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SOME THEORETICAL ISSUES IN NEUTRON ELECTRIC DIPOLE MOMENT

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ABSTRACT

Recent works on various contributions to the neutron electric dipole moment are reported. In particular, the relative importance of the contributions due to $\theta-$ term, chromo-electric dipole moments of quarks and gluon, and dimension-8 gluonic operators are analyzed.

The mechanism of CP violation in a given theory typically involves heavy particles like the top quark in the Kobayashi Maskawa model, the right handed boson in left-right models, the Higgs boson in Higgs-mediated theories or the susy particles in supersymmetric theories. At the energy scale of the W -boson mass or below, these heavy particles can be integrated out and the resulting effective theory may contain many new operators that can be used to account for the CP violating effect of these heavy particles at low energy. Most of these operators are higher dimensional. Here we are interested in the operators that will contribute to one of the most well-measured CP violating quantities: the neutron electric dipole moment.

For these CP-violating operators to contribute to the neutron electric dipole moment, D_n, one expects the leading ones to be flavor neutral and strongly interacting. A catalogue of such operators includes the strong-CP $\theta-$operator[1],

$$O_\theta = (g^2/64\pi^2)G^a_{\mu\nu}G^a_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}, \qquad (1)$$

the electric and chromo-electric dipole moment of the light quark, $O^\gamma_q = (e/2)F_{\mu\nu}\bar{q}i\sigma^{\mu\nu}\gamma_5 q$ and $O^c_q = (g/2)G^a_{\mu\nu}\bar{q}i\sigma^{\mu\nu}\gamma_5(\lambda^a/2)q$, or of the heavy quarks, $O^\gamma_Q = (e/2)F_{\mu\nu}\bar{Q}i\sigma^{\mu\nu}\gamma_5 Q$ and

$$O^c_Q = (g/2)G^a_{\mu\nu}\bar{Q}i\sigma^{\mu\nu}\gamma_5(\lambda^a/2)Q, \qquad (2)$$

the dimension-6, chromo-electric dipole moment operator [2,3,4],

$$O_g = (g^3/6)f_{abc}G^a_{\mu\alpha}G^{b\alpha}{}_\nu G^c_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}, \qquad (3)$$

the dimension-8, purely gluonic operators of Morozov[5,6],

$$\begin{aligned} O_{8,1} &= g^4\tfrac{1}{12}\tilde{G}^a_{\mu\nu}G^{a\mu\nu}G^b_{\alpha\beta}G^{b\alpha\beta} \quad , \\ O_{8,2} &= g^4\tfrac{1}{12}\tilde{G}^a_{\mu\nu}G^{b\mu\nu}G^a_{\alpha\beta}G^{b\alpha\beta} \quad , \\ O_{8,3} &= g^4\tfrac{1}{12}d^{abe}d^{ecd}\tilde{G}^a_{\mu\nu}G^{b\mu\nu}G^c_{\alpha\beta}G^{d\alpha\beta} \end{aligned} \qquad (4)$$

d^{abc} is the totally symmetric tensor of $SU(3)$. There are also two dimension-8 operators with one electromagnetic field strength, $F^{\mu\nu}$, and three $G^{a\mu\nu}$'s [7], three with two $F^{\mu\nu}$'s and two $G^{a\mu\nu}$'s [8] and one with four $F^{\mu\nu}$'s. In this report, we shall ignore these photon-gluonic operators because they typically do not give rise to the leading constraint. The discussion on the light quark operators, O^γ_q and O^c_q, can be found in [7].

The heavy quark operators, O^γ_Q, O^c_Q, are typically induced at the one loop level when their heavier partners are integrated out at high energy. Therefore one has to use renormalization group (RG) machinery to evolve them to lower energy. The most typical example is that of the b quark. The RG effects are suppressive for these operators. At the threshold of this heavy quark, typically higher dimensional gluonic operators are induced. This mechanism for inducing the dimension-6 O_g is widely discussed in the literature[9]. The result can be translated into constraints on the coefficients of O^γ_Q and O^c_Q at high energy. For the b quark, it is about 2×10^{-6}[7] in unit of M_W^{-1}. Such a value can be realized in SUSY or charged Higgs models. For the c quark, it is around 10^{-7} and it can be realized in the SUSY model too. Recently, we

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calculated the contribution to the dimension-8 operators[6,8]. I shall briefly summarize below.

The effective operators of dimension 6 and 8 after integrating out the quark of mass m can be written as,

$$\begin{aligned} S_{\text{eff}} &= S_{QCD} + S_{\text{light quarks}} + \\ & \int d^4x \left[C_6 O_6 + \sum_{i=1}^{3} C_{8,i} O_{8,i} \right] \quad , \end{aligned} \tag{5}$$

$$\begin{aligned} C_6 = \frac{C}{32\pi^2 m}, \quad C_{8,1} = -\frac{C}{96\pi^2 m^3}, \\ C_{8,2} = 0, \quad C_{8,3} = -\frac{C}{64\pi^2 m^3}, \end{aligned} \tag{6}$$

where C is the coefficient of the CEDM operator in Eq. (2). In the subsequent RG evolution, the three operators mix and the three eignvalues of the anomalous dimension matrix are 4.82, −42.98, and −20.84. The operator that is RG enhanced is mainly composed of $O_{8,1}$. After these operators are evolved further down to hadronic scale, only one of them remains significant. The hadronic matrix element and the size of the NEDM can be estimated using naive dimensional analysis[10]. The resulting numerical ratio between $|D_N(O_6)|$ and $|D_N(O'_{8,i})|$ gives

$$|D_N(O'_{8,1})|/|D_N(O_6)| \simeq 3.6 \quad , \tag{7}$$

using the same set of input parameters as in Ref.[2]. The result shows the potential importantance of the contribution from the dimension 8 operators. Since it is known that the O_6 contribution to NEDM can be close to the current experimental bound, the induced O'_8 operators will place a strong constraint on parameters of CP violation.

There is still another important effect of the induced heavy quark CEDM at high energy. It arises because the dimension-5 CEDM operator can induce the dimension-4 operator, $\mathcal{O}_\theta$, through the operator mixing in the RG evolution[5,6]. As we shall discuss below, this mechanism gives rise to the strongest constraint on the CEDM of b (and possibly c) quarks.

The RG equation can be found in the literature [5,1]. The eigenstates of the RG equation are given by

$$\begin{aligned} \hat{\mathcal{O}}_g &= \mathcal{O}_g + \frac{36\pi\alpha_s}{7+2n} m_b \mathcal{O}_b^c - \frac{144\pi^2}{5(7+2n)} m_b^2 \mathcal{O}_\theta \quad , \\ \hat{\mathcal{O}}_b^c &= \mathcal{O}_b^c - \frac{24\pi}{23-2n} \frac{m_b}{\alpha_s} \mathcal{O}_\theta \quad , \\ \hat{\mathcal{O}}_\theta &= \mathcal{O}_\theta \quad . \end{aligned} \tag{8}$$

with eigenvalues of the operator $\mu(d/d\mu)$ equal to $-\frac{18\alpha_s}{2\pi}$, $-\frac{2\alpha_s}{6\pi}$ and zero respectively.

If the CP violating effective Lagrangian $\mathcal{L}_{CP}$ at the electroweak scale can be written as

$$\begin{aligned} \mathcal{L}_{CP} &= \tilde{d}_g G_F \mathcal{O}_g(M_W) \\ & + \tilde{d}_b G_F m_b \mathcal{O}_b^c(M_W) + (\theta)_{M_W} \mathcal{O}_\theta(M_W) \quad , \end{aligned} \tag{9}$$

where $\tilde{d}_g$ and $\tilde{d}_b$ are defined to be dimensionless, then we can easily derive the effective Lagrangian at the scale m_b^+ just above the b quark threshold using the above RG equations.

When integrating through the b quark threshold, the induced $\mathcal{O}_g$ operator can be calculated through the usual matching condition. Then we obtain the CP nonconserving interaction at the hadronic scale μ below the charm quark mass,

$$\begin{aligned} \mathcal{L}_{CP} = & \left[\gamma G_F/\alpha_s(\mu)^2\right] \mathcal{O}_g(\mu) \\ & + \left[(\theta)_{M_W} + \theta_{\text{ind}}\right] \mathcal{O}_\theta(\mu) \quad , \end{aligned} \tag{10}$$

where, numerically,

$$\begin{aligned} \gamma &= 1.1 \times 10^{-2} \tilde{d}_g - 10^{-4} \tilde{d}_b, \\ \theta_{\text{ind}} &= 2.2 \times 10^{-3} \tilde{d}_b - 3.1 \times 10^{-4} \tilde{d}_g \quad , \end{aligned} \tag{11}$$

for the QCD scale $\Lambda = 150$ MeV. The neutron electric dipole moment is related to the parameters in the effective Lagrangian of Eq. (9) as

$$\begin{aligned} D_n &= \left[(1.3 \times 10^4 (\theta)_{M_W} + 29\tilde{d}_b - 4\tilde{d}_g)\, \xi_\theta\, r \right. \\ & \left. + (0.74\tilde{d}_g - 0.0067\tilde{d}_b)\, \xi_g \right] \times 10^{-20}\, e\,\text{cm}. \end{aligned} \tag{12}$$

where $r = (m_d/9\,\text{MeV}) m_u/(m_u + m_d)$, and ξ_g and ξ_θ are coefficients associated with the unknown nonperturbative QCD dynamics.

For typical light quark masses of $m_u = 5$ MeV and $m_d = 9$ MeV, r is 0.36. The coefficients ξ_g and ξ_θ are defined such that they both have value one in the estimate using naive dimensional analysis[10] (NDA). The renormalization point for the NDA rule[2] is chosen such that $g(\mu) = 4\pi/\sqrt{6}$. However for the NEDM from $\mathcal{O}_\theta(\mu)$, more elaborated analyses[11] are available and all of them give $|\xi_\theta| \geq 1$. For example using the current algebra technique[13], one finds $|\xi_\theta| \simeq 7.7$. For the NEDM from the three gluon operator $\mathcal{O}_g(\mu)$, an equally naive scaling argument[14] gives $|\xi_g| \simeq 1/30$. These different values serve to indicate the uncertainty involved in the estimates of matrix elements.

Comparing the contributions to NEDM from the RG induced θ term to those from $\mathcal{O}_g(\mu)$ at the hadron scale μ, we find

$$\frac{\delta D_n(\mathcal{O}_g \to \mathcal{O}_\theta)}{\delta D_n(\mathcal{O}_g \to \mathcal{O}_g)} \simeq 2\,\frac{\xi_\theta}{\xi_g} \quad , \tag{13}$$

$$\frac{\delta D_n(\mathcal{O}_b^c \to \mathcal{O}_\theta)}{\delta D_n(\mathcal{O}_b^c \to \mathcal{O}_g)} \simeq 1600\frac{\xi_\theta}{\xi_g} \quad , \tag{14}$$

where the arrows denote the RG evolution from M_W down to the hadronic scale μ. This, together with the estimates for ξ_θ and ξ_g discussed before, implies that the NEDM associated with $\mathcal{O}_b^c(M_W)$ and $\mathcal{O}_g(M_W)$ may be dominated by the RG induced θ term, *instead of* $\mathcal{O}_g(\mu)$ considered previously[9], unless a significant cancellation occurs between $(\theta)_W$ and the RG induced $\theta_{\rm ind}$ at the hadronic scale. This is particularly true for $\mathcal{O}_b^c(M_W)$. It can be understood from the fact that the θ term is not renormalized once RG induced while the coefficient of the threshold induced $\mathcal{O}_g$ is strongly suppressed by the subsequent renormalzation effect.

For models models with a Peccei–Quinn symmetry [12] our analysis poses no problem since in these models the θ at low energy can rotated away by a Peccei–Quinn transformation. One may also wonder what is the relationship between the RG induced contribution and the radiatively induced contribution that is usually calculated through the argument of the determinant of the quark mass matrix $ArgDetM$. One can argue that the two contributions are really independent. Note that the calculation of $ArgDetM$ necessarily involves light quark masses while the RG calculation we gave above is insensitive to the light quark masses. In the limit that the light quark is zero, the effect of *theta* is of course vanishing because the parameter is unphysical. This consequence is explicit in the calculation of $ArgDetM$ but not in the RG calculation above. However that does not present any inconsistency. It is well known that even unphysical operators get induced in the formal RG evolution.

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