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SUPERFLUID-LIKE TURBULENCE IN COSMOLOGY

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ABSTRACT

A network of vortices in a superfluid system exhibits turbulent behavior. We argue that the universe may have experienced such a phase of superfluid-like turbulence due to the existence of a coherent state with non-topological charge and a network of global strings. The unique feature of a distribution of turbulent domains is that it can yield non-gravitationally induced large-scale coherent velocities. It may be difficult, however, to relate these velocities to the observed large-scale bulk motion.

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1. Introduction

The idea that the cosmic plasma could be in a state of turbulence during the radiation dominated epoch is almost half a century old[1]. It was initially hoped that a primeval turbulent velocity field could generate the needed perturbation for galaxy formation, explain the peculiar velocities and angular momentum of galaxies and explain the origin of intergalactic magnetic fields. Peebles, however, claims[2] that this concept of turbulent behavior is probably ruled out, since the amount of turbulence needed to explain the residual matter velocities on galactic scales leads to structure formation too early on.

On the other hand, in the standard instability picture large-scale structure forms gravitationally out of small initial perturbations and at the same time initiates the growth of peculiar velocities[3]. In fact, based on this very process, one can deduce the mass density of the universe from the observed large–scale streaming velocities. This has lately been one of the most attractive means of 'weighing the universe' and hints strongly towards a density parameter[4] $\Omega \gtrsim 0.5$. ($\Omega = \rho/\rho_c$ is the ratio of the actual to the critical density.)

We would like to challenge the link between perturbations, velocities and mass density and reintroduce *non-gravitationally* induced velocities that are remnants of a cosmic turbulent phase. But instead of seeking again turbulence in the primeval plasma, we propose a late–time *superfluid*–like epoch[5], which could in principle take place after decoupling. Inspired by low–temperature ^{4}He, we argue that part of the matter content of the universe can undergo bose condensation. This is equivalent to the 'injection' of a non–topological charge into the vacuum. In the presence of global strings, alias superfluid vortices, this non–topological charge is identified with angular momentum and leads to turbulent domains on the scale of the inter–string distance.

2. Background

There are at least three different approaches for baryogenesis which involve some kind of a bose condensate[6]. (Bose condensation can, of course, also arise in completely different circumstances.) Condensation is here related to the existence of a coherent background field with nonzero charge (particle) density. We will not follow any specific theory, but consider only the general concept.

Let us focus on a complex scalar field ϕ with a global U(1) symmetry. We assume that the symmetry is spontaneously broken and ϕ acquires a non-vanishing vacuum expectation value, $<\phi>=\nu$, where

$$\phi = \frac{\nu}{\sqrt{2}}\, e^{i\Theta(t)} \;. \tag{1}$$

The coherent ϕ-field can 'hide' a density of ϕ-particles through a time-dependent Goldstone field Θ. The number (charge) density is given by the time-component of the conserved Noether current of the U(1) symmetry,

$$n_\phi \equiv j^0 = \nu^2 \dot{\Theta} \;. \tag{2}$$

This is reminiscent of the interior of non-topological solitons and Q-balls. Cosmic expansion conserves the total charge, $Q = n_\phi V$, inside a comoving volume.

3. Vortices

Our system generically also contains global strings, one-dimensional topological defects. However, in the presence of strings, the spatially homogeneous state (1) must be modified to

$$\phi = \frac{\nu}{\sqrt{2}}\, e^{i[n\theta+\Theta(t)]} \;, \tag{3}$$

where we have taken the string to lay along the z-axis, θ is the polar angle in the x-y plane and n denotes the winding number (the *topological* charge). Neglecting the core structure of the string, the non-topological charge Q is unchanged.

The crucial observation is that ϕ now describes a fluid. In the presence of a charge density, the gradient of the Goldstone field becomes time-like (it is space-like in the absence of any charge) and describes a velocity field[7], the Goldstone mode playing the role of the velocity potential. The system now also contains angular momentum[5],

$$J = n\, Q \;, \tag{4}$$

correlated over a volume $V_\xi \sim \xi^3$, where ξ is of order the inter-string distance. We thus end up with turbulent domains of volume V_ξ. Note that angular momentum is the product of the topological and non-topological charges and thus *both* charges are needed to obtain turbulence.

The distance ξ is fixed by the dynamics of the string network. In the absence of any background charge n_ϕ, global strings move relativistically and ξ scales with the horizon, $\xi \propto t$. However, once imbedded in the coherent background, the strings experience a damping Lorentz-like force, the 'Magnus' force, and become highly non–relativistic[7,8]. Like vortices in superfluid ^{4}He, they are frozen into the 'fluid'. As a consequence, ξ growths only conformally and hence conserves the turbulent structure on the comoving scale V_ξ.

4. Cosmological Implications

• Turbulent motion on astrophysical scales is a source of perturbations for large–scale structure formation[5]. The unique feature of turbulence, however, is associated with the angular motion of the ϕ-field. The coherent state may be stable at low temperatures or decay into free particles. But even in the latter case, angular momentum has to be conserved during the decay process. *Coherent bulk motion has to be transferred into the free ϕ-particles.*

• Can this flow be identified with the observed large–scale streaming velocities of $v \sim 600$ km/sec on scales $l \sim 60$ Mpc? We estimate the charge density to be $n_\phi \simeq \rho/m$, where m is the characteristic mass scale of the ϕ-field. Relating the angular momentum in a correlation volume, $J \simeq \rho V_\xi \, lv$, to the non-topological charge (see eq. (4)) yields

$$m \simeq \frac{n}{l\,v} \simeq 5 \cdot 10^{-29}\,\mathrm{eV}\; n \,. \tag{5}$$

• If $n \sim \mathcal{O}(1)$, ϕ can obviously not be identified with baryons. On the other hand, if ϕ constitutes some non-baryonic dark matter, we are left with the difficulty of transferring the bulk motion into the baryonic component. After all, the large-scale streaming velocities are intrinsic to the baryonic matter. This transfer may be possible through tidal gravitational interactions between the baryons and the dark matter, as turbulent volumes naturally present an ensemble of quadrupole moments.

• Alternatively we can imagine $n \gg 1$. In fact, global strings produced at low temperatures at a time $t_{\rm in}$ (and temperature $T_{\rm in}$) have an initial correlation length $\xi_{\rm in}$ well below the horizon scale[9],

$$t_{\rm in}/\xi_{\rm in} \sim m_{\rm pl}/T_{\rm in} . \tag{6}$$

($m_{\rm pl} = 1.2 \cdot 10^{19}$ GeV is the Planck mass.) The initial total winding number on a scale r, with $\xi_{\rm in} \ll r \lesssim t_{\rm in}$, is then expected to be

$$n_{\rm in} \simeq (r/\xi_{\rm in})^{1/2} \gg 1 . \tag{7}$$

Let us consider the winding number inside the horizon, $r \simeq t$. Since the correlation length ξ in the vortex–superfluid system is only conformally stretched, cosmic evolution results in a further accumulation of winding number, $n(t) \propto t^{1/4}$ ($t^{1/6}$) during the radiation (matter) dominated era. Although the resulting winding number can be very large, it is nevertheless impossible to push the mass m up to at least the GeV scale.

• Previously we pictured the universe as consisting of a distribution of turbulent domains of a *single* size ξ. Note, however, that eq. (7) is correct for any $r \gg \xi$. This leads to a hierarchical distribution of turbulent patches or quadrupole moments. There is turbulence on *any* scale r, though the distribution is not exactly scale-invariant, as the initial winding number $n_{\rm in}$ is scale-*dependent* (see eq. (7)). An ensemble of gravitational quadrupole moments is known to generate an interesting spectrum of perturbations on large scales[10].

5. Conclusions

Although turbulent behavior in the primeval plasma is probably ruled out, we argue that the universe may experience a different kind of turbulence, similar to the turbulent dynamics of a vortex tangle in superfluid ^{4}He. Such a phase naturally emerges from the injection of non-topological charge into the spontaneously broken vacuum in the presence of global strings. Turbulent domains endow the dark matter with coherent large-scale motion, though it may be difficult to account for the observed bulk velocities on supercluster scales. Note, however, that we have only considered a very simple model. It could well be that a more realistic model can lead to higher turbulent velocities.

Even with relatively small velocities, a distribution of turbulent domains may have interesting implications for cosmology. They are worth pursuing, but this is beyond the scope of this work.

6. Acknowledgement

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7. References

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