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**Calculable Nonminimal Coupling of
Composite Scalar Bosons to Gravity**

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Abstract

The nonminimal coupling to gravity $-\xi R\phi^2/2$ of a composite scalar field ϕ is calculated in the Nambu-Jona-Lasinio model. We find $\xi = 1/6$. The result is exact in a leading large- N approximation, or in a fully improved one-loop renormalization group approximation. We briefly discuss some related cosmological implications.



I. Composite Scalar Fields and Gravitation

The Nambu–Jona-Lasinio (NJL) model [1] is one of the few four-dimensional quantum field theories that can be treated analytically in a study of relativistic, composite boundstates. This model, which is closely related to the BCS theory of superconductivity, describes the spontaneous breaking of a chiral symmetry in which a fermion ψ forms a vacuum condensate $\langle \bar{\psi}\psi \rangle \neq 0$ while its mass m_ψ is dynamically generated. The pairing force driving the formation of the condensate is postulated to exist at some high energy scale Λ . The scale of the generated condensate, or equivalently the mass of the fermion, can be arbitrarily small, though $m_\psi \ll \Lambda$ results only from a fine-tuning of the strength of this pairing force coupling constant. In addition to the appearance of massless Nambu–Goldstone bosons a scalar composite state, ϕ , composed of $\bar{\psi}\psi$, appears in the spectrum.

While the NJL model is generally applied to study spontaneous symmetry breaking, in the present paper we are interested in the induced nonminimal coupling of the boundstate object ϕ to gravity. Thus we are using the NJL model as a laboratory to discuss the relationship between compositeness and gravitational interactions. Remarkably, we find a simple result in the usual fermion bubble approximation: $\xi = 1/6$, *i.e.*, ϕ is conformally coupled to gravity, even though scale breaking dynamics exists at high energies Λ . Moreover, $\xi = 1/6$ is an attractive renormalization group fixed point in the infra-red in this approximation. This implies that, even if there are corrections to $\xi = 1/6$ from irrelevant operators at Λ , the observed low energy coupling is quickly attracted to a physical or “observed” value of $\xi = 1/6$ as one evolves into the infra-red. Remarkably, even when more physics is included beyond the simple fermion bubble approximation by using the full one-loop renormalization group, this result persists. This is closely related to previous results which analyzed the RG behavior of ξ for large curvature [2].

Technically, the usual treatment of the NJL model involves the solution to coupled, self-consistent Schwinger-Dyson equations for propagators and vertex functions, and is valid only in a large- N limit, where N is the number of fermion degrees of freedom ("colors") flowing in a Feynman loop. Here one keeps only the effects of fermion loops (this is called fermion bubble approximation) and one finds the mass of ϕ is exactly $m_\phi = 2m_\psi$. This is not to imply that ϕ is a "loosely bound state;" indeed, the ϕ particle appears point-like on all scales $m_\phi \leq \mu \leq \Lambda$. When additional interactions are kept, or one goes beyond the large- N limit, then $m_\phi \neq 2m_\psi$.

Rather than carry out the more technically complicated analysis utilizing the Schwinger-Dyson formalism, we will follow [3] and carry out an equivalent, but much more transparent and easier analysis which makes use of the renormalization group. The key to using the renormalization group is identifying the appropriate boundary conditions that apply at the scale Λ that are a consequence of compositeness. Indeed, the renormalization group can be used as a dynamical tool to analyze the NJL model in fermion bubble approximation, but it can also be readily generalized to include *all of the effects of the physics in the full theory* to generate reliable, precise predictions of its consequences. This goes beyond the limited approaches of large- N fermion bubble sums, or planar QCD calculations. In fact, it is not clear how to perform comparably detailed calculations in the more cumbersome Schwinger-Dyson formalism. The renormalization group also provides the easiest means of understanding the physics of the theory.

There has been considerable interest in dynamical symmetry breaking of the electroweak interactions in which a top quark condensate plays the role of the order parameter [3,4]. The simplest models discussed thus far are generalizations of the NJL model. Here the Higgs boson is composed of $\bar{t}t$, thus the physics of the NJL model may be relevant to the scales of current interest in elementary particle physics. In the minimal version one predicts $m_{top} \sim 230$ GeV and $m_{Higgs} \sim 260$ GeV [3]. How-

ever, we show presently that there is a third, albeit experimentally mute prediction, *i.e.*, the nonminimal coupling to gravity of such a composite Higgs is determined with $\xi = 1/6$.

In the context of cosmology, it has been recognized that nonminimal fields could be employed to solve some problems associated with inflation. Several authors [5] have suggested that the inflaton could be the Grand Unified Theory Higgs if one assumed a large negative curvature coupling parameter, $\xi \approx -10^4$. One could then show that radiative corrections to the Higgs potential would not generate excess metric fluctuations violating microwave background limits even if the Higgs self-coupling is rather large, $\lambda > 0.01$. The net result is that matter fields can be naturally incorporated in slow-roll inflation [6] by altering the gravitational sector. Furthermore, in the extended inflation model [7], it has been suggested that the bubble nucleation scenario of old inflation [8] could be resurrected if one considered two scalar fields, the inflaton and a Brans-Dicke field. Here, however, small negative values of the curvature parameter are favored $-0.01 < \xi < 0$.

It is clear from the present analysis that a composite boson as occurs in the pure NJL model will not lead to a cosmologically acceptable ξ . In the NJL case $\xi = 1/6$ is a constant with scale, and there is no renormalization group evolution of this coupling constant. However, we emphasize that $\xi = 1/6$ will generally obtain at low energies for any initial value of ξ at high energies, in any theory containing scalar bosons as a consequence of the infrared fixed point behavior of the renormalization group equations. In fact, even in a more detailed composite scheme large irrelevant operators at the high energy scale Λ can give an essentially arbitrary initial value to $\xi(\Lambda)$, which then evolves toward $\xi = 1/6$ at low energies. Since this evolution is slow (*i.e.*, logarithmic), perhaps one can exploit the phase during which $\xi \neq 1/6$ in the context of inflationary cosmology. Conversely, it is imperative to consider the evolution toward $\xi = 1/6$ in any scheme, since large $|\xi|$ is unstable under renormalization. We will not

address the cosmological issues further in the present paper, and turn instead to the analysis of the NJL model.

II. Analysis of the NJL model

A. Fermion Bubble Approximation

Let ψ be a fermion field with left- and right-handed projections $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$. Consider, for discussion, the following theory at some “high energy scale” Λ :

$$\mathcal{L} = i\bar{\psi}D\psi + G(\bar{\psi}_L^a\psi_{Ra})(\bar{\psi}_R^b\psi_{Lb}) + \dots \quad (2.1)$$

Here (a, b) are indices (*e.g.*, quark “color”) and run from 1 to N . The first term contains a covariant derivative D for the usual gauge invariant and generally covariant fermion kinetic term, $\mathcal{L}_{kinetic}$. The ellipsis refers to gauge boson and gravitational kinetic terms. We have introduced here a four-fermion interaction with a coupling constant G which reflects some new pairing force at the scale Λ , *i.e.*, $G = g_\psi^2/\Lambda^2$. Above the scale Λ , this term “softens” into some gauge boson exchange interaction. For $G > 0$ the pairing force is attractive.

The full Lagrangian \mathcal{L} admits a chiral symmetry,

$$\psi_L \rightarrow e^{i\alpha}\psi_L, \quad \psi_R \rightarrow e^{-i\alpha}\psi_R, \quad (2.2)$$

which forbids a mass term in the Lagrangian of the form $m\bar{\psi}\psi = m\bar{\psi}_R\psi_L + h.c.$ Nonetheless, if the coupling constant g_ψ is sufficiently strong, then the vacuum state will form a “chiral condensate,” $\langle \bar{\psi}_L\psi_R + h.c. \rangle \neq 0$, and the chiral symmetry will be spontaneously broken.

To analyze the model we may introduce a non-dynamical auxiliary H field to

rewrite eq.(2.1) equivalently as,

$$\mathcal{L} = \mathcal{L}_{kinetic} + (\bar{\psi}_L^a \psi_{R\alpha} H + h.c.) - M_\Lambda^2 H^\dagger H, \quad (2.3)$$

where we identify

$$G = 1/M_\Lambda^2. \quad (2.4)$$

Note that eq.(2.3) must be viewed as an effective Lagrangian at the scale Λ ; Λ and M_Λ are independent quantities. By “effective Lagrangian at a scale μ ” we mean that all the dynamics above the scale μ has been integrated out, but all dynamics below μ must be computed. Notice that the identification of eq.(2.3) with eq.(2.1) is only possible for $G > 0$.

The structure of eq.(2.3) will change significantly, due to radiative corrections from Feynman loops, when we consider the effective Lagrangian at any other scale, $\mu < \Lambda$. The technique for descending from Λ to μ is known as the “block-spin renormalization group,” [9] and consists in the present case of integrating out all loops with internal momenta $\Lambda \geq p \geq \mu$. We will see that eq.(2.3) defines the renormalization group boundary conditions for the full solution to the theory of eq.(2.1) at a scale μ . The auxiliary field introduced at the scale Λ will become the propagating physical (Higgs) field at low energies $\mu \ll \Lambda$.

The block spin renormalization group transformation performed on eq.(2.1) in flat space generates the following effective Lagrangian at a scale μ [3]:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{kinetic} + (\bar{\psi}_L \psi_R H + h.c.) \\ & + Z_H |D_\mu H|^2 - M_\mu^2 H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2 + \dots \end{aligned} \quad (2.5)$$

We have included the standard induced gauge invariant kinetic terms of the Higgs doublet. The wave-function normalization constant, Z_H , and the induced quartic

interaction arising from fermion loops have been calculated to be [3],

$$Z_H = \frac{N}{16\pi^2} \ln(\Lambda^2/\mu^2), \quad (2.6)$$

$$\lambda_0 = \frac{2N}{16\pi^2} \ln(\Lambda^2/\mu^2). \quad (2.7)$$

The mass term M_μ^2 is quadratically divergent,

$$M_\mu^2 = M_\Lambda^2 - \frac{N}{8\pi^2} (\Lambda^2 - \mu^2). \quad (2.8)$$

The evolution of this term is ultimately a matter of our choice of defining the two parameters M_Λ and Λ . One can “fine-tune” the theory by demanding an approximate cancellation between the large terms, M_Λ^2 and $N\Lambda^2/(8\pi^2)$ in eq.(2.8). We then see that M_μ^2 can become negative as $\mu \rightarrow 0$. This triggers the instability in the vacuum at that scale, leading to the formation of a symmetry breaking phase at low energies.

Conventionally one renormalizes the kinetic terms of a field theory at any scale, μ , with a condition that they have free-field theory normalization. Indeed, this is an intermediate step in the block-spin RG transformation as described by Kogut and Wilson [9]. In the previous discussion we chose not to insist upon this because of the singular behavior of Z_H as in eq.(2.6). However, we can transfer this singularity to a condition on coupling constants in the conventional normalization. That is, we may exercise our freedom of rescaling the various fields, H and ψ , to define the coefficient of $|D_\mu H|^2$ to be unity. In the present case $H \rightarrow H/\sqrt{Z_H}$, and the conventionally normalized Lagrangian becomes

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{kinetic} + (g_\psi \bar{\psi}_L \psi_R H + h.c.) \\ & + |D_\mu H|^2 - m_\mu^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 + \dots \end{aligned} \quad (2.9)$$

where the physical coupling constants, m_μ^2 , g_ψ , and λ , are given by

$$m_\mu^2 = M_\mu^2/Z_H, \quad g_\psi^2 = \frac{1}{Z_H}, \quad \lambda = \frac{1}{Z_H^2} \lambda_0. \quad (2.10)$$

It is clear from eqs.(2.10) that as $\mu \rightarrow \Lambda$, g_ψ and λ diverge while g_ψ^2/λ approaches a constant.

Let us now examine the low energy symmetry breaking phase. We assume that we have tuned the theory to produce the low energy potential in H ,

$$V(H) = -m_H^2 H^\dagger H + \frac{\lambda}{2} (H^\dagger H)^2, \quad (2.11)$$

where $m_H = m_{\mu=0}$ and eq.(2.11) is the low energy limit of the potential contained in eq.(2.9). The Higgs field, $H = v + \phi/\sqrt{2}$, may be expanded about its vacuum expectation value v which is given by

$$v^2 = m_H^2/\lambda. \quad (2.12)$$

Moreover, by substituting the shift in eq.(2.11) we can read off the induced mass of the ϕ excitation,

$$m_\phi^2 = 2v^2\lambda. \quad (2.13)$$

The VEV of H implies an induced mass for ψ ,

$$m_\psi = g_\psi v. \quad (2.14)$$

Combining the above equations, we find that the mass of the scalar field is just twice that of the fermion,

$$m_\phi^2/m_\psi^2 = 2\lambda/g_\psi^2 = 2\lambda_0/Z_H = 4. \quad (2.15)$$

We have thus derived this familiar result entirely from a block-spin renormalization group analysis, and we have essentially kept track only of the logarithmically evolving

terms, Z_H and λ_0 , or equivalently λ and g_ψ . The quadratically evolving M_μ^2 is only tuned to produce the vacuum instability in the infra-red limit.

It is useful to derive this result again from the perspective of the differential renormalization group equations. To obtain this renormalization group description of the NJL model we utilize the partial β -functions which are calculated using only the fermion loops:

$$16\pi^2 \frac{dg_\psi}{dt} = Ng_\psi^3, \quad (2.16)$$

$$16\pi^2 \frac{d\lambda}{dt} = (-4Ng_\psi^4 + 4Ng_\psi^2\lambda). \quad (2.17)$$

The appropriate boundary conditions are typically dictated by the behavior of eqs.(2.16, 2.17) as $\mu \rightarrow 0$. Alternatively, these may be replaced by limits as $\mu \rightarrow \Lambda$,

$$Z_H \rightarrow 0|_{\mu \rightarrow \Lambda}, \quad (2.18)$$

$$\lambda_0 \rightarrow 0|_{\mu \rightarrow \Lambda}, \quad (2.19)$$

or equivalently, in the conventional normalization,

$$g_\psi(\mu) \rightarrow \infty|_{\mu \rightarrow \Lambda}, \quad (2.20)$$

$$\lambda \rightarrow \infty|_{\mu \rightarrow \Lambda}, \quad (2.21)$$

Now, solving eq.(2.16) gives

$$\frac{1}{g_\psi^2(\mu)} = \frac{N}{16\pi^2} \ln(\Lambda^2/\mu^2), \quad (2.22)$$

where we use the boundary condition, $1/g_\psi^2(\Lambda) = 0$. Eq.(2.17) may then be solved by

hypothesizing an ansatz of the form $\lambda = cg_\psi^2$. Substituting into eq.(2.17) one finds,

$$16\pi^2 \frac{dg_\psi}{dt} = \frac{1}{2c}(4c - 4)Ng_\psi^3, \quad (2.23)$$

which must be consistent with eq.(2.16). Thus $c = 2$ and

$$\frac{1}{\lambda(\mu)} = \frac{N}{32\pi^2} \ln(\Lambda^2/\mu^2). \quad (2.24)$$

Note that the solutions eqs.(2.22, 2.24) are equivalent to those of eq.(2.6, 2.7) with the identifications of eq.(2.10). Again we recover the NJL relation

$$m_\phi^2/m_\psi^2 = 2\lambda/g_\psi^2 = 4 \quad (2.25)$$

from eqs.(2.16, 2.17).

B. Incorporating Gravity

We now assume that the field theory of eq.(2.1) is placed in a weak background gravitational field,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (2.26)$$

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Again we may proceed to eq.(2.3) with the auxiliary field H as the effective Lagrangian at the scale Λ .

In addition to the diagrams of Fig.(1) that led to eq.(2.9), we will now include the emission and absorption of gravitons. For example, the diagram of Fig.(2) represents the insertion of the fermionic stress-energy tensor,

$$\frac{1}{2}T_{\mu\nu} = \frac{i}{8}\bar{\psi} \left[\delta_\mu^\alpha \gamma_\nu + \delta_\nu^\alpha \gamma_\mu + g_{\mu\nu}(-2\cancel{\partial} - 4im) \right] \psi. \quad (2.27)$$

It contributes $O(h_{\mu\nu})$ terms to the covariant kinetic and mass terms

$$\sqrt{g} \left[g_{\mu\nu} (\partial^\mu H)^\dagger (\partial^\nu H) - M^2 H^\dagger H \right], \quad (2.28)$$

which are related to the \sqrt{g} and $g_{\mu\nu}$ factors. However, the diagram of Fig. (2) also generates a term $q^2 g_{\mu\nu} - q_\mu q_\nu$, which is an induced coupling of the form $RH^\dagger H$, where R is the scalar curvature. Thus, the effective Lagrangian at the scale μ is now found to be:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{kinetic} + (\bar{\psi}_L \psi_R H + h.c.) \\ & + Z_H |D_\mu H|^2 - M_\mu^2 H^\dagger H - \xi_0 R H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2 + \dots \end{aligned} \quad (2.29)$$

The result of explicitly calculating Fig.(2) is

$$\xi_0 = \frac{1}{6} \frac{N}{(16\pi^2)} \ln(\Lambda^2/\mu^2) \quad (2.30)$$

We remark that the $\xi_0 R H^\dagger H$ can be inferred directly from the diagram of Fig.(2) in the symmetric phase of the theory ($m_\psi = 0$), or from Fig.(3) in the broken phase, when $\langle H \rangle = v \neq 0$ and the nonminimal term becomes $\sqrt{2}\xi_0 R v \phi$. The results for ξ_0 are, of course, the same.

In conventional normalization, the Lagrangian is

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{kinetic} + (g_\psi \bar{\psi}_L \psi_R H + h.c.) \\ & + |D_\mu H|^2 - m_H^2 H^\dagger H - \xi R H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 + \dots \end{aligned} \quad (2.31)$$

and therefore the physically observed nonminimal coupling constant, ξ is given by

$$\xi = \frac{\xi_0}{Z_H} \equiv \xi_0 g_\psi^2 = \frac{1}{6}. \quad (2.32)$$

Eq.(2.32) is the central result of this paper. We see that the compositeness conditions as derived from the block-spin RG analysis are now supplemented by:

$$\xi \rightarrow \frac{1}{6} \Big|_{\mu \rightarrow \Lambda} \quad (2.33)$$

The composite scalar field in the NJL model is conformally coupled to gravity.

It is interesting to study this result in the differential renormalization group (RG). By introducing the differential operator,

$$D \equiv 16\pi^2 \frac{\partial}{\partial \ln \mu}, \quad (2.34)$$

the RG equation for $\xi = \xi_0/Z_H$ can be derived by considering,

$$D\xi = D(\xi_0)/Z_H - \xi_0 D(Z_H)/Z_H^2. \quad (2.35)$$

In the fermion bubble approximation, we know from (2.10) that $g_\psi^2 = 1/Z_H$ and hence,

$$Dg_\psi^2 = 2Ng_\psi^4. \quad (2.36)$$

Eq.(2.35) then becomes,

$$D\xi = -\frac{N}{3}g_\psi^2 + 2N\xi g_\psi^2, \quad (2.37)$$

and the solution for the curvature coupling parameter,

$$\xi(\mu) = 1/6, \quad (2.38)$$

is a constant for all scales. More generally, as one descends toward the infra-red, $\xi = 1/6$ is an attractive fixed point. Therefore, no matter what is the initial value for ξ at the large scale Λ , given enough RG running time ξ will eventually reach $1/6$ for small μ . Of course, the RG running only occurs for scales $\mu > m_H$, since for $m_H > \mu$ the fermion loops decouple.

The RG equation (2.37) is exact to all orders of N when only effects of the Higgs-Yukawa coupling constant are kept, as well as any gauge interactions with fermions which are not shared by ϕ (*e.g.*, QCD effects). In more elaborate discussions of the RG equations for ξ it is always found that the right hand side is proportional to $(\xi - 1/6)$ [2] at the one-loop level. Hence, the general result that $\xi = 1/6$ is expected to be valid modulo two-loop effects.

III. Conclusions

We have exploited the effectiveness of the RG to give an analysis of the induced coupling of a composite scalar particle to gravitation. The appearance of quasi-infrared fixed points is interesting, and desensitizes the prediction to the details at the composite scale Λ . This analysis sheds light on the issue of nonminimal coupling, which is usually viewed as an arbitrary users choice, but which is potentially dictated by either the composite nature of a scalar boson or by the full dynamics of the coupling to other fields.

However, if one considers an arbitrary, composite or noncomposite scalar field, then the renormalization group equation for ξ , eq.(2.37), remains valid although one then disregards the boundary condition (2.33). As one descends in energy scale, ξ will approach its infrared fixed point value of $1/6$. On the other hand, if one evolves to higher energy scales, ξ can become quite large and negative. Hence, the running of the curvature coupling parameter could be important for inflationary cosmology which serves as a probe of very small length scales.

Some mysteries remain. For example, why does the leading the classically conformally invariant result $\xi = 1/6$ obtain? There are, afterall, scale breaking effects at the scale Λ as well as m_H . It is important to note that generally trace anomalies are measures of β -functions, and this theory has a non-vanishing matter trace anomaly;

hence, we might expect propagation of the scale breaking effects into ξ . It would be illuminating to study the full trace-anomaly structure in relationship to the RG structure of the theory to better understand when scale invariant ξ values will obtain, and why particular non-scale invariant results also can occur. We feel there may be deeper relationships between fine-tuning, approximate scale-invariance, nonminimal coupling to gravity, and dynamical symmetry breaking than we have appreciated.

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Fig.(1) Fermion loops induce Z_H and λ_0 for the auxilliary Higgs field in the NJL model.

Fig.(2) A massless fermion loop induces the nonminimal coupling between the graviton and the composite Higgs in the symmetric phase of the theory.

Fig.(3) The coupling of the graviton to the Higgs can equivalently be computed in the broken phase where the fermion is massive, $m_\psi = g_\psi \langle H \rangle = g_\psi v \neq 0$.

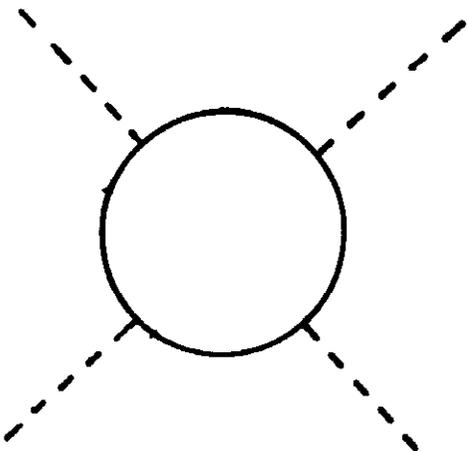
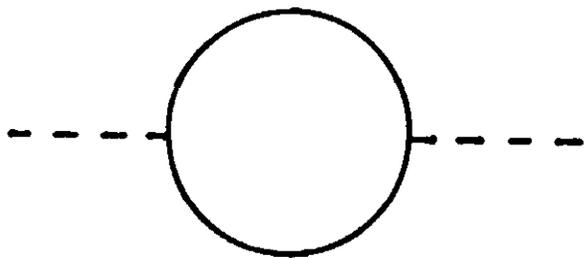


Fig. (1)

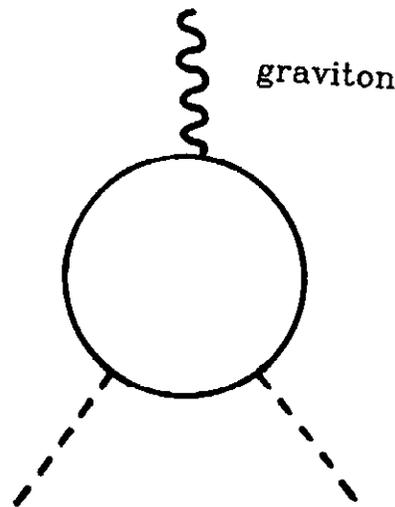
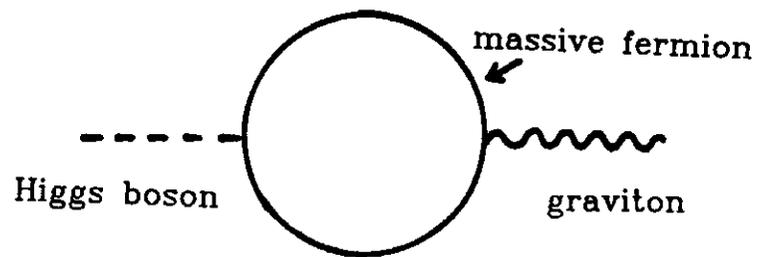


Fig. (2)



Higgs boson

graviton

Fig. (3)