Fermi National Accelerator Laboratory

FERMILAB-Pub-91/30-T
February 5, 1991

# Rates for top quark production

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Total rates for the production of top quarks at Tevatron energies are presented and compared with earlier work.

# 1. The theory of top quark production

The short distance cross section for the production of a heavy quark of mass $m$ is known up to order $\alpha_S^3$ [1,2]. At the parton level the total cross section may be written as a systematic expansion in the running coupling.

$$\hat{\sigma}_{ij}(s,m^2) = \frac{\alpha_S^2(\mu)}{m^2} f_{ij}\Big(\rho, \frac{\mu^2}{m^2}\Big) \tag{1.1}$$

Eq.(1.1) completely describes the short distance cross-section for the production of a heavy quark of mass $m$ in terms of the functions $f_{ij}$, where the indices $i$ and $j$ specify the types of the annihilating partons. The dimensionless functions $f_{ij}$ have the following perturbative expansion,

$$f_{ij}\Big(\rho, \frac{\mu^2}{m^2}\Big) = f_{ij}^{(0)}(\rho) + 4\pi\alpha_S(\mu)\Big[f_{ij}^{(1)}(\rho) + \overline{f}_{ij}^{(1)}(\rho)\ln\Big(\frac{\mu^2}{m^2}\Big)\Big] + O(\alpha_S^2) \tag{1.2}$$

where $\rho$ is defined as $4m^2/\hat{s}$. The functions $f_{ij}^{(1)}$ and $\overline{f}_{ij}^{(1)}$ are completely known[1]. The full calculation involves both real and virtual corrections to the Born cross-section. For full details I refer the reader to refs. [1,2]. In order to calculate the $f_{ij}$ in perturbation theory one must perform both renormalisation and factorisation of mass singularities. The subtractions required for renormalisation and factorisation are done at mass scale $\mu$. All dependence on the scale $\mu$ is shown in Eq.(1.2).

The quantities $f^{(1)}$ depend on the scheme used for renormalisation and factorisation. Therefore one must first specify the choices made in the definition of $f^{(1)}$. The results of ref. [1] are obtained in an extension of the $\overline{MS}$ renormalisation and factorisation scheme [3]. At one loop order, the renormalisation scheme is completely specified as follows. Graphs containing a light parton loop are renormalised using the normal $\overline{MS}$ subtraction scheme. The following renormalisation conditions are chosen for $\Gamma^{(2)}(p,m)$, the two point function of the heavy quark field,

$$\Gamma^{(2)}(p,m)\big|_{p^2=m^2} = 0 \tag{1.3}$$

$$\frac{d}{d\hat{p}}\Gamma^{(2)}(p,m)\big|_{p^2=0} = 1, \quad \hat{p} = \gamma^\mu p_\mu. \tag{1.4}$$

Eq.(1.3) implies that the mass $m$ corresponds to a pole in the renormalised propagator. Eq.(1.4) fixes the wave function renormalisation for the heavy quark field.

Eqs. (1.3,1.4) are sufficient to show that the anomalous dimensions associated with the mass renormalisation and the renormalisation of the heavy quark field are equal to zero. The renormalisation constant for the gluon-$Q$-$\overline{Q}$ vertex is then fixed by the Taylor-Slavnov identity. This completely specifies the treatment of primitively divergent graphs with heavy quarks on external lines. Graphs containing internal loops of heavy quarks are subtracted at zero momentum. In this scheme heavy quarks are decoupled at low energy [4]. The light partons continue to obey the same renormalisation group equation as they would have done in the absence of the heavy quarks. Thus the results of ref. [1] should be used in conjunction with the running coupling as defined in Eq.(11) and together with light parton densities evolved using the two loop $\overline{MS}$ evolution equations.

I now consider a more physical factorisation scheme which can be defined for the parton distribution functions [8,10]. In this scheme the quark distributions are defined directly in terms of the DIS structure function $F_2$. The $O(\alpha_S)$ corrections are completely absorbed into the definitions of the distribution functions. The 'physical' $f_{ij}^{(1p)}$ and $f_{gq}^{(1p)}$ are defined as follows,

$$f_{ij}^{(1p)}(\rho) = f_{ij}^{(1)}(\rho) - \sum_k \int_\rho^1 dz_1 f_{ik}^{(0)}(\frac{\rho}{z_1}) c_{kj}(z_1) - \sum_k \int_\rho^1 dz_2 f_{kj}^{(0)}(\frac{\rho}{z_2}) c_{ki}(z_2) \tag{1.5}$$

where the $c_{ij}(z)$ are given by [8],

$$\begin{aligned}
c_{qq}(z) \equiv c_{\overline{q}\overline{q}}(z) &= \frac{4}{3}\frac{1}{8\pi^2}\Bigg\{(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{2}\left[\frac{1}{1-z}\right]_+ \\
&\quad -(1+z^2)\frac{\ln z}{1-z} + 3 + 2z - \Big(\frac{9}{2}+\frac{\pi^2}{3}\Big)\delta(1-z)\Bigg\} \\
c_{qg}(z) &= \frac{1}{2}\frac{1}{8\pi^2}\left\{\Big(z^2+(1-z)^2\Big)\ln\Big(\frac{1-z}{z}\Big) - 1 + 8z(1-z)\right\} \\
c_{gq}(z) \equiv c_{g\overline{q}}(z) &= -c_{qq} \\
c_{gg}(z) &= -2n_{\mathrm{lf}} c_{qg}
\end{aligned} \tag{1.6}$$

and the plus distributions are given by,

$$\int_0^1 dz f(z)\Big[g(z)\Big]_+ = \int_0^1 dz(f(z)-f(1))g(z) \tag{1.7}$$

The specification of $c_{gg}$ and $c_{gq}$ is not fixed by deep inelastic scattering. The choice shown above follows the procedure of DFLM [10]. It is a possible choice which has the advantage of preserving the momentum sum rule. Note that the form of $c_{qg}$ given in Eq.(37) of ref. [1] is in error. The expresssion used in refs.[8,1] is not exactly in the $\overline{MS}$ scheme because initial gluon spins were averaged in 4 rather than in $n$ dimensions.

## 2. Phenomenological results

Accurate values of the top quark cross section are needed to set limits on the top quark mass from present data. In addition, once the top quark has been observed, the measured cross section can be used to estimate the mass. For the setting of limits one is interested in the lowest theoretically acceptable value of the top quark cross section. The estimate of the mass from an observed top quark signal requires the best prediction for the central value.

The estimate of theoretical errors is an *ad hoc* procedure, for which little justification can be given. For definiteness, I shall vary $\Lambda_5$ in the range $60 < \Lambda_5 < 250$ MeV. The subscript on $\Lambda$ indicates the number of active flavours. In addition, I shall consider variations of the renormalisation and factorisation scale in the range $m/2 < \mu < 2m$.

The sets of parton distributions which I shall use in the phenomenological analysis are due to DFLM[10] and HMRS[11]. The DFLM sets are non-leading fits to the data and are available in three forms with $\Lambda_4 = 160, 260, 360$ MeV ($\Lambda_5 = 100, 170, 250$ MeV). These sets are in the 'physical' scheme specified by Eq.(1.6). The HMRS sets are in the $\overline{MS}$ scheme. I now describe the four distributions due to HMRS which I use. The first set (HMRSE) are based predominantly on the measurements of EMC[12] and have $\Lambda_4 = 100$ MeV, ($\Lambda_5 = 60$ MeV). The second set (HMRSB) is based on the measurements of BCDMS[13] which yield a central value for $\Lambda_4 = 200$ MeV, ($\Lambda_5 = 122$ MeV). Since July 1990 a further two BCDMS-type sets have been available. They are fits to the same data as the above-mentioned BCDMS set but with $\Lambda$ constrained to be either $\Lambda_4 = 100$ MeV ($\Lambda_5 = 60$ MeV) or $\Lambda_4 = 300$ MeV ($\Lambda_5 = 205$ MeV). These latter sets represent the variation of the parton distributions as $\Lambda$ is changed. They can be used to bound the theoretical

| $m_t$ [GeV] | DFLM<br>$\Lambda_5 = 170$ MeV<br>$\mu = m$<br>$\sigma$[pb] | DFLM<br>$\Lambda_5 = 250$ MeV<br>$\mu = m/2$<br>$\sigma$[pb] | DFLM<br>$\Lambda_5 = 100$ MeV<br>$\mu = 2m$<br>$\sigma$[pb] |
|---|---|---|---|
| 40 | 9390. | 11550. | 7310. |
| 60 | 1260. | 1490. | 995. |
| 80 | 285. | 335. | 229. |
| 100 | 88.3 | 103. | 72.1 |
| 120 | 33.8 | 39.0 | 27.8 |
| 140 | 14.8 | 16.7 | 12.3 |
| 160 | 7.08 | 7.87 | 5.99 |
| 180 | 3.60 | 3.94 | 3.11 |
| 200 | 1.92 | 2.05 | 1.68 |
| 220 | 1.05 | 1.12 | 0.940 |
| 240 | 0.591 | 0.619 | 0.536 |
| 260 | 0.336 | 0.349 | 0.309 |
| 280 | 0.194 | 0.199 | 0.181 |
| 300 | 0.112 | 0.114 | 0.106 |

Table 1: Total cross section for top quark production.

predictions for the cross sections in a similar way to the DFLM set.

I shall begin by comparing results with the earlier results [5] of Altarelli, Diemoz, Martinelli and Nason, (ADMN) using the DFLM structure functions. My results for $\Lambda_5 = 250$ and 170 MeV are in approximate agreement with the values found by ADMN. My cross section for $\Lambda_5 = 100$ MeV is higher than the results of ADMN. The discrepancy is due to the fact that the results of ADMN for $\Lambda_5 \approx 100$ MeV were derived using an outdated set of the DFLM structure functions[1]. The cross section for the smallest value of $\Lambda$ effectively sets the lower limit, so my estimate of the lower bound on top quark production differs from the value of ADMN. The cross section limits given in ref. [5] do *not* follow from the $\Lambda_5 \approx 100$ MeV DFLM structure functions.

[1]I am grateful to G. Martinelli for information on this point.

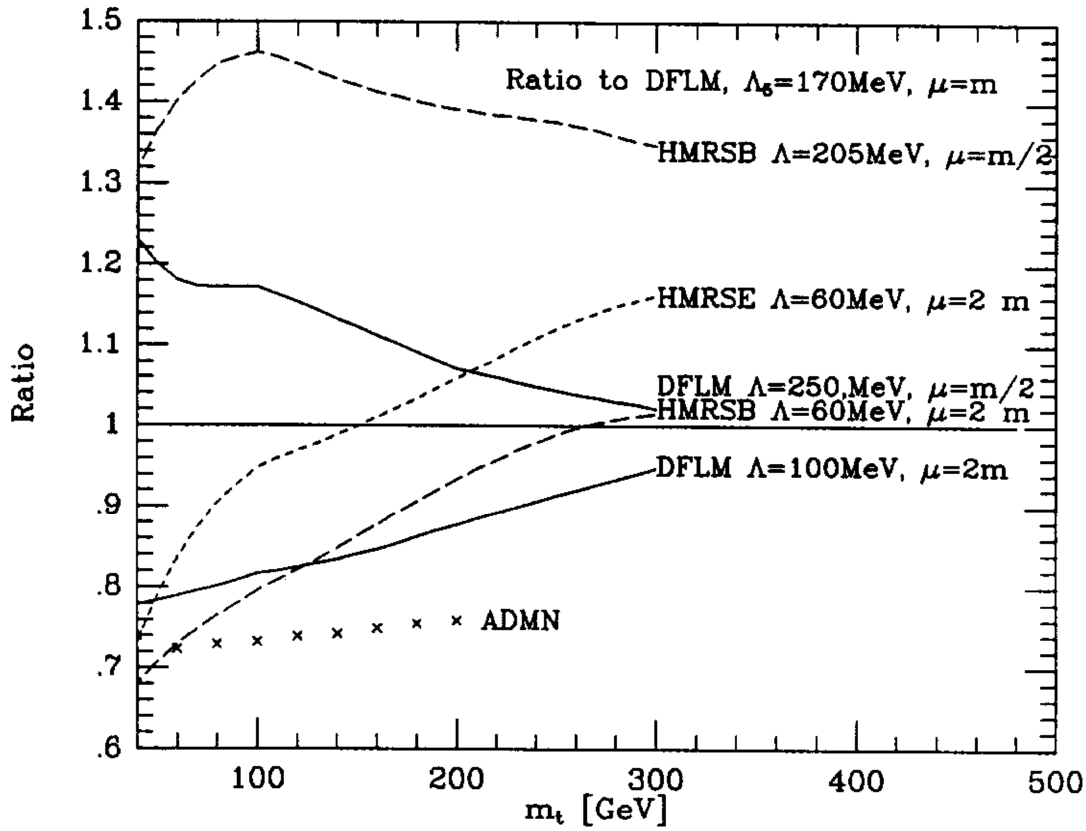


Figure 1: Ratio of extreme values to DFLM central value

| $m_t$ [GeV] | $\sigma$[pb] | $m_t$ [GeV] | $\sigma$[pb] | $m_t$ [GeV] | $\sigma$[pb] |
|---|---|---|---|---|---|
| 40 | 6392. | 130 | 18.3 | 220 | 0.940 |
| 50 | 2228. | 140 | 12.3 | 230 | 0.708 |
| 60 | 920. | 150 | 8.51 | 240 | 0.536 |
| 70 | 428. | 160 | 5.99 | 250 | 0.406 |
| 80 | 218. | 170 | 4.29 | 260 | 0.309 |
| 90 | 120. | 180 | 3.11 | 270 | 0.236 |
| 100 | 70.3 | 190 | 2.28 | 280 | 0.181 |
| 110 | 43.2 | 200 | 1.68 | 290 | 0.139 |
| 120 | 27.8 | 210 | 1.25 | 300 | 0.106 |

Table 2: Estimate of lower limit on top quark production at $\sqrt{S} = 1.8$ TeV.

Fig. 1 shows my results on the top quark cross section. The results are plotted relative to the central prediction using the DFLM structure functions. Results using the DFLM structure functions are denoted by solid lines in Fig. 1. For the purposes of setting limits on the top quark mass, I shall estimate the lower limit of the top quark cross section to be given by the envelope of minimum values given in Fig. 1 including also the HMRS structure function result where appropriate. Note that this is an inherently more conservative procedure than was adopted in ref. [5], because a) my analysis allows a larger range of $\Lambda$ than ADMN, and b) ADMN estimate errors by adding deviations from the central value due to independent variations of $\Lambda$ and $\mu$ in quadrature. The lower limits derived by ADMN are also shown plotted in Fig. 1. The lower limits of ADMN are too conservative, especially at higher values of the top mass, by as much as 18%. My estimate of the lower limit is given in Table. 2. The effect which these limits have on the top mass is shown in Fig. 2. Fig. 2 also shows the cross section coresponding to the upper extremum in Fig. 1. For $m_t \approx 80$ GeV, the change in the limit from the results of ADMN (denoted by crosses) is quite small. This region is shown in detail in Fig. 3. For $m_t \approx 160$ GeV the limit on the top quark mass is increased by about 3 GeV.

I now turn to the estimate of central values. These are as displayed in Fig. 4, again related to the central value of DFLM. Above $m_t = 160$ GeV, the *central* predictions using HMRS lie above the uncertainty band suggested by the DFLM structure

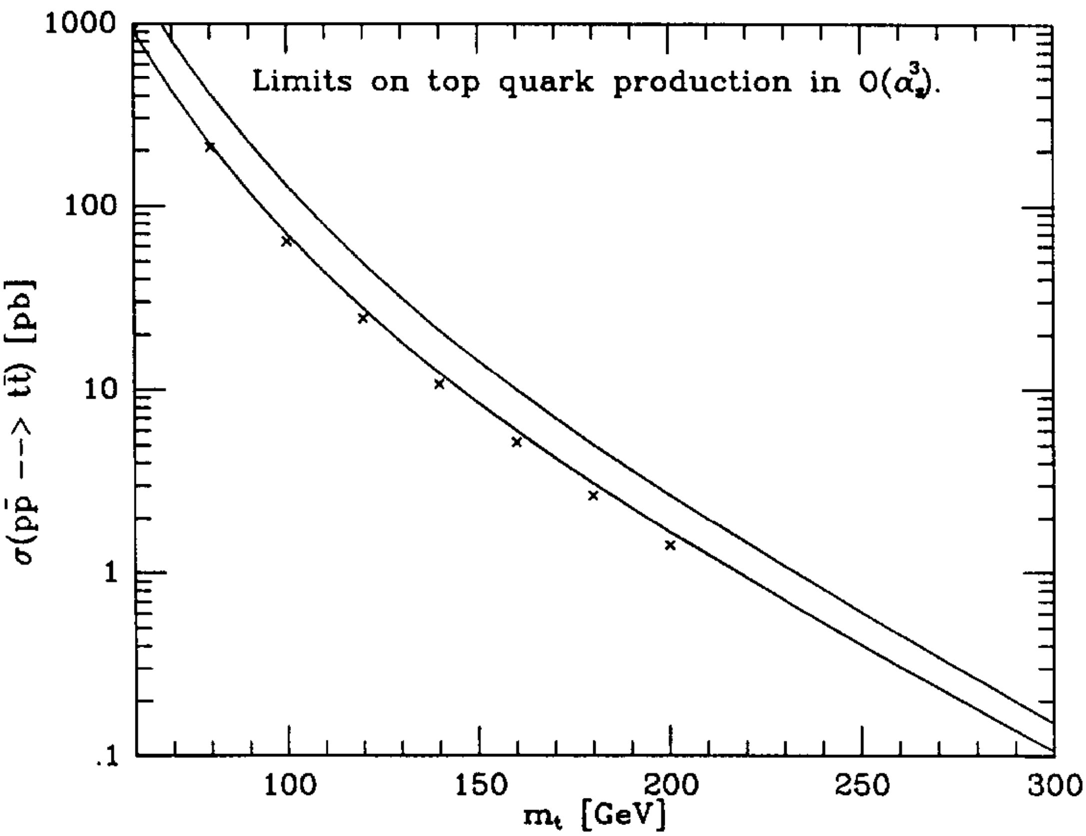


Figure 2: Upper and lower limits on top quark production

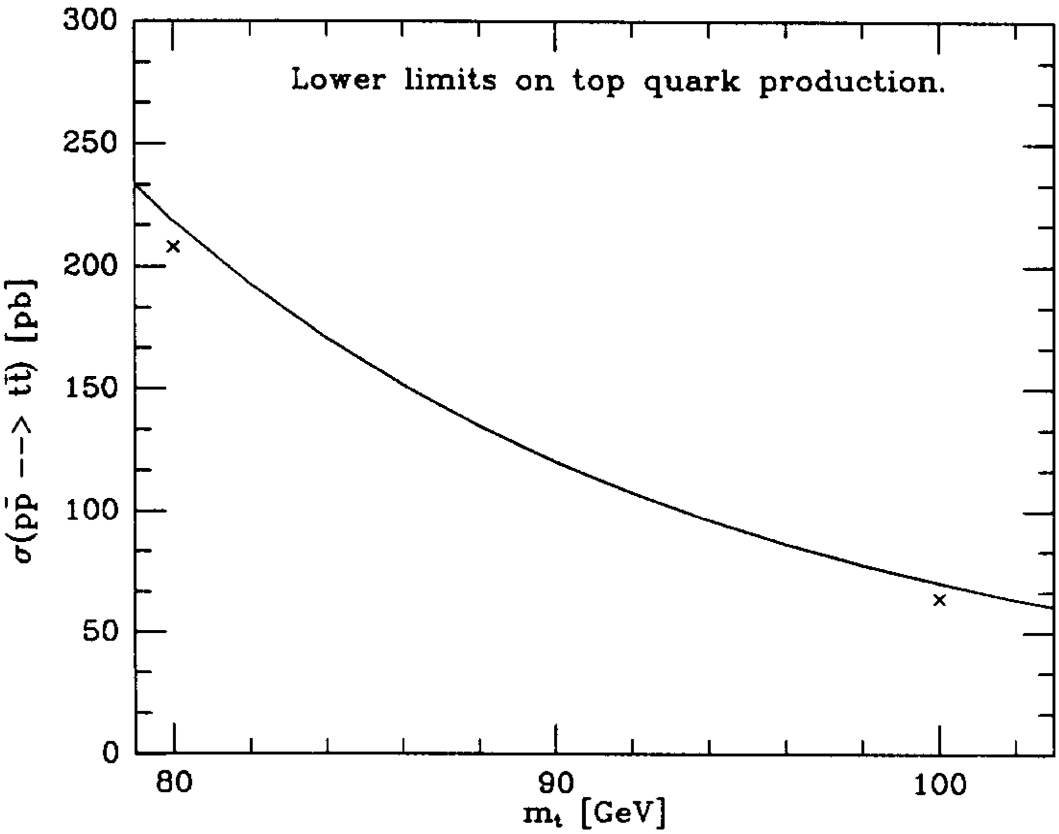


Figure 3: Lower limits in the region of current interest

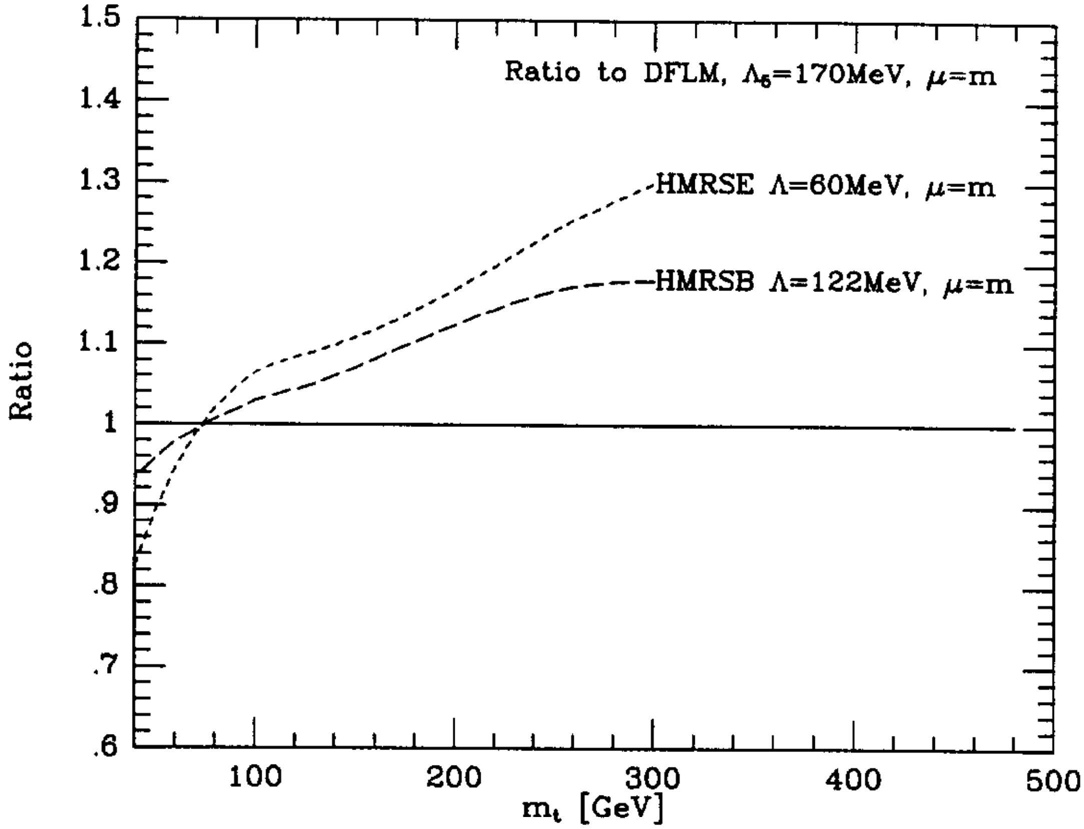


Figure 4: Ratio of the central values from HMRS-B and HMRS-E to the central value from DFLM.

functions. The central values given by HMRS structure functions are given in Table 3 for $\sqrt{s} = 1.8$ TeV. The corresponding results at $\sqrt{s} = 2$ TeV are given in Table 4.

## 3. Conclusions

I have a re-examined the predictions for top quark production using the best available theoretical and phenomenological information. For $m_t > 60$ GeV the estimates of ref. [5] are found to be too conservative because of use of an outdated structure function. New lower limits on the top quark cross section are given in Table 2.

By use of HMRS structure functions I also conclude that the upside errors are not

| $m_t$ [GeV] | HMRSE<br>$\Lambda_5 = 60$ MeV<br>$\mu = m$<br>$\sigma$[pb] | HMRSB<br>$\Lambda_5 = 122$ MeV<br>$\mu = m$<br>$\sigma$[pb] |
|---|---|---|
| 40 | 7740. | 8760. |
| 60 | 1190. | 1232. |
| 80 | 291. | 287. |
| 100 | 94. | 90.9 |
| 120 | 36.6 | 35.3 |
| 140 | 16.2 | 15.6 |
| 160 | 7.91 | 7.65 |
| 180 | 4.12 | 3.98 |
| 200 | 2.24 | 2.16 |
| 220 | 1.26 | 1.20 |
| 240 | 0.72 | 0.69 |
| 260 | 0.42 | 0.39 |
| 280 | 0.25 | 0.23 |
| 300 | 0.14 | 0.13 |

Table 3: Central values of total cross section for top quark production with HMRS structure functions at $\sqrt{s} = 1.8$ TeV.

| $m_t$ [GeV] | HMRSE<br>$\Lambda_5 = 60$ MeV<br>$\mu = m$<br>$\sigma$[pb] | HMRSB<br>$\Lambda_5 = 122$ MeV<br>$\mu = m$<br>$\sigma$[pb] |
|---|---|---|
| 40 | 9560. | 11000. |
| 60 | 1520. | 1595. |
| 80 | 380. | 378. |
| 100 | 125. | 120. |
| 120 | 48.9 | 47.0 |
| 140 | 21.9 | 21.1 |
| 160 | 10.8 | 10.4 |
| 180 | 5.72 | 5.52 |
| 200 | 3.18 | 3.07 |
| 220 | 1.84 | 1.77 |
| 240 | 1.09 | 1.04 |
| 260 | 0.66 | 0.63 |
| 280 | 0.40 | 0.38 |
| 300 | 0.25 | 0.23 |

Table 4: Central values of total cross section for top quark production with HMRS structure functions at $\sqrt{s} = 2$ TeV.

accurately estimated by using the DFLM structure functions alone. The top cross section may be larger than previously estimated using the DFLM structure functions. This conclusion would also be supported by the larger values of $\Lambda$ measured recently in $e^+e^-$ annihilation[14].

The effects detailed in this paper are quite small. However, since they lead to a slightly more optimistic picture of top quark production at Tevatron energies, they may be of interest to the experimental community.

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