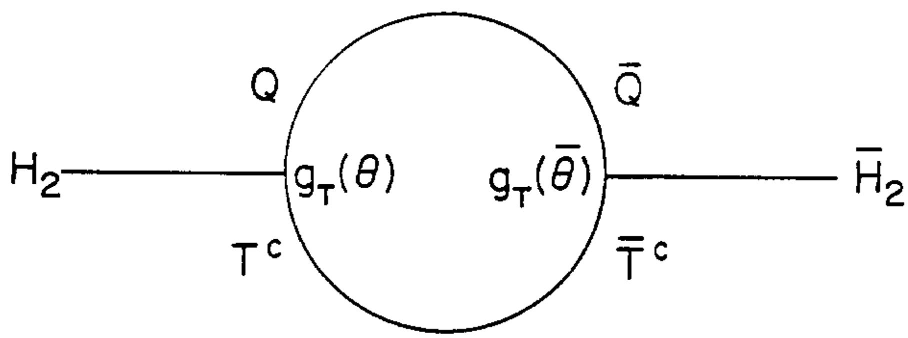
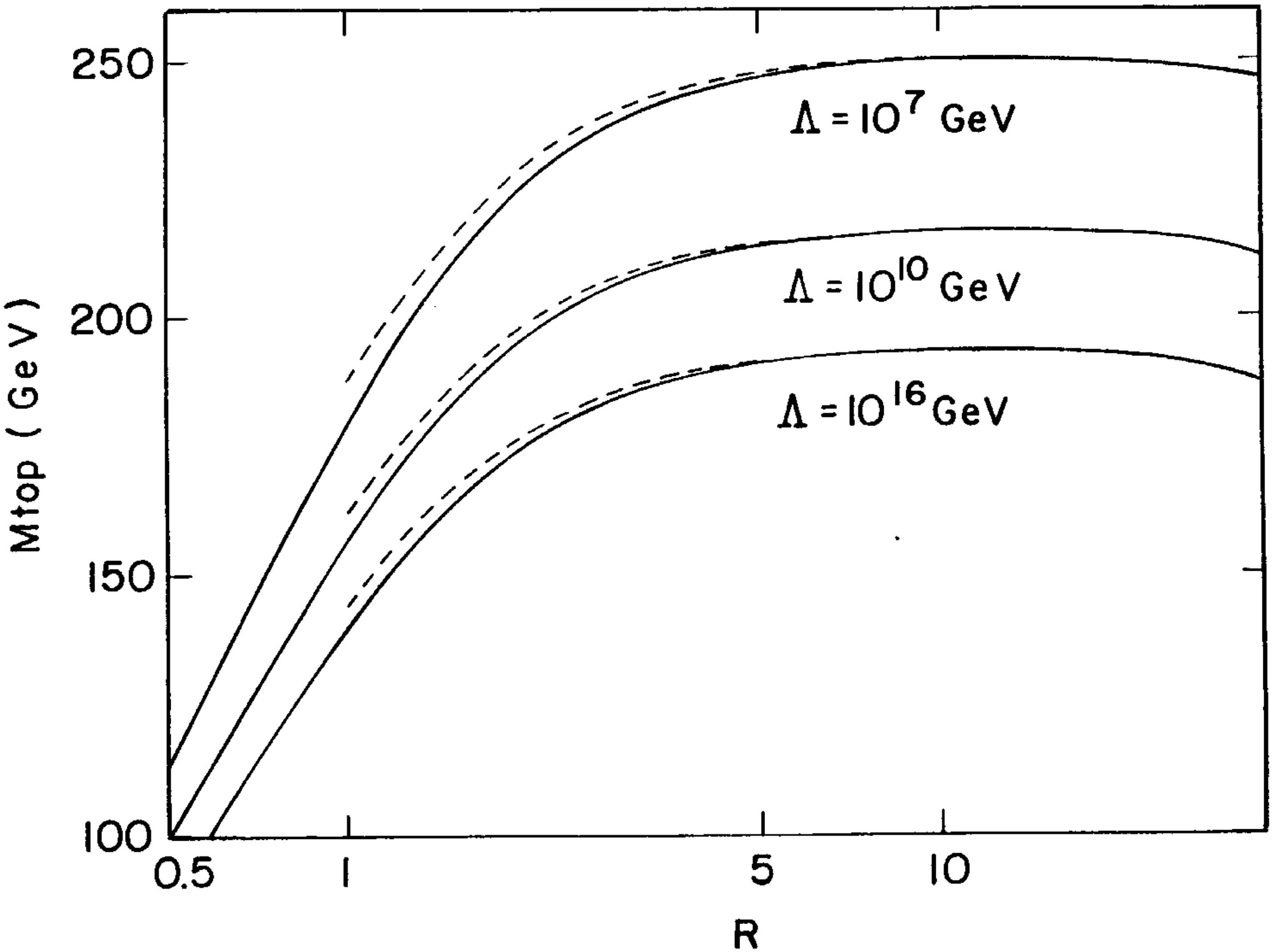
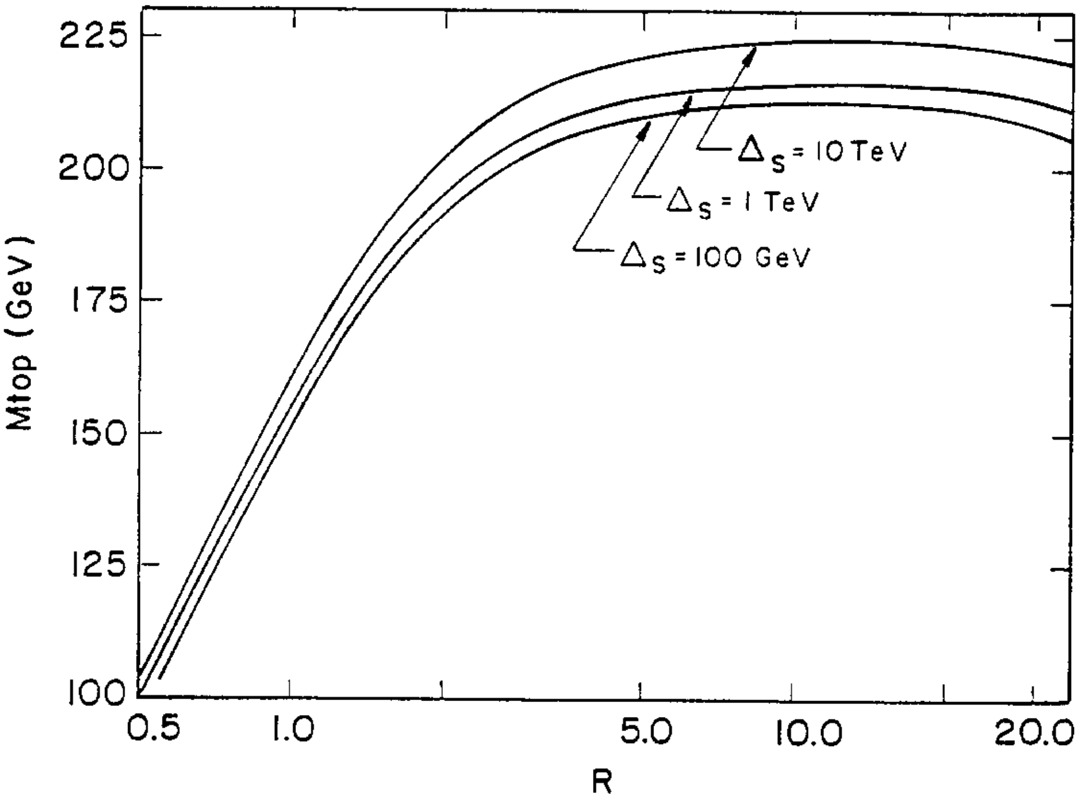


Figure 3

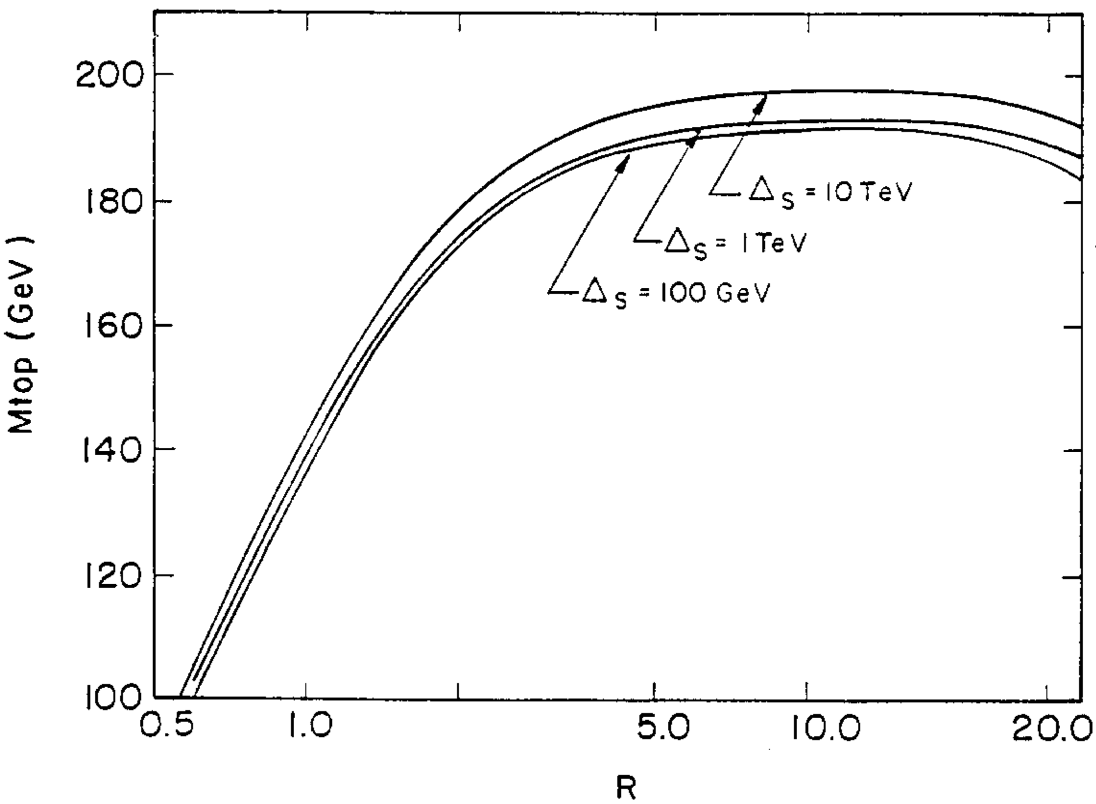


Mtop (GeV)
250
200
150
100
0.5
1
5
10
R
$\Lambda = 10^7$ GeV
$\Lambda = 10^{10}$ GeV
$\Lambda = 10^{16}$ GeV

Figure 4


225
200
175
150
125
100
Mtop (GeV)
0.5
1.0
5.0
10.0
20.0
R
Δ_S = 10 TeV
Δ_S = 1 TeV
Δ_S = 100 GeV

(a)


200
180
160
140
120
100
Mtop (GeV)
0.5
1.0
5.0
10.0
20.0
R
Δ_S = 10 TeV
Δ_S = 1 TeV
Δ_S = 100 GeV

(b)

Figure 5.1

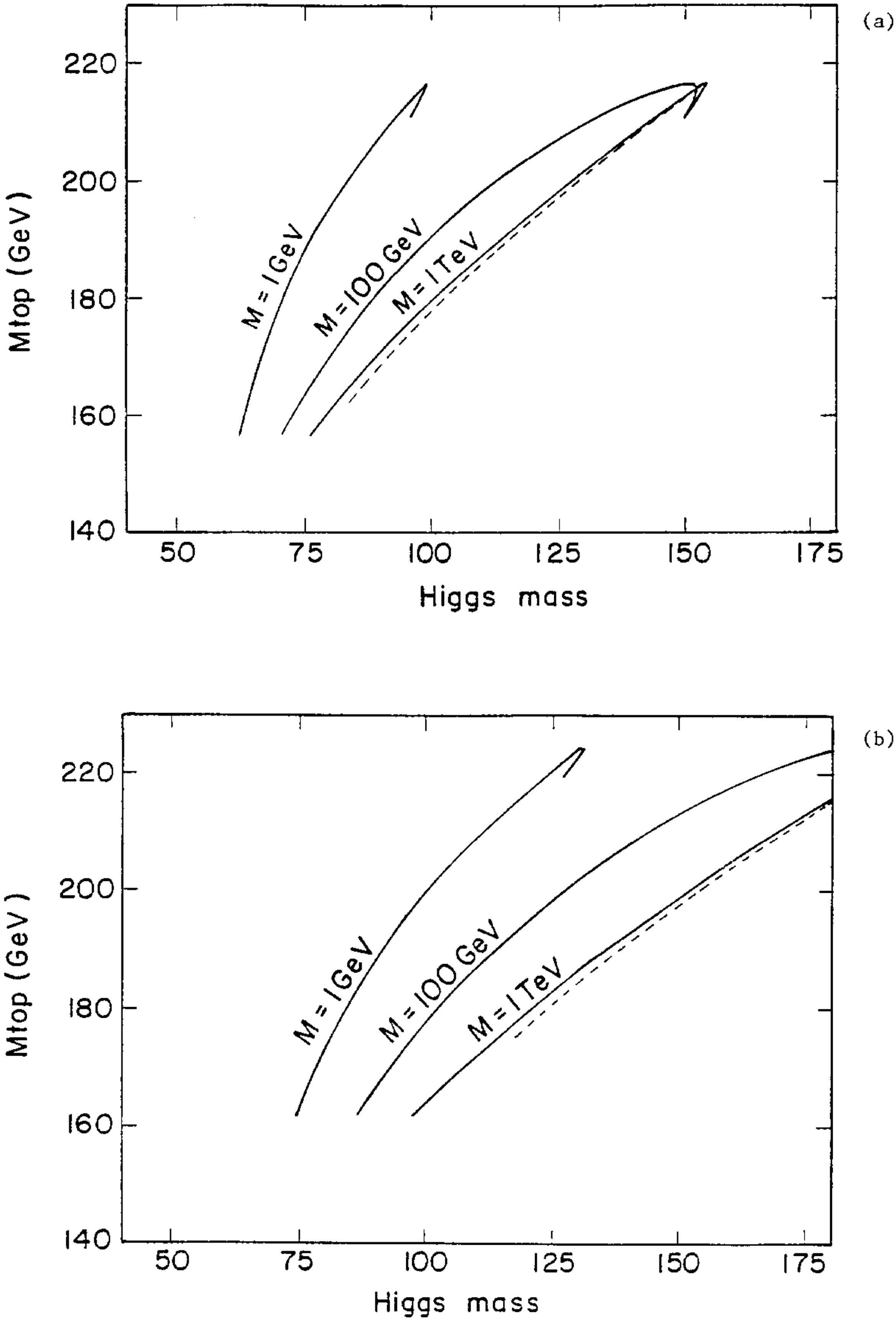
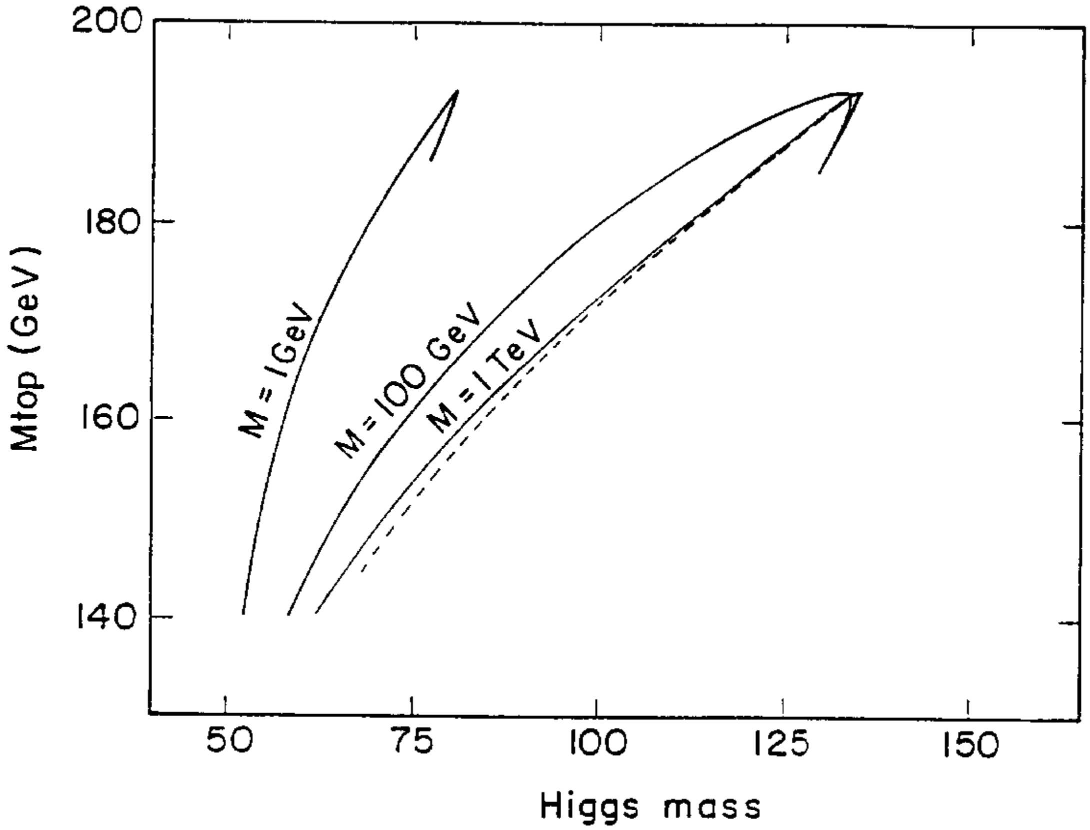
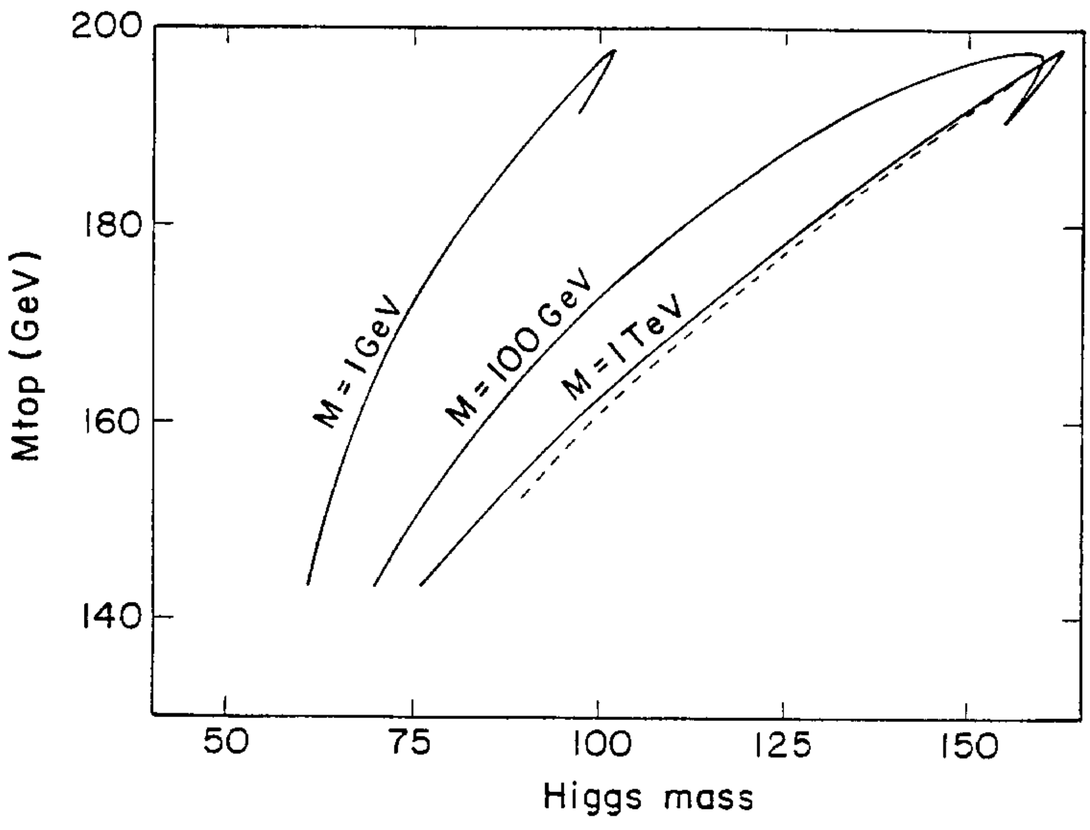

(a)
220
200
180
160
140
Mtop (GeV)
M = 1 GeV
M = 100 GeV
M = 1 TeV
50
75
100
125
150
175
Higgs mass
(b)
220
200
180
160
140
Mtop (GeV)
M = 1 GeV
M = 100 GeV
M = 1 TeV
50
75
100
125
150
175
Higgs mass

Figure 5.2

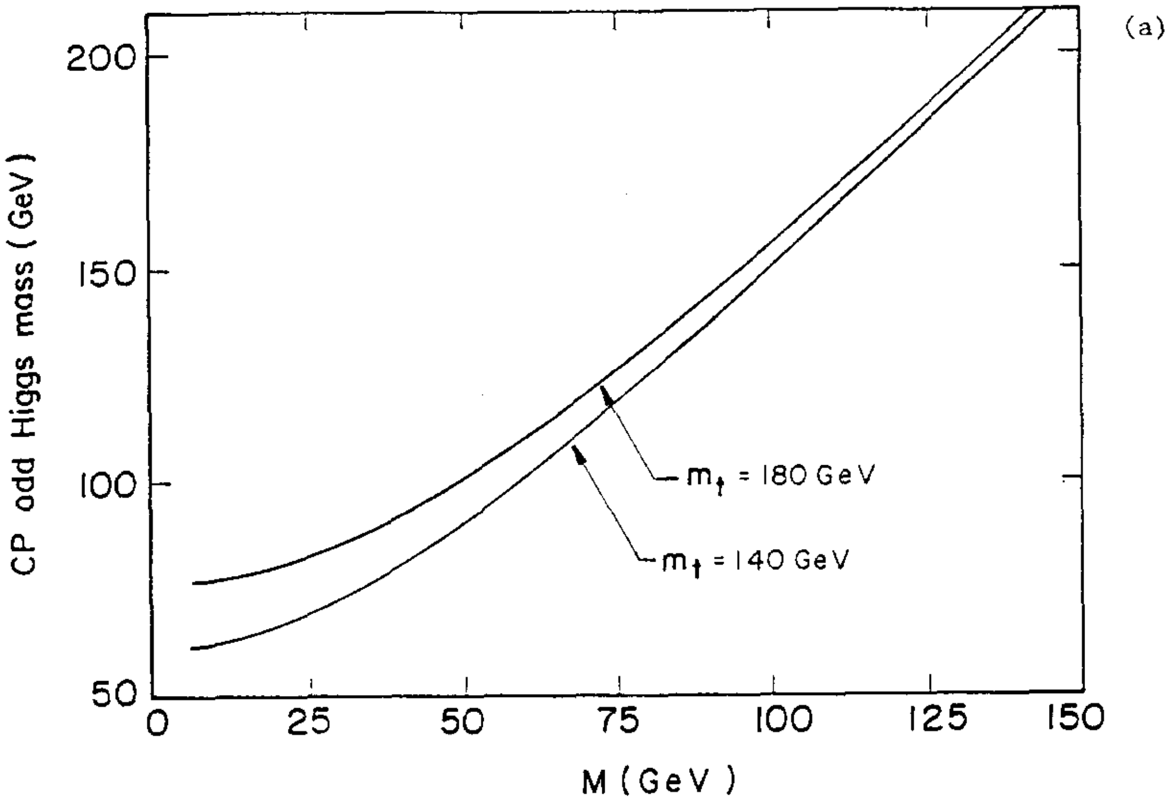

200
180
160
140
Mtop (GeV)
50
75
100
125
150
Higgs mass
M = 1 GeV
M = 100 GeV
M = 1 TeV

(a)

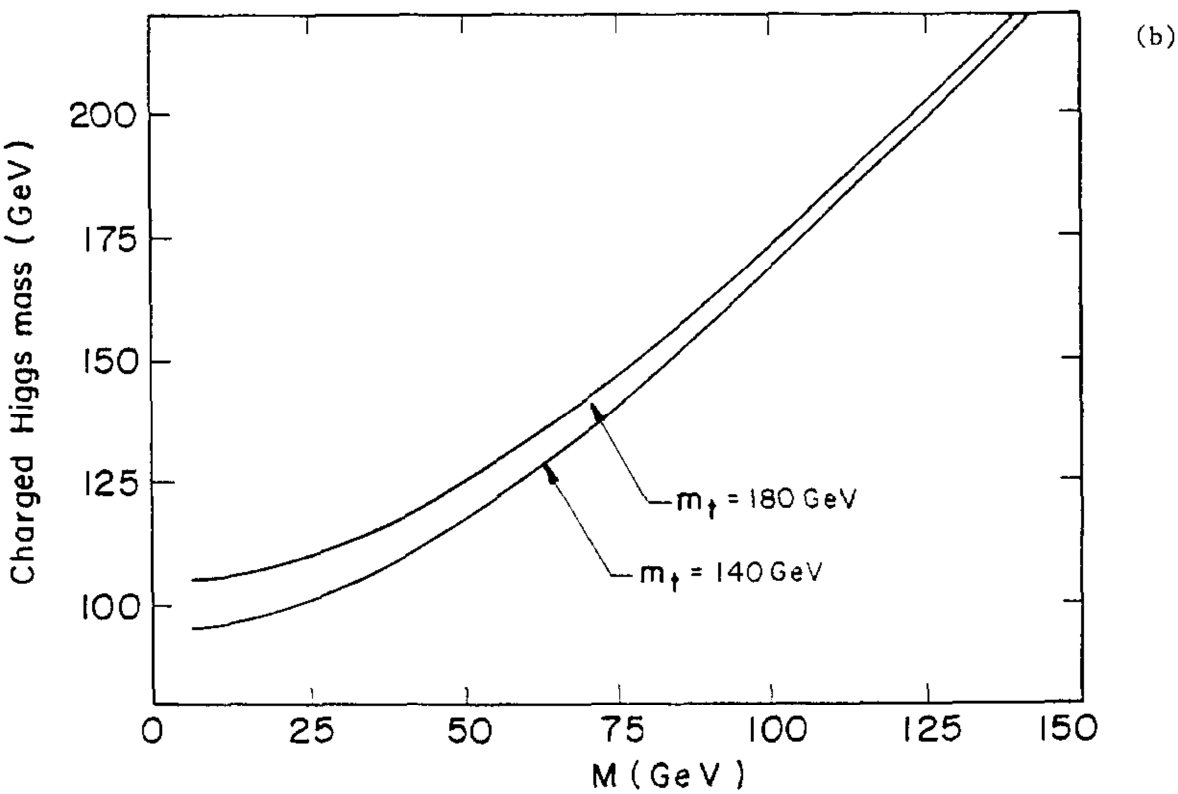

200
180
160
140
Mtop (GeV)
50
75
100
125
150
Higgs mass
M = 1 GeV
M = 100 GeV
M = 1 TeV

(b)

Figure 6



(a)
CP odd Higgs mass (GeV)
200
150
100
50
0
25
50
75
100
125
150
M (GeV)
m_t = 180 GeV
m_t = 140 GeV



(b)
Charged Higgs mass (GeV)
200
175
150
125
100
0
25
50
75
100
125
150
M (GeV)
m_t = 180 GeV
m_t = 140 GeV