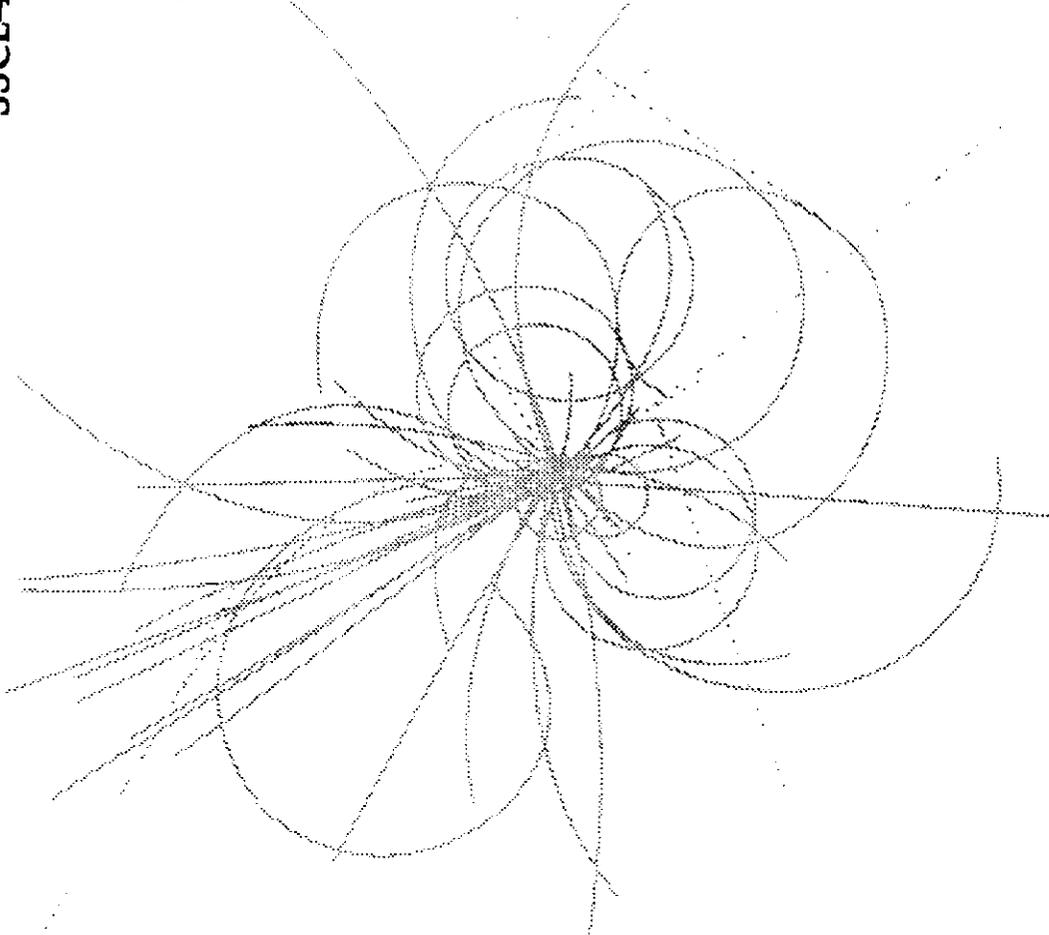


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Abstract

When a beam undergoes a collective oscillation, its emittance tends to grow because of variations in betatron tune which are due to energy spread and chromaticity and to non-linear lattice characteristics. In this paper we estimate the emittance growth in the SSC caused by the measured ground motions at the SSC site and by beam-beam effects in the Jostlein beam-centering scheme [1].

I. EMITTANCE GROWTH FROM CHROMATICITY

The vector trajectory of a pencil beam kicked through an angle α at the point s_0 , where the betatron function has the value β_0 , is (using normalized coordinates $\eta \equiv x/\sqrt{\beta}$, and $\eta' \equiv d\eta/d\phi$, ϕ being the betatron phase beginning at s_0):

$$\bar{\eta} \equiv \eta + i\eta' = i\alpha\sqrt{\beta_0}e^{-i\phi}. \quad (1)$$

However, after N turns, the pencil beam has spread into an arc whose rms width [2] in the phase coordinate is $\sigma_\phi = (2\sigma_E\xi/v_s)\sin\pi v_s N$, where σ_E is the rms energy spread, ξ the chromaticity, and $v_s = f_s/f_0$ is the relative synchrotron frequency.

If there is only a single kick, this spreading in phase space is reversed after one half of a synchrotron period, and, in the linear approximation, at integral numbers of turns the pencil beam again collapses to a point, with no net smearing having occurred. However, if the beam is repeatedly kicked at random times, the elementary spreading effects occur in many directions in phase space and will tend to add as in a two-dimensional random walk. The characteristic time for this process is one half of a synchrotron period, which for the SSC at 20 TeV is 0.125 sec, or 430 turns.

A. Non-Linear Tune Variation

If the betatron tune varies with amplitude as

$$v = v_0 + \mu x^2, \quad (2)$$

then any radial locus of points in phase space is continually transforming in a spiral, adding one turn to the spi-

ral every $(\mu x^2)^{-1}$ turns. Thus, a circular beam of phase-space radius A that is kicked into a collective oscillation of amplitude a will eventually smear out into a circle of radius $A + a$. For the SSC μ is expected [3] to be about $-4.8 \times 10^{-5} \text{ mm}^{-2}$ and the rms beam width 0.12 mm, so that the characteristic time for non-linear smearing will be about 1.4×10^6 turns, or 7 minutes.

B. Amount of Emittance Growth

If the amplitude of collective oscillation is a , Syphers [4] has shown that the emittance dilution factor F at equilibrium is

$$F \equiv \epsilon/\epsilon_0 = 1 + (a/\sigma)^2/2, \quad (3)$$

where σ is the rms beam width. With random excitation, the collective amplitude after N turns is expected to be $a_1\sqrt{N}$, where a_1 is the rms increase in amplitude per turn. (The case of sinusoidal excitation will be treated in a later section.)

II. AMPLITUDE OF COLLECTIVE OSCILLATIONS PRODUCED BY GROUND MOTION

If a single quadrupole (of focal length f_Q) at the point s_0 in an otherwise perfect machine is suddenly moved by the amount δx , the resulting trajectory of a bunch that had been traveling on the ideal central orbit is given by equation (1) with the angle α given by $\alpha = \delta x/f_Q$. However, the bunch will no longer be on the closed orbit, which now becomes

$$\bar{\eta}_{co} = (\delta x/f_Q)(\sqrt{\beta_0}/2 \sin \pi v)e^{-i(\phi - \pi v)}. \quad (4)$$

The collective oscillation is the difference between equations (1) and (4):

$$\bar{\eta}_{coll} = -(\delta x/f_Q)(\sqrt{\beta_0}/2 \sin \pi v)e^{-i(\phi + \pi v)}. \quad (5)$$

Thus, it can be seen that the amplitude of the collective oscillation induced by the *elementary* motion of a quadrupole is equal in magnitude to the corresponding contribution to the induced closed orbit.

If now the quadrupole is moved again during the revolution time of the beam, the consequent change in the *closed orbit* will add *linearly* to that produced by the first motion. However, the *two* contributions to the *collective oscillation* must be added *vectorially*, since they will differ in phase by $2\pi v$.

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Because of the fixed relationships in amplitude and in phase between the contributions to the collective oscillation and to the closed orbit produced by the elementary motion of a quadrupole, we can conclude that the collective amplitude produced by the elementary motion of the 1,000 quadrupoles in each SSC ring is equal to the corresponding increment in the overall closed orbit. We have already computed the closed orbits for each ring and combined them to find the beam separations at the crossing points for several types of ground waves at many frequencies [5]. For each case we can conclude that the rms incremental amplitude of collective oscillation is equal to $(2\sqrt{2})^{-1}$ of the maximum incremental beam separation. Thus, at a typical ground-wave frequency ($\omega_g/2\pi$) we obtain

$$a_1 = \omega_g x_g / (2\sqrt{2}f_o), \quad (6)$$

where x_g is the beam separation due to ground motion at that frequency at a reference crossing point, and f_o is the revolution frequency in the SSC (3.44 kHz). The resultant emittance dilution factor then is

$$F = 1 + \omega_g^2 x_g^2 T / (16f_o \sigma^2), \quad (7)$$

where T is the time period of interest.

III. GROWTH FROM MEASURED GROUND MOTIONS

Measurements [6] were made at tunnel depth at the SSC site of ground motions due to trains crossing overhead, blasts from nearby quarries, and ambient background. All showed very ragged, irregular wave forms so that we feel that equation (7) should apply.

A. Growth Due to Train Crossings

The maximum beam separation due to trains [5] was $0.10 \mu\text{m}$ caused by ground waves in the 3-Hz band plus $0.066 \mu\text{m}$ from a band near 7 Hz. The rms width of the beams at this interaction point is $4.8 \mu\text{m}$. For a time T of 120 seconds, the dilution factor (adding the two contributions in quadrature) is $1 + 1.1 \times 10^{-3}$, which would not be observable. If the quadrupole displacements should be amplified by a factor of, say, 5 due to the magnet support structures, the emittance growth would be $1 + 2.8 \times 10^{-2}$ —still not an important effect.

B. Growth Due to Quarry Blasts

The maximum beam separation due to quarry blasts was found to be $0.93 \mu\text{m}$ due to ground waves in the 1-Hz band plus $1.12 \mu\text{m}$ from the 3-Hz region. The geophone detectors showed relatively large ground motion for about 30 seconds in a typical section of the ring. The resulting dilution factor is $1 + 1.14 \times 10^{-2}$, which is not a worrisome level. However, an amplification factor of 5 due to the quadrupole supports would raise this dilution factor to $1 + 0.28$, which would be significant, since the luminosity would drop by

the same factor permanently if corrections could not be made promptly.

C. Growth Due to Ambient Ground Motion

Measurements of the ambient ground motion at several tunnel positions around the ring at different times varied over two orders of magnitude. The average amplitude was $0.015 \mu\text{m}$, the typical frequencies were in the region of 3 Hz, and the wave-forms were ragged and irregular. Since we know little else about these waves, we have tried to make a conservative estimate of their effects by assuming that they are due to random plane waves having an unfavorable direction and negligible attenuation. For such waves our program [5] predicts a maximum separation of about $0.03 \mu\text{m}$ at a low-beta crossing. For a 12-hour period the resulting emittance dilution factor is $1 + 1.1 \times 10^{-2}$, which is about the same as for the worst-case quarry blast. Again an amplification factor of 5 would raise this dilution factor to a significant level. It would be harder to correct in this case because the growth of the collective oscillation per characteristic smearing period is so much smaller that it would be difficult to measure. More complete data, including wave direction and velocity, are needed.

IV. GROWTH DUE TO A BEAM-CENTERING SCHEME AND THE BEAM-BEAM EFFECT

In the Jostlein [1] beam-centering scheme, one beam is rotated about the other at an interaction point, and the resulting variation in luminosity serves to measure the amount and direction by which the two beam centers miss each other. In this situation each beam acts on the other to first order like a moving quadrupole of focal length $f_Q = \beta_o / (4\pi \Delta v_{bb})$, where Δv_{bb} is the head-on beam-beam tune shift. On the n^{th} turn each beam receives a kick:

$$d\bar{\eta} = ia_n \equiv i \left(b_m \sqrt{\beta_o} / f_Q \right) \sin(2\pi v_m n), \quad (8)$$

where b_m is the amplitude of the sinusoidal beam modulation at relative frequency $v_m \equiv f_m / f_o$ cycles per revolution.

Consider a pencil of beam whose vector amplitude at turn zero is \bar{A} . At turn N its amplitude will be:

$$\begin{aligned} \bar{\eta}_N &= \left(\dots \left(\left((\bar{A} + ia_0) e^{-2\pi v_0} + ia_1 \right) e^{-i2\pi v_1} \right. \right. \\ &\quad \left. \left. + ia_2 \right) e^{-i2\pi v_2} + \dots + ia_N \right) e^{-i2\pi v_N} \\ &= \bar{A} e^{-i2\pi \sum_0^N v_k} + i \sum_0^N a_n e^{-i2\pi \sum_n^N v_k}, \end{aligned} \quad (9)$$

where v_k is the net betatron tune on the k^{th} turn.

A. Chromaticity Effects

Consider first the variation of betatron tune with energy:

$$v_k = v_o + \delta E \xi \sin(2\pi v_k k + \theta), \quad (10)$$

where θ is the phase in synchrotron phase space at turn zero. In this case equation (9) can be summed approximately to give the spreading in phase space, relative to an on-momentum particle, as:

$$\begin{aligned} \bar{\eta}_N = & \left[2\pi b_m \sqrt{\beta_o} \delta E \xi / (f_Q \sin \pi v_s) \right] e^{-i2\pi v_o N} \\ & \left\{ [\cos(2\pi v_s N + \theta) \sin \pi v_+ N e^{i\pi v_+ N} / (2 \sin \pi v_+) \right. \\ & - \sin \pi(v_+ + v_s) N e^{i\pi(v_+ + v_s)N + i\theta} / (4 \sin \pi[v_+ + v_s]) \\ & - \sin \pi(v_+ - v_s) N e^{i\pi(v_+ - v_s)N - i\theta} / (4 \sin \pi[v_+ - v_s])] \\ & \left. - [\text{same with } v_+ \rightarrow v_-] \right\}, \end{aligned} \quad (11)$$

where $v_{\pm} \equiv v_o \pm v_m$. This expression describes a complicated motion that repeatedly goes through zero with a period of $1/v_s$ turns. It represents a breathing motion whose maximum amplitude, evaluated at the collision point, is approximately

$$|x_{\max}| = 2\pi v_m b_m \beta_o \delta E \xi / (f_Q \sin \pi v_s \sin^2 \pi v_o). \quad (12)$$

Thus the motion is bounded. For the case $f_m = 20$ Hz, $\beta_o = 0.5$ m, $\delta E = \sigma_E = 6 \times 10^{-5}$, $\xi = 5$, $\Delta v_{bb} = 0.004$ ($\rightarrow f_Q = 10$ m), $v_s = 1.16 \times 10^{-3}$, and $v_o = 123.4$, $|x_{\max}| = 0.57 b_m$. Thus, if the amplitude of modulation is a small fraction of the beam size, the increased emittance due to the chromaticity tune spread is negligible, provided that no resonances are involved.

Simulations in which particles are followed according to equations (9) and (10) are in excellent agreement with these conclusions.

B. Non-Linear Tune-Shift Effects

Using the non-linear tune dependence of equation (2) in equation (9) and assuming relatively small quadrupole motion, we obtain for the vector amplitude at the N^{th} turn the approximate expression:

$$\begin{aligned} \bar{\eta}_N = & e^{-i2\pi N v_A} \left\{ \bar{A} + \left[b_m \sqrt{\beta_o} / (2f_Q) \right] \right. \\ & \left. \left[e^{i\pi v_+ N} \sin \pi v_+ N / \sin \pi v_+ \right. \right. \\ & \left. \left. - e^{i\pi v_- N} \sin \pi v_- N / \sin \pi v_- \right] \right\}, \end{aligned} \quad (13)$$

where $v_A = v_o + \mu A^2$, and here $v_{\pm} \equiv v_A \pm v_m$. Thus, all particles of the same absolute initial amplitude $|A|$ have

the same relative motion, so that no smearing occurs, and the beam motion induced by the sinusoidal motion of the "beam-beam quadrupole" averages to zero, again provided that no resonances are involved. Again, simulations using equations (2) and (9) are in excellent agreement with these conclusions.

C. Possible Disagreement with Observations

These results on the effect of sinusoidal modulation of one beam relative to the other at a crossing point may be in disagreement with the experimental observations [7] at CERN, where beam effects were observed at modulation frequencies in the range 2 to 10 kHz, well below the betatron frequency band (13.5–14.5 kHz) in the SPS collider. However, they also observed in the beams higher harmonics of the modulation frequencies, and they found that the modulation equipment produced higher harmonics of the excitation frequencies. Since the deleterious beam effects could have been due to harmonics that fell in the betatron frequency band, it is still not clear experimentally whether sinusoidal modulation in the presence of the beam-beam effect causes appreciable emittance degradation. No such effects were seen at modulation frequencies below 2 kHz.

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