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DARK MATTER AND THE EQUIVALENCE PRINCIPLE

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ABSTRACT

It is generally assumed that the dark matter in galaxies and clusters is subject only to gravitational forces. Yet, if the dark matter is non-baryonic, it can interact with additional long-range fields that are invisible to experimental tests of the equivalence principle. We discuss the astrophysical and cosmological implications of an additional long-range force of gravitational strength coupled only to the dark matter. If the interaction is repulsive, the masses of galaxy groups and clusters may have been systematically underestimated; it is possible that the universe is closed or flat ($\Omega \geq 1$) but has simply been misinterpreted as open. Such an interaction also gives rise to a new pseudo-tidal force between the baryonic core and the dark halo of a galaxy in a rich cluster. We also study the implications for the growth of large-scale density perturbations.

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The notion that there may be hidden matter in the universe has a long history. In the last century, anomalies in the orbit of Uranus were ascribed to the gravitational pull of an unseen planet, leading to the discovery of Neptune. More recently, the observed flat rotation curves of galaxies and the application of the virial theorem to clusters of galaxies have revealed the presence of large amounts of dark matter (constituting perhaps 90% of the total mass in these systems).[1] Several lines of argument suggest that much of the dark matter in galaxies and clusters is not baryonic, while particle physics models provide a gallery of exotic elementary particles as dark matter candidates. [2] [3]

In keeping with the principle of equivalence, it is generally assumed that the dark matter gravitates like the visible baryons. However, since the existence of dark matter is inferred solely from its gravitational effects, and its nature is otherwise unknown, this assumption is open to question. Although a new long-range force of gravitational strength coupled to *ordinary* matter is experimentally ruled out by recent 'fifth-force' experiments [4], there may be an additional long-range interaction which couples to a quantum number carried exclusively by *non-baryonic* matter. Such an additional force clearly evades laboratory tests of the equivalence principle. Its effects would only be manifest in systems where the dark matter is dynamically important, that is, in the outer regions of galaxies and in clusters. In this essay, we investigate the implications of additional long-range forces acting between non-baryonic dark matter particles.

Additional long-range interactions have been proposed in the context of a variety of particle theories. For example, in extended supergravity models [5], a vector field coupled to particles of mass m and effective charge $\sim m/m_{pl}$ [#1] gives rise to a repulsive force of gravitational strength. Alternatively, pseudo-Nambu-Goldstone bosons with scalar couplings, called schizons, can arise naturally in extensions of the standard electroweak model [6]. As a concrete example, consider the phenomenological schizon model with Lagrangian

$$L = \bar{\psi} i \gamma_\mu \partial^\mu \psi + m_\psi \bar{\psi}\psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\epsilon}{f}\phi\bar{\psi}\psi - \frac{1}{2}m_\phi^2 \phi^2, \tag{1}$$

where the fermion ψ of mass $m_\psi \sim 1 - 10$ eV constitutes the dark matter ($\Omega_\psi \simeq$

#1 $m_{pl} = G_N^{-1/2} = 1.2 \times 10^{19}$ GeV is the Planck mass.

1), ϕ is the schizon, and f is a global symmetry breaking scale. In the non-relativistic limit, the static potential between two separated fermion sources with masses M_1 and M_2 is given by

$$V_\phi = \frac{-\epsilon^2}{4\pi m_\psi^2 f^2}\frac{M_1 M_2}{r}e^{-m_\phi r} \ . \tag{2}$$

Thus, on scales $r \ll m_\phi^{-1}$, the relative magnitude of the scalar force is $\alpha = G_\phi/G_{\rm N} = \epsilon^2 m_{pl}^2/m_\psi^2 f^2$; for $\epsilon \sim m_\psi$ and $f \sim m_{pl}$, it has roughly gravitational strength. In these models, the scalar mass is of order [6] $m_\phi \sim m_\psi^2/f$, so the range of the force is astronomical, $\lambda = m_\phi^{-1} \sim 1 - 100$ kpc or more.

In general, the potential energy of two non-baryonic masses M_1 and M_2 at separation r may be parameterized by

$$V(r) = -G_{\rm N}\frac{M_1 M_2}{r}(1+\alpha e^{-r/\lambda}), \tag{3}$$

where the range of the additional interaction is fixed by the Compton wavelength of the exchanged vector ($\alpha < 0$) or scalar ($\alpha > 0$) particle, $\lambda = 1/m_{v,s}$. We study constraints on the coupling α for a range of wavelengths λ. Since there are gravitationally bound systems of dark matter, we require $\alpha > -1$ for $\lambda \gtrsim 10$ kpc.

The dark matter density is usually expressed in terms of the mass–to–light ratio $\Upsilon = \langle M/L \rangle_V$ in the V band.[#2] The observation of high velocity stars in the solar neighborhood implies the total mass–to–light ratio for the Milky Way is at least $\Upsilon_{mw} \gtrsim 30\Upsilon_\odot$. This is consistent with mass–to–light ratios inferred from rotation curves in other spiral galaxies. Dynamical measurements of the mass within galaxies rely on baryonic tracers (stars or gas) of the gravitational potential. Since we assume the new interaction does not couple to baryons (the experimental bound [4] on a long-range force *between baryons* is $\alpha_b \lesssim 10^{-4}$), the masses inferred for individual galaxies, $M_{\rm inf}(r) \sim v^2 r/G_{\rm N}$, are the true masses in the sense that they are independent of α.

#2 From the observed mean luminosity density [7] $j_V \simeq 1.7 \times 10^8 h L_\odot$ Mpc^{-3}, the density parameter can be expressed as $\Omega = 6 \times 10^{-4} h^{-1}\Upsilon/\Upsilon_\odot$, where the Hubble constant $H_0 = 100h$ km/sec/Mpc. Thus, the critical mass–to–light ratio for an $\Omega = 1$ universe is $\Upsilon_c = 1600 h\Upsilon_\odot$.

In systems of galaxies such as binaries and groups, galaxies themselves are used as test particles; if the galaxy mass is dominated by non-baryonic dark matter, one must take into account the additional force on the dark mass. Consider two galaxies in a binary system with separation $r \ll \lambda$ radially approaching each other with speed v_r; from Kepler's law

$$r v_r^2 \propto G_{\rm N} M_{\rm inf} = G_{\rm N}(1+\alpha) M_{\rm true} \ , \tag{4}$$

where $M_{\rm true}$ is the true (dark) mass of the binary system and $M_{\rm inf}$ is the mass one erroneously infers without knowledge of the additional force.[#3] Thus,

$$\frac{\Omega_{\rm true}}{\Omega_{\rm inf}} = \frac{\Upsilon_{\rm true}}{\Upsilon_{\rm inf}} = \frac{1}{1+\alpha}. \tag{5}$$

A repulsive force ($\alpha < 0$) would delude us into believing that the dark matter density is smaller than it actually is; for example, a flat universe with $\Omega_{\rm true} = 1$ could masquerade as an open universe with $\Omega_{\rm inf} < 1$, perhaps reconciling the theoretical prejudice for a flat universe with the observational indications (on scales of groups and clusters) that $\Omega \sim 0.2$.

To first approximation, the Local group of galaxies can be thought of as a binary system dominated by our Galaxy and Andromeda (M31), with a separation $r = 700$ kpc. Given the relative approach speed of the two galaxies, the application of Kepler's law implies $\Upsilon_{\rm inf}({\rm LG}) = 76 - 130\Upsilon_\odot$ [7]. Since M31 is expected to have a ratio of dark to luminous matter and a stellar population similar to those of the Milky Way, the true mass–to–light ratio of the Local group should at least equal that of our galaxy, $\Upsilon_{\rm true}({\rm LG}) \gtrsim 30\Upsilon_\odot$. From eq. (5) this implies the upper bound $\alpha \lesssim 3.3$ for $\lambda \gtrsim 1$ Mpc.

So far, we have assumed that the luminous baryons are gravitationally enslaved to their dark halos. For galaxies in binaries and dense clusters, the fact that spiral disks and elliptical cores are not completely stripped of their halos provides another constraint on α. Consider a galaxy of mass m orbiting at a

#3 In the second equality of eq. (4) , we neglected the contribution from the baryonic mass; this approximation only fails for $\alpha \lesssim -0.9$.

distance R from a mass M (another galaxy or a cluster) and a dark matter particle a distance $r \ll R$ from the center of m. In the standard case ($\alpha = 0$), the halo particle will be stripped from the galaxy if it is beyond the tidal radius, $r > r_t \sim R(m/M)^{1/3}$. However, if $\alpha \neq 0$, additional *non-tidal* stripping arises from the fact that the orbital speeds of baryons and dark matter particles *at the same point* in the field of a central mass M do not coincide. In this case, the halo particle is marginally bound if it is at a distance $r = r_s$ given by

$$\frac{G_{\rm N}\, m}{r_s^2} \simeq \left| \frac{G_{\rm N} M}{R^2} - \frac{G_{\rm N}(1+\alpha)M}{(R+r_s)^2} \right| \simeq \left| -\frac{\alpha G_{\rm N} M}{R^2} + 2\, G_{\rm N}(1+\alpha)M\, \frac{r_s}{R^3} \right| . \quad (6)$$

In the limit $\alpha \to 0$, we recover the usual result $r_s = r_t$ for the tidal radius. For $\alpha \neq 0$ (and $\lambda \gg R$), the first term in eq. (6) dominates and

$$\frac{r_s}{R} \simeq \left| \frac{m}{\alpha M} \right|^{1/2} . \quad (7)$$

If the mass–to–light ratio for our Galaxy (m) is the same as that for M31 (which is about twice as luminous as the Milky Way), then $m \simeq M/2$. The mass–to–light ratio inferred above from local high velocity stars implies that the Galaxy extends to at least $r_s > 40$ kpc [7]; recalling $R = 0.7$ Mpc for the Local group, we find the relatively weak constraint $|\alpha| \lesssim 150$.

In clusters of galaxies, the situation is more subtle. In the cores of rich clusters, the outer halos of most galaxies are thought to be tidally stripped off by the cluster potential [8], while the halos of galaxies farther out (at distances $R \gtrsim 1h^{-1}$ Mpc) appear to be intact. Requiring that a typical galaxy far from the core not be stripped of its halo leads from eq. (7) to the constraint $|\alpha| \lesssim 10$. Since the bulk of cluster galaxies appear to retain at least partial halos, the inferred mass–to–light ratio in clusters is given roughly by eq. (5) . For rich clusters, $\Upsilon_{\rm inf} \simeq 400h\Upsilon_\odot$; thus, if the dark matter on cluster scales is to close the universe, we require $\alpha \simeq -3/4$.

Cosmology is the final arena where the effects of an additional dark matter interaction would be played out. Despite the form of eq. (3) , the gravitational constant $G_{\rm N}$ is *not* replaced by a function of α in the Einstein equations for a homogenous and isotropic universe. This is most easily seen by considering the scalar example of eq.(1) . The homogeneous field $\phi = \phi(t)$ leads to only two effects: a cosmological density of coherent scalar particles $\rho_\phi(t)$ that behaves like non-relativistic matter, and a time-dependent mass for the dark fermions. For $f \lesssim m_{pl}$ and temperatures $T \lesssim m_\psi$, both effects have negligible impact on the density of the universe.[#4] Consequently, we can assume that the standard cosmology is unaltered by the additional interaction.

The new force will, however, dramatically affect the growth of inhomogeneities. Consider the fractional density perturbation in the non-baryonic component, $\Delta(\mathbf{x}) = (\rho_{nb}(\mathbf{x}) - \langle\rho_{nb}\rangle)/\langle\rho_{nb}\rangle$. We focus on small-amplitude fluctuations inside the horizon, so we can apply linear perturbation theory in the Newtonian approximation. The dark matter obeys the usual perturbed fluid equations [9] augmented by the additional potential ϕ,

$$\nabla_x^2\phi - m_{v,s}^2\phi = 4\pi\alpha G_{\rm N}\rho_{nb}(\mathbf{x})\Delta a^2 \ . \tag{8}$$

Here, the gradient is taken with respect to the comoving coordinate $\mathbf{x}$ and a is the cosmic scale factor. For a spatially flat universe ($\Omega = \Omega_b + \Omega_{nb} = 1$), the Fourier transform of the perturbation amplitude satisfies

$$\ddot{\Delta}_k + \frac{4}{3t}\dot{\Delta}_k - \frac{2}{3t^2}\left[\Omega_b\delta_k + \left(1 + \frac{\alpha}{1+(m_{v,s}/k_p)^2}\right)\Omega_{nb}\Delta_k\right] = 0 \ , \tag{9}$$

where $k_p \propto a^{-1}$ is the physical wavenumber of the perturbation, and the fractional perturbation in the baryonic component, δ, obeys

$$\ddot{\delta}_k \ + \ \frac{4}{3t}\,\dot{\delta}_k \ - \ \frac{2}{3t^2}\left[\ \Omega_b\delta_k \ + \Omega_{nb}\Delta_k \ \right] \ = \ 0 \ . \tag{10}$$

Since $\Omega_{nb} \gg \Omega_b$, the approximate solution for short wavelength modes ($k_p \gg m_{v,s}$) is $\Delta_k \propto t^p$, where $p = -(1/6)[1 \pm (25+24\alpha)^{1/2}]$. As expected, the perturbation growth rate is enhanced for an attractive interaction ($\alpha > 0$), and retarded

#4 Similar considerations apply for vector fields. There are scenarios where the scalar energy density can play an important cosmological role, but we do not consider them here.

by a repulsive force, relative to the usual $\triangle \sim t^{2/3}$ behavior. For standard gravity, the asymptotic growing mode satisfies $\delta = \triangle$, so the baryonic fluctuations track the dark matter. For $\alpha \neq 0$, the asymptotic ratio $\delta/\triangle$ is a function of α which is larger than one for a repulsive interaction. Thus an additional force automatically generates a bias between the dark matter and the light.

On large scales, $k_p \ll m_{v,s}$, we retrieve the standard result $\triangle \sim t^{2/3}$ for the growth rate. This introduces a feature into the perturbation spectrum at a wavelength comparable to the Compton radius of the exchanged particle; for $\alpha < 0$, perturbations on wavelengths larger than $m_{v,s}^{-1}$ undergo a late period of 'standard' growth while those on smaller scales only experience retarded growth, resulting in more *relative* power on large scales. For $\alpha > 0$, the power on small scales is enhanced relative to large scales. The implications for large-scale structure formation and the microwave background anisotropy are clearly worth pursuing but they are beyond the scope of this essay.

We have touched upon only a few of the many interesting phenomena which arise if the dark matter violates the principle of equivalence and interacts with itself via 'hidden' long-range forces. Although constrained by the mass–to–light ratios in galaxies and binaries, such an interaction can alter the apparent density of dark matter and profoundly change the spectrum and amplitude of large-scale density fluctuations.[#5] If nothing else, this idea provides a useful theoretical palimpsest against which the hazy depths of gravity stand out in sharper relief.

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#5 Other important issues not discussed here include the formation of dark halos and their response to baryonic infall, and large-scale peculiar velocities.

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