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Charge Quantization Of Wormholes And The Finiteness of Newton's Constant.

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Abstract

We derive, from first principles, the equations of K. Lee which exhibit wormhole solutions. The interpretation of such solutions becomes more transparent: they are local extrema of the action which contribute to transition amplitudes between states of definite charge. Hence the charge carried by the wormhole is quantized. We briefly review Coleman's mechanism for the vanishing of the cosmological constant, with emphasis on the problem of the vanishing of Newton's constant G. A mechanism is proposed that could naturally make $1/G$ a bounded function of the wormhole parameters.

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1 Introduction

For some time now it has been suspected that the solution to the cosmological constant problem would bring about a revolution of thought in physics[1]. The recent proposal by Coleman[2], while perhaps not revolutionary, does introduce new and speculative (and exciting) ideas. It is worth exploring its physical consequences and testing its formal basis.

Coleman's proposal builds on earlier work by Baum[3] and Hawking[4]. It improves on it by making the argument valid for essentially all boundary conditions for the wave function of the universe. Moreover, it explains how the cosmological constant, and for that matter all coupling constants, may be dynamically adjustable parameters.

There are two ways in which Coleman's proposal might collapse. Because it relies on a variety of technical assumptions, disproving any one of them may prove the mechanism for the vanishing of Λ untenable. We have in mind here, for example, the relevance of Euclidean Quantum Gravity or the assumption that wormholes make the dominant topology changing contributions(see, *e.g.*, ref. [5]). Of course, one could imagine the main ingredients of the argument to survive closer examination even if some of the details of the analysis might require modification. On the other hand, if one uses these main ingredients to derive observable consequences which contradict observation, then it is hard to see how the proposal is not plainly doomed. Here we are thinking of, for example, the observation that Newton's Constant may be automatically set to zero[6]. It is this issue that we address in this paper.

At the heart of Coleman's argument is the wormhole summation formula[7,8]. We will assume that only wormhole configurations which are actually saddle points of the action play a role in determining the effect of topology changing configurations on the effective action at long wavelengths. This will be discussed in more detail in Section 3. With this in mind we start in Section 2 by giving the wormhole solution found by K. Lee[9] a proper first principles formulation. As a result we will find that the charge carried by the wormhole is integrally quantized. Section 3 begins with a review of the wormhole summation formula. We will then use the results of Section 2 and rather naïve arguments to characterize the form of the vertex operators for wormhole insertions[10] in Lee's theory. We will argue then that large wormholes may be destabilized by small ones.

The problem of Newton's Constant sliding to zero is reviewed in Section 4. We then propose a mechanism that may explain the resolution of this problem for the 'bare' constant. In Section 5 we turn to the quantum corrections to this result. It seems that the quantum corrections have the effect of driving particle masses up to the wormhole scale, unless protected by some symmetry. But they normally don't induce vanishing of Newton's Constant.

Our results are briefly summarized in Section 6. This is important because, by the end of Section 5 we will have introduced a number of hypothesis and proposed several ideas, and the reader might have trouble keeping it all straight (especially in the light of our admittedly poor style of presentation).

2 Charge Quantization of Lee's Wormholes

Several wormhole configurations have been discussed in the literature [10,11,9]. We will focus here on those introduced by K. Lee [9]. As we will see, these configurations are extrema of the effective action that enters in the calculation of transition amplitudes between states of definite charge. Therefore it is at least plausible that they may give a non-negligible contribution to the sum over configurations. An added advantage to studying this particular case is that the theory considered by Lee is extremely simple. It is simply the theory of gravitation and two matter fields, a minimally coupled complex scalar. The action is taken to be invariant under phase redefinitions of the scalar field, and this $U(1)$ symmetry is spontaneously broken.

The size of these wormholes is determined by the charge Q they carry. In ref. [9] this was taken to be quantized to integral values. In our view, no good justification was given for this assertion. In fact, in the very similar cases studied by Giddings and Strominger[11] 'string effects' were invoked to justify this assumption. It is the main object of this section to put this quantization condition on firmer basis. As a byproduct we will derive Lee's equations from first principles. To be somewhat more precise, we will derive a set of equations which agree with those in ref [9] only after one introduces a spherically symmetric ansatz. The equations are those satisfied by configurations which extremize the effective action that occurs in the functional integral computation of the transition matrix element between states of definite charge Q. It will then be obvious that the charge is integrally quantized.

We begin by considering a simple example[9,12] that well exemplifies the features of the full fledged theory that we are interested in. Consider a particle moving in two spatial dimensions under the influence of a central potential $V(r)$. The dynamics of the system is described by the lagrangean

$$L = \frac{1}{2}\dot{r}^2 + \frac{1}{2}r^2\dot{\theta}^2 - V(r). \tag{1}$$

Now the Feynman path integral can be used to calculate the propagator

$$< r_2\theta_2 \mid e^{-iH(t_2-t_1)} \mid r_1\theta_1 >= \int_{b.c.} [dr][rd\theta]e^{iS} \tag{2}$$

where the action S is given by

$$S = \int_{t_1}^{t_2} dt\, L \tag{3}$$

and 'b.c.' stands for the boundary conditions

$$r(t_i) = r_i \qquad \theta(t_i) = \theta_i \qquad i = 1, 2. \tag{4}$$

Since angular momentum is conserved it is of interest to compute the propagator between states of definite angular momentum ℓ. We can choose to label our states by $\mid r\ \ell >$ instead of $\mid r\ \theta >$. Now, since angular momentum is the momentum conjugate to the angular variable θ one has

$$< r_1\ \theta \mid r_2\ \ell >= 2\pi\delta(r_2 - r_1)e^{i\ell\theta} \tag{5}$$

so one can change basis easily. For the propagator between states of definite ℓ one has

$$< r_2\ \ell_2 \mid e^{-iH(t_2-t_1)} \mid r_1\ \ell_1 >= \int d\theta_1 d\theta_2 \int_{b.c.} [dr][rd\theta]e^{i(\theta_1\ell_1-\theta_2\ell_2)}e^{iS} \tag{6}$$

The integrals over θ can be done. A simple trick makes the computation trivial. Using the identity

$$\theta_2 = \int_{t_1}^{t_2} dt\dot{\theta} + \theta_1 \tag{7}$$

to remove the explicit dependence on θ_2, the integration over θ_2 removes the boundary condition on θ at time t_2. The trick is to change variables from θ to $\dot\theta$, so that the $\dot\theta$ integration has no boundary conditions. The θ_1 integral yields a δ-function of angular momentum conservation. We finally obtain

$$< r_2\, \ell_2 \mid e^{-iH(t_2-t_1)} \mid r_1\, \ell_1 >= \delta(\ell_2 - \ell_1) \int_{b.c.} [dr] e^{iS_{eff}} \tag{8}$$

where the effective action S_{eff} for the radial coordinate is given by

$$S_{eff} = \int_{t_1}^{t_2} dt\, L_{eff} \;=\; \int_{t_1}^{t_2} dt \left(\frac{1}{2}\dot r^2 - V_{eff}(r)\right) \tag{9}$$

$$V_{eff}(r) \;=\; V(r) + \frac{\ell_1^2}{2r^2} \tag{10}$$

This is as expected. The reader might be dissatisfied with the formal manipulations just performed, and in particular with the simplifying trick introduced above. We have performed the same calculation using the canonical expression for the Feynman propagator, *i.e.*, functionally integrating over coordinates and their conjugate momenta. The same answer is obtained. We briefly sketch this computation. The propagator is given by

$$\int_{b.c.} [dr][d\theta] \int [dp_r][dp_\theta] e^{i\int dt [p_r \dot r + p_\theta \dot\theta - H]} \tag{11}$$

where the hamiltonian H, a function of the coordinates and the conjugate momenta, is

$$H = p_r \dot r + p_\theta \dot\theta - L \tag{12}$$

and

$$p_r = \frac{\partial L}{\partial \dot r} = \dot r \qquad p_\theta = \frac{\partial L}{\partial \dot\theta} = r^2 \dot\theta \tag{13}$$

Using the identity

$$\int_{t_1}^{t_2} dt\, p_\theta \dot\theta = \theta_2 p_\theta(t_2) - \theta_1 p_\theta(t_1) - \int_{t_1}^{t_2} dt\, \theta \dot p_\theta \tag{14}$$

one finds that in the calculation of the matrix element between states of definite angular momentum the integrations over θ yield the factor

$$\delta(\ell_1 - p_\theta(t_1))\delta(\ell_2 - p_\theta(t_2)) \prod_t \delta(\dot p_\theta) \tag{15}$$

It is interesting that only paths which conserve angular momentum contribute to the transition amplitude. The delta functions make the remaining integral over p_θ trivial and one recovers the result of eq. (8).

It is straightforward to rotate into imaginary time, $t \to -i\tau$. One can simply take the result in eq. (8) and perform the rotation:

$$< r_2\, \ell_2 \mid e^{-H(\tau_2-\tau_1)} \mid r_1\, \ell_1 > \;=\; \delta(\ell_2-\ell_1)\int_{b.c.}[dr]e^{-I_{eff}} \tag{16}$$

$$I_{eff} \;=\; \int_{t_1}^{t_2} dt\,\left(\frac{1}{2}\dot{r}^2 + V_{eff}(r)\right) \tag{17}$$

The dot, of course, refers now to a τ-derivative. This, again, is the correct answer, but the derivation seems to have sidestepped completely the ambiguities encountered in ref. [9]. There it was pointed out that if one simply eliminates θ from the imaginary time action I using the constraint $r^2\dot{\theta} = \ell$ then the equation of motion that obtains is incorrect. To understand this better we can perform the calculation again, but now rotating first into imaginary time, and only then performing the functional integrals over the angular variable. Note though that the argument of the exponential in eq. (11) becomes complex:

$$e^{i\int dt\left[p_r\dot{r}+p_\theta\dot{\theta}-H\right]} \to e^{i\int d\tau\left[p_r\dot{r}+p_\theta\dot{\theta}+iH^E\right]} \tag{18}$$

Here H^E is the imaginary time hamiltonian. The change in basis is performed as before (*cf.*, eq.(5)). Therefore the integrals over θ still yield the delta functions for conservation of angular momentum (15). Hence we arrive at the same (correct) answer. We learn that the ambiguity found in [9] is resolved by asking the proper question ('what is the transition amplitude between states of definite angular momentum?'), and that the corresponding constraint is enforced in the imaginary time path integral by making the action complex.

The above discussion may be transcribed almost word by word to the case of interest. Let's consider first the theory of a complex scalar field with a $U(1)$ symmetric potential. It is described by the lagrangean density

$$\mathcal{L} \;=\; \partial_\mu\phi^*\partial_\mu\phi - V(\sqrt{2}\,|\phi|) \tag{19}$$

$$\;=\; \frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}\rho^2(\partial_\mu\theta)^2 - V(\rho) \tag{20}$$

The conserved charge is

$$Q = -i\int d^3x\,(\phi^*\overleftrightarrow{\partial}_0\phi) \tag{21}$$
$$= \int d^3x\,\rho^2\dot{\theta} \tag{22}$$
$$= \int d^3x\,\pi_\theta \tag{23}$$

where we have introduced the conjugate momenta

$$\pi_\rho = \frac{\partial\mathcal{L}}{\partial\dot{\rho}} = \dot{\rho} \tag{24}$$
$$\pi_\theta = \frac{\partial\mathcal{L}}{\partial\dot{\theta}} = \rho^2\dot{\theta} \tag{25}$$

Since the radial degree of freedom ρ will play no role, we will cavalierly omit it in the following, and will restore it only at the end of the discussion. We can introduce a basis of eigenstates of the field operator $\hat{\theta}$, and we can easily relate it to a basis of eigenstates of the conjugate momentum operator $\hat{\pi}_\theta$:

$$<\theta\mid\pi_\theta>= e^{i\int d^3x\,\theta(\vec{x})\pi_\theta(\vec{x})} \tag{26}$$

This is just what was done in (5). Moreover

$$e^{i\hat{Q}\alpha}\mid\pi_\theta>= e^{i\alpha\int d^3x\,\pi_\theta(\vec{x})}\mid\pi_\theta> \tag{27}$$

so charge is diagonal in this basis. Let us compute the matrix elements between eigenstates of $\hat{\pi}_\theta$:

$$<\pi_{\theta 2}\mid e^{-iH(t_2-t_1)}\mid\pi_{\theta 1}>= \int[d\theta_2][d\theta_1]e^{i\int d^3x\,(\theta_1(\vec{x})\pi_{\theta 1}(\vec{x})-\theta_2(\vec{x})\pi_{\theta 2}(\vec{x}))}\mathcal{G}(\theta_1,\theta_2;t_2-t_1) \tag{28}$$

Here the functional integrals are over functions of (three-)space and $\mathcal{G}$ stands for the path integral expression for the transition amplitude between eigenstates of $\hat{\theta}$,

$$\mathcal{G}(\theta_1,\theta_2;t_2-t_1) = \int_{b.c.}[d\theta]e^{i\int d^4x\,\frac{1}{2}\rho^2(\partial_\mu\theta)^2} \tag{29}$$

Here the time integral has limits t_1 and t_2 and the boundary conditions are $\theta(t_i) = \theta_i$. To proceed any further we must realize that since only global charge is conserved we can only expect to obtain delta functions as in (15) corresponding to the zero mode (with respect to the operator $\vec{\nabla}^2$) in θ. Therefore it is useful to introduce the decompositions[12]

$$\theta_i(\vec{x}) = \tilde{\theta}_i(\vec{x}) + \Theta_i \qquad \theta(x) = \tilde{\theta}(x) + \Theta(t) \tag{30}$$

where the zero modes are denoted by Θ_i and Θ. Now, all the integrations over the zero modes parallel precisely what we did in the simple quantum mechanical case. Therefore we can write the result immediately. Restoring the dependence on ρ we have finally

$$< \rho_2\pi_{\theta 2} \mid e^{-iH(t_2-t_1)} \mid \rho_1\pi_{\theta 1} >= \delta(Q_1 - Q_2) \int [d\tilde{\theta}_1][d\tilde{\theta}_2] e^{i\int d^3x\,(\theta_1\pi_{\theta 1} - \theta_2\pi_{\theta 2})}$$
$$\times \int_{b.c.} [d\rho][\rho d\tilde{\theta}] \frac{\prod_t \int d^3x\, \rho}{\sqrt{\prod_t \int d^3x\, \rho^2}} e^{iS_{eff}} \tag{31}$$

where

$$S_{eff} = S(\rho, \tilde{\theta}) - \frac{1}{2}\int dt\, \frac{(Q_1 - \int d^3x \rho^2 \dot{\tilde{\theta}})^2}{\int d^3x \rho^2} \tag{32}$$

and

$$S(\rho, \tilde{\theta}) = \int d^4x \left[\frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\rho^2(\partial_\mu \tilde{\theta})^2 - V(\rho)\right] \tag{33}$$

The factor $1/\sqrt{\prod_t \int d^3x\, \rho^2}$ in eq. (31) is simply the determinant that results from the Θ integration, and will play no role in determining the configurations that dominate the integral in the semiclassical approximation. The factor $\prod_t \int d^3x\, \rho$ is the zero mode component of the Jacobian of the transformation from cartesian to polar coordinates (*i.e.*, $[d\,\mathrm{Re}\,\phi][d\,\mathrm{Im}\,\phi] = [d\rho][d\theta]\prod_x \rho$). The effective action is no longer manifestly Lorentz invariant, but this is just a reflection of the matrix element that is being considered. Note also that there is still an explicit integration over the nonzero modes at the initial and final times, weighed by the usual Fourier transform factor. In some sense it would have been simpler to consider eigenstates of $\hat{\rho}$ and $\hat{\tilde{\theta}}$ of definite charge

Again it is straightforward to rotate into imaginary time. Again the effective action is complex, enforcing the constraint automatically. The result is

$$< \rho_2\pi_{\theta 2} \mid e^{-H(\tau_2-\tau_1)} \mid \rho_1\pi_{\theta 1} >= \delta(Q_1 - Q_2) \int [d\tilde{\theta}_1][d\tilde{\theta}_2] e^{i\int d^3x\,(\theta_1\pi_{\theta 1} - \theta_2\pi_{\theta 2})}$$
$$\times \int_{b.c.} [d\rho][\rho d\tilde{\theta}] \frac{\prod_t \int d^3x\, \rho}{\sqrt{\prod_t \int d^3x\, \rho^2}} e^{-I_{eff}} \tag{34}$$

where

$$I_{eff} = I(\rho,\tilde{\theta}) + \frac{1}{2}\int d\tau \, \frac{(Q_1 - i\int d^3x\rho^2\dot{\tilde{\theta}})^2}{\int d^3x\rho^2} \tag{35}$$

$$I(\rho,\theta) = \int d^4x_E \left[\frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}\rho^2(\partial_\mu\tilde{\theta})^2 + V(\rho)\right] \tag{36}$$

This expression is the the starting point for the semiclassical approximation of the transition amplitude in question. In looking for the saddle points of the effective euclidean action we must keep in mind that the equations we obtain are complex, *i.e.*, each complex equation yields two real equations. From variations with respect to ρ we obtain

$$-\partial^2\rho + \rho(\partial_\mu\tilde{\theta})^2 + V'(\rho) - \rho\left[\frac{Q_1^2 - \left(\int d^3x\,\rho^2\dot{\tilde{\theta}}\right)^2}{\left(\int d^3x\,\rho^2\right)^2}\right] - 2\rho\dot{\tilde{\theta}}\left[\frac{\int d^3x\,\rho^2\dot{\tilde{\theta}}}{\int d^3x\,\rho^2}\right] = 0 \tag{37}$$

$$\frac{Q_1}{\left(\int d^3x\,\rho^2\right)^2}\left[2\rho\dot{\tilde{\theta}}\int d^3x\,\rho^2 - 2\rho\int d^3x\,\rho^2\dot{\tilde{\theta}}\right] = 0 \tag{38}$$

while from variations with respect to $\tilde{\theta}$ we have

$$-\partial_\mu(\rho^2\partial_\mu\tilde{\theta}) + \frac{\partial}{\partial\tau}\left[\rho^2\frac{\int d^3x\,\rho^2\dot{\tilde{\theta}}}{\int d^3x\,\rho^2}\right] = 0 \tag{39}$$

$$Q_1\frac{\partial}{\partial\tau}\left(\frac{\rho^2}{\int d^3x\,\rho^2}\right) = 0 \tag{40}$$

The structure of these equations is quite simple. If $V'(\rho_0) = 0$ for $\rho \neq 0$, then there is an obvious solution $\rho = \rho_0$, $\tilde{\theta} = 0$. Let us look for nontrivial solutions, so let us assume that $1/\int d^3x\,\rho^2$ does not vanish. Thus, we are looking for localized solutions. Then eq. (38) states that the τ-derivative of $\tilde{\theta}$ is pure zero mode. Since there are no zero modes in $\tilde{\theta}$ then $\tilde{\theta}$ is constant in (imaginary)-time, *i.e.*, $\dot{\tilde{\theta}} = 0$. Now, eq. (40) states that ρ^2 is separable, that is $\rho(\tau,\vec{x}) = \mathrm{T}(\tau)X(\vec{x})$. One may normalize X to $\int d^3x\,X^2 = 1$. Eqs. (38) and (40) are easy to solve because they simply enforce the charge conservation constraint. The remaining two equations are equations for the functions T and X:

$$-\ddot{\mathrm{T}}X - \mathrm{T}\vec{\nabla}^2X + \mathrm{T}X(\vec{\nabla}\tilde{\theta})^2 + V'(\mathrm{T}X) - \frac{Q_1^2X}{\mathrm{T}^3} = 0 \tag{41}$$

$$\vec{\nabla}(X^2\vec{\nabla}\tilde{\theta}) = 0 \tag{42}$$

Notice that the last term on the left hand side of eq. (41) has the expected minus sign, indicating a repulsive force at the origin of field space. If a solution exists, then as $\tau_2 - \tau_1 \to \infty$ it must have infinite action (relative to the trivial nonlocalized solution), for $I_{eff} > \frac{1}{2}\int_{\tau_1}^{\tau_2} d\tau \left[T^2 \int d^3x\, (\vec{\nabla}X)^2 + T^{-2}Q_1^2\right]$.

It is instructive to compare these equations with those proposed by Lee[9]. In that case one has the usual conservation of charge current,

$$\partial_\mu(\rho^2\partial_\mu\theta) = 0 \tag{43}$$

with the constraint

$$\int d^3x\, \rho^2\dot{\theta} = Q_1 \tag{44}$$

plus the additional wrong sign pseudo-equation of motion

$$-\partial^2\rho - \rho(\partial_\mu\theta)^2 + V'(\rho) = 0. \tag{45}$$

To make the comparison more direct we introduce a new variable into our equations defined by $\theta \equiv \tilde{\theta} + \Theta(\tau)$ where Θ is defined to be an integral of

$$\dot{\Theta}\mathrm{T}^2 = Q_1. \tag{46}$$

Then, in terms of θ and $\rho = \mathrm{T}X$ eq. (42) reduces to the current conservation equation (43), the defining expression (46) reproduces the constraint (44), while eq.(41) yields

$$-\partial^2\rho + \rho(-\dot{\theta}^2 + (\vec{\nabla}\theta)^2) + V'(\rho) = 0. \tag{47}$$

Thus, while eq. (45) is not quite correct, it will reduce to the correct expression when one is interested in solutions with θ being only a function of τ. As we will see, this is precisely what happens when looking for spherically symmetric wormholes. The reason for the discrepancy is quite apparent. While the functional integral automatically picks out paths with constant *global* charge (*cf.*, eq. (15)), the condition of Lee corresponds to paths with constant local charge.

Including gravity into the above theory is not difficult. We must assume that, by analogy with eq. (29), the transition amplitude $\mathcal{G}(\phi_1, \phi_2; \Sigma_1, \Sigma_2)$ between eigenstates of the field operator $\hat{\phi}$ on given spacelike hypersurfaces Σ_i is given again by a path integral with corresponding boundary conditions. The lagrangean density for the scalar field is just the covariant version of (19). The charge is now defined by[13,14]

$$Q \equiv \int_{\Sigma} d\Sigma^{\mu} j_{\mu} \tag{48}$$

Here $d\Sigma^{\mu} = n^{\mu} d\Sigma$, where n^{μ} is a future-directed unit vector orthogonal to the spacelike hypersurface Σ, $d\Sigma = d^3x\sqrt{h}$ is the volume element in Σ and the conserved charge current is $j_{\mu} = -i(\phi^* \overleftrightarrow{\partial}_{\mu}\phi)$. Whereas in flat space charge conservation is the statement that Q is time independent, in a curved space the corresponding statement is that Q is independent of Σ[1].

We learnt from the flat space case that the charge conservation constraint was enforced automatically by a term in the effective action which is not explicitly invariant under Lorentz transformations. Correspondingly we expect general covariance not to be explicit in the constraint term in the effective action when gravity is included. To make as many symmetries explicit as possible we choose to parametrize the metric in terms of lapse and shift functions[15],

$$ds^2 = -(N^2 - N^a N^b h_{ab})dt^2 + 2h_{ab}N^a dt dx^b + h_{ab}dx^a dx^b. \tag{49}$$

For each hypersurface of constant time we can construct normal unit vectors

$$n^2 = -1 \tag{50}$$

$$n^{\mu} = (1/N, -N^a/N) \tag{51}$$

$$n_{\mu} = (-N, 0, 0, 0) \tag{52}$$

As we will consider transition amplitudes $\mathcal{G}(\phi_1, \phi_2; \Sigma_1, \Sigma_2)$ between eigenstates of the field operator $\hat{\phi}$ on given spacelike hypersurfaces Σ_i, we will choose a gauge with $t = t_i =$ constant defining Σ_i.

Conjugate momenta operators are defined as before and the change in basis is again given by eq. (26), where the integral is now over the spacelike hypersurface Σ_i. The last ingredient we need before we can transcribe the result in (31) to this case is the decomposition (30) into zero and non-zero modes in the

[1]One needs to assume that Σ is Cauchy[14]. The statement then follows from Gauss' theorem and the equations of motion.

path integral. For any fixed background metric this presents no new difficulties. As in the flat space case the decomposition is analogous to the expansion in modes in canonical quantization[13]. One can imagine then performing first the integration over the matter fields and then the one over the metric.

The resulting effective action is

$$S_{eff} = S^E + S(\rho, \tilde{\theta}) + \frac{1}{2}\int dt\, \frac{(Q_1 - \int d\Sigma^\mu \rho^2 \partial_\mu \tilde{\theta})^2}{\int d\Sigma\, \frac{1}{N}\rho^2} \tag{53}$$

where S^E is Einstein's action and $S(\rho, \tilde{\theta})$ is the generally covariant action for the matter fields. The result is not explicitly invariant under general coordinate transformations since we have computed a very particular transition amplitude in a particular gauge. There remains explicit symmetry under time redefinitions and under general spatial-coordinate transformations. Under the coordinate transformation $x \to y = y(x)$ with $\partial x^0/\partial y^a = 0$ and $\partial x^a/\partial y^0 = 0$, one has

$$N \quad\to\quad N' = \frac{\partial x^0}{\partial y^0} N \tag{54}$$

$$N^a \quad\to\quad N'^a = \frac{\partial x^0}{\partial y^0}\frac{\partial y^a}{\partial x^b} N^b \tag{55}$$

$$h_{ab} \quad\to\quad h'_{ab} = \frac{\partial x^c}{\partial y^a}\frac{\partial x^d}{\partial y^b} h_{cd} \tag{56}$$

and S_{eff} remains invariant.

It is not quite clear to us how one should continue this answer into imaginary time. We will use as a guide the requirement that the resulting expression for the effective action should reduce to the one obtained earlier in the flat space case. We thus perform the continuation according to

$$I_{eff} = iS_{eff}(N \to iN, N^a \to N^a, h_{ab} \to h_{ab}). \tag{57}$$

We then have

$$I_{eff} = I^E + I(\rho, \tilde{\theta}) + \frac{1}{2}\int dt\, \frac{(Q_1 - i\int d\Sigma^\mu \rho^2 \partial_\mu \tilde{\theta})^2}{\int d\Sigma\, \frac{1}{N}\rho^2} \tag{58}$$

with the Einstein action $I^E = -\frac{1}{16\pi G}\int d^4x \sqrt{g} R$ and the matter action $I(\rho, \tilde{\theta}) = \int d^4x \left\{\frac{1}{2} g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho + \frac{1}{2}\rho^2 g^{\mu\nu}\partial_\mu\tilde{\theta}\partial_\nu\tilde{\theta} + V(\rho)\right\}$.

This is the central result of this section. It allows us to look for saddle point configurations that dominate[2] the transition amplitude between states of definite charge. ρ variations yield (assuming $Q_1/(\int d\Sigma N^{-1}\rho^2)^2 \neq 0$)

$$-\Box\rho + \rho g^{\mu\nu}\partial_\mu\tilde{\theta}\partial_\nu\tilde{\theta} + V'(\rho) - \frac{Q_1^2 + \left(\int d\Sigma^\mu\,\rho^2\partial_\mu\tilde{\theta}\right)^2}{\left(\int d\Sigma\,\frac{1}{N}\rho^2\right)^2}N^{-2}\rho = 0 \tag{59}$$

$$Nn^\mu\partial_\mu\tilde{\theta} = \frac{\int d\Sigma^\mu\,\rho^2\partial_\mu\tilde{\theta}}{\int d\Sigma\,\frac{1}{N}\rho^2} \tag{60}$$

Since $g^{\mu\nu}\partial_\mu\tilde{\theta}\partial_\nu\tilde{\theta} = (n^\mu\partial_\mu\tilde{\theta})^2 + h^{ab}\partial_a\tilde{\theta}\partial_b\tilde{\theta}$ one can use (60) to rewrite eq. (59) as

$$-\Box\rho + \rho h^{ab}\partial_a\tilde{\theta}\partial_b\tilde{\theta} + V'(\rho) - \frac{Q_1^2/N^2}{\left(\int d\Sigma\,\frac{1}{N}\rho^2\right)^2}\rho = 0 \tag{61}$$

This displays the repulsive core potential produced by the charge-carrying zero mode. The $\tilde{\theta}$ equations are

$$-\partial_\mu(\rho^2\sqrt{g}g^{\mu\nu}\partial_\nu\tilde{\theta}) + \partial_\mu\left(\sqrt{h}n^\mu\rho^2\frac{\int d\Sigma^\sigma\,\rho^2\partial_\sigma\tilde{\theta}}{\int d\Sigma\,\frac{1}{N}\rho^2}\right) = 0 \tag{62}$$

$$\partial_\mu\left(\sqrt{h}n^\mu\rho^2\frac{1}{\int d\Sigma\,\frac{1}{N}\rho^2}\right) = 0 \tag{63}$$

In contrast to the flat space case, this is not the whole story, for we still have to consider the equations obtained from varying the metric $g_{\mu\nu}$. These are Einstein's equations

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi GT^{\mu\nu} \tag{64}$$

where for the energy-momentum tensor $T^{\mu\nu}$ we find

$$\begin{aligned}T^{\mu\nu} &= (g^{\mu\lambda}g^{\nu\sigma} - \frac{1}{2}g^{\mu\nu}g^{\lambda\sigma})(\partial_\lambda\rho\partial_\sigma\rho + \rho^2\partial_\lambda\tilde{\theta}\partial_\sigma\tilde{\theta}) - g^{\mu\nu}V(\rho)\\ &+\frac{\int d\Sigma^\sigma\,\rho^2\partial_\sigma\tilde{\theta}}{\int d\Sigma\,\frac{1}{N}\rho^2}(\rho^2\partial_\lambda\tilde{\theta})N^{-1}(n^\lambda g^{\nu\mu} - n^\nu g^{\lambda\mu} - n^\mu g^{\nu\lambda})\\ &+\frac{Q_1^2 - \left(\int d\Sigma^\sigma\,\rho^2\partial_\sigma\tilde{\theta}\right)^2}{\left(\int d\Sigma\,\frac{1}{N}\rho^2\right)^2}N^{-2}\rho^2(\frac{1}{2}g^{\mu\nu} - n^\mu n^\nu)\end{aligned}$$

[2]More precisely, it is fluctuations about these configurations that dominate the path integral. The measure for any one configuration is typically vanishingly small[16].

$$+iQ_1\left[\frac{1}{\int d\Sigma\,\frac{1}{N}\rho^2}(\rho^2\partial_\lambda\tilde\theta)N^{-1}(n^\lambda g^{\nu\mu}-n^\nu g^{\lambda\mu}-n^\mu g^{\nu\lambda})\right.$$
$$\left.-2\frac{\int d\Sigma^\sigma\,\rho^2\partial_\sigma\tilde\theta}{\left(\int d\Sigma\,\frac{1}{N}\rho^2\right)^2}\rho^2N^{-2}(\frac{1}{2}g^{\mu\nu}-n^\mu n^\nu)\right] \tag{65}$$

The imaginary part of Einstein's equation is

$$(\rho^2\partial_\lambda\tilde\theta)N^{-1}(n^\lambda g^{\nu\mu}-n^\nu g^{\lambda\mu}-n^\mu g^{\nu\lambda})=2\frac{\int d\Sigma^\sigma\,\rho^2\partial_\sigma\tilde\theta}{\int d\Sigma\,\frac{1}{N}\rho^2}\rho^2N^{-2}(\frac{1}{2}g^{\mu\nu}-n^\mu n^\nu) \tag{66}$$

We see, by contracting with $g^{\mu\nu}$, that eq. (60) is a special case of (66). Contracting with n^μ one has

$$\partial_\mu\tilde\theta=n_\mu\frac{\int d\Sigma^\sigma\,\rho^2\partial_\sigma\tilde\theta}{N\int d\Sigma\,\frac{1}{N}\rho^2}. \tag{67}$$

In fact this equation is equivalent to (66). Using the explicit form (50) of the normal vector in our gauge this implies $\partial_a\tilde\theta=0$. This is a surprising and welcome simplification. For one thing, it decouples $\tilde\theta$ from the ρ equation of motion (*cf.*, eq. (61)), and eq. (62) is trivially satisfied. In fact, $\tilde\theta$ decouples from the real part of Einstein's equation as well: using (66) the energy-momentum tensor simplifies to

$$T^{\mu\nu}=(g^{\mu\lambda}g^{\nu\rho}-\frac{1}{2}g^{\mu\nu}g^{\lambda\rho})\partial_\lambda\rho\partial_\rho\rho-g^{\mu\nu}V(\rho)+\frac{Q_1^2}{\left(\int d\Sigma\,\frac{1}{N}\rho^2\right)^2}N^{-2}\rho^2(\frac{1}{2}g^{\mu\nu}-n^\mu n^\nu) \tag{68}$$

This equation displays also a repulsive-core 'potential' for the metric. A simple picture of Lee's wormhole emerges. As the metric tends to shrink space down to reduce Einstein's action in going from Σ_1 to Σ_2, there is a large enhancement in the contribution to the action from the zero mode in θ, for $\dot\theta\sim Q/V$ with V the volume of the cross-section of the wormhole. It is the competition between these two terms that gives rise to the wormhole solution.

How are these equations related to those of K. Lee[9]? We can compare the answers by using the same trick as in the flat space case. Since our equations don't involve the θ field anymore, we may introduce a spurious θ field satisfying

$$\partial_\mu\theta=n_\mu\frac{Q_1}{N\int d\Sigma\,\frac{1}{N}\rho^2}. \tag{69}$$

Then eq. (61) takes the (compellingly covariant-looking) form

$$-\Box\rho - \rho g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta + V'(\rho) = 0 \tag{70}$$

which is one of Lee's equations. Current conservation in ref. [9] is just (63). The energy momentum tensor (68) can also be recast in the form of Lee:

$$T^{\mu\nu} = (g^{\mu\lambda}g^{\nu\rho} - \frac{1}{2}g^{\mu\nu}g^{\lambda\rho})(\partial_\lambda\rho\partial_\rho\rho - \rho^2\partial_\lambda\theta\partial_\rho\theta) - g^{\mu\nu}V(\rho) \tag{71}$$

The point is that Lee has solved the correct equations *provided his solution also satisfies our eq.* (69). It is easy to see that the spherically symmetric ansatz studied by Lee automatically satisfies this constraint.

The quantization condition on the charge of the wormhole now follows trivially from the quantization of the charge of the definite charge states whose transition amplitudes we are computing. The point is made clear, if it is at all needed to dwell on this, by referring back to the simple example at the beginning of this section. The change of basis in (5) shows quite explicitly that single valuedness of the expression on the right hand side and periodicity in θ by 2π implies quantization of angular momentum ℓ. The same is true, word by word, for our field-theoretic examples.

3 Vertex Operators and the Wormhole Summation Formula

In this section we will extract some features of the form of the vertex operators for wormhole insertions[10]. The results are quite expected and the methods quite bold and simplistic. But we feel that it must be included since these features are the basis for the proposal of the next section, where a solution to the problem of the vanishing of G is advanced. We will briefly review[7,8] the wormholes summation formula of Coleman, Giddings and Strominger[3]. We do this to recall the role played by the vertex operator in Coleman's proposal for

[3]Excellent reviews of the summation formula have been given by Preskill[17]and by Klebanov *et al.*[18]. The original papers [7,8] make additional assumptions valid only in the semiclassical approximation[17]. Their discussion is somewhat complicated by their consideration of spontaneous emission/absorption of baby universes.

the solution of the cosmological constant problem[2], and to properly set up the discussion of the next section.

The primary object under consideration is some transition amplitude between states $\psi(h,\phi)$ of the universe characterized by a three-metric h_{ab} and some matter fields ϕ. These universe states are large and smooth. By assumption, the transition amplitude is computed by a euclidean quantum gravity path integral. Regardless of what makes such an integral a well defined object, one may still attempt to compute the effect of integrating out short distance fluctuations which induce topology changes. Of course, one may always write down such configurations, but it is not at all clear that they will have a nonnegligible effect, *i.e.*, the measure associated with them can, and probably is, vanishingly small. But if such configurations have finite action and correspond to a saddle point of the action, then it is not implausible that their effect will not vanish. This is the case with instantons and bounces for field theories in flat space, and we have no reason to suspect it would not be in a consistent theory of gravity. We will refer to these finite action saddle points as 'wormholes'.

We feel it is important to stress that topology changing configurations that are not local extrema of the action may not have macroscopic effects. A simple analogy can be used to make this point clearer. Consider a field theory, on flat space, with a global symmetry group G. By this we mean, of course, that the lagrangean exhibits this symmetry. It is easy enough to find finite action configurations which violate this symmetry, *i.e.*, configurations with a 'sink' or a 'source' of charge. Still, and quite obviously so, the Greens' functions of the theory are symmetric under G. So the macroscopic effects of such configurations are negligible, inasmuch as symmetry violations are concerned.

In order to investigate the effect of wormholes on the transition amplitude, we imagine doing the path integral over short wavelength fluctuations first, holding the long wavelength ones fixed. This amounts to considering the integration of short wavelength metric and matter fields in the presence of smooth background metric and matter fields. We are led to summing over all possible insertions of wormholes onto a fixed background metric that interpolates smoothly between the states $\psi(h,\phi)$. We associate with the insertion of a wormhole end onto the background at a point x a vertex operator[4] $K(x)$. This is a local function of the background fields. Inserting both ends of the wormhole on all possible points gives a contribution to the path integral of $\beta^2 \equiv \frac{1}{2}(\int d^4x \sqrt{g}K(x))^2$. The factor of $\frac{1}{2}$ is included because the wormhole ends are indistinguishable. Similarly the contribution from n wormholes is $\beta^{2n}/n!$. Summing over n and neglecting wormhole-wormhole interactions[19] (the dilute gas approximation), one has $\sum_0^\infty \beta^{2n}/n! = \exp(\beta^2)$. Therefore, the effect of wormholes is to shift the action for the background fields by β^2,

$$I \to I_{eff} = I - \frac{1}{2}\left(\int d^4x \sqrt{g}K(x)\right)^2 \tag{72}$$

Notice that the shift in the action is bilocal. This can be made to look local by introducing a 'wormhole parameter' α, through the trick

$$\exp(\beta^2) = \sqrt{2/\pi}\int_{-\infty}^{\infty} d\alpha \, \exp(-\frac{1}{2}\alpha^2 - \sqrt{2}\,\alpha\beta). \tag{73}$$

The usefulness of this is made clear by the following consideration. If one is interested in computing the expectation value $< \mathcal{O} >$ of some operator $\mathcal{O}$ one must compute a ratio of path integrals as in

$$< \mathcal{O} >= \frac{\int_{disc}[dg][d\phi]e^{-I_{eff}}\mathcal{O}}{\int_{disc}[dg][d\phi]e^{-I_{eff}}1} \tag{74}$$

The path integral includes all disconnected graphs, or more properly put, all disconnected universes. Now, for local actions simple combinatorics lead to the disconnected components (the graphs with no external legs) to cancel in the ratio, and one is left with the ratio of connected graphs. But the effective action in (72) is not local. By introducing the wormhole parameters one sidesteps this difficulty. One has

$$I_{eff}(\alpha) = I + \alpha\int d^4x\sqrt{g}K(x), \tag{75}$$

and then

$$\frac{\int_{disc}[dg][d\phi]e^{-I_{eff}(\alpha)}\mathcal{O}}{\int_{disc}[dg][d\phi]e^{-I_{eff}(\alpha)}1} = \frac{\int_{conn}[dg][d\phi]e^{-I_{eff}(\alpha)}\mathcal{O}}{\int_{conn}[dg][d\phi]e^{-I_{eff}(\alpha)}1} \equiv < \mathcal{O} >_\alpha . \tag{76}$$

[4] Also called a probability amplitude for insertion of a wormhole[7]. We have absorbed the wormhole action factor, $\exp(-I_w)$, in the vertex operator.

Of course, one pays the price of having to perform the integral over the wormhole parameter. Defining[2]

$$Z(\alpha) \equiv \int_{disc} [dg][d\phi] e^{-I_{eff}(\alpha)} \tag{77}$$

the expectation value is then

$$< \mathcal{O} >= \frac{\int d\alpha e^{-\frac{1}{2}\alpha^2} Z(\alpha) < \mathcal{O} >_\alpha}{\int d\alpha e^{-\frac{1}{2}\alpha^2} Z(\alpha)} \tag{78}$$

This is Coleman's master formula. The expectation value $< \mathcal{O} >_\alpha$ is what we would have computed if we didn't have wormholes and the action was $I_{eff}(\alpha)$. With wormholes we must average over all such universes with probability distribution $\mathcal{P}(\alpha) \propto \exp(-\frac{1}{2}\alpha^2)Z(\alpha)$. The effect of the wormholes is twofold. On the one hand, it introduces a shift into the action for the longer wavelengths, as in eq. (72). On the other hand, by linking two otherwise disconnected large smooth manifolds, it forces us to consider the sum over disconnected components once the wormholes are integrated out.

The wormhole-induced shift in the lagrangean density (*cf.*, eq. (75)) is just α times the vertex operator $K(x)$. We would like to obtain the form of the vertex operator in the theory discussed in the preceeding section. Of course, in that theory there are many different wormholes, at least one for each nonnegative integer corresponding to the charge Q carried through the wormhole. Let us consider what modifications to the above discussion are needed in accounting properly for the multiple wormhole case.

The effect of multiple types of wormholes of a given size can be accounted for by introducing an *a priori* different vertex operator $K_i(x)$ for each different wormhole type (labeled by i). Correspondingly we may introduce wormhole parameters α_i, and the above discussion goes through replacing the wormhole parameter by an array of wormhole parameters. When one considers wormholes of vastly different sizes, though, the situation is quite different. Consider a theory which, at tree level, exhibits wormholes of two different sizes $\ell \ll L$. As one carries out the program of integrating out fluctuations from the small scale up, an effective action for fluctuations on scales much larger than ℓ is produced. This depends on the small-wormhole parameter α_ℓ. After integrating out fluctuations with wavelengths between ℓ and L it is not clear that the resulting action admits wormhole solutions of size L for any value of α_ℓ. As we will see shortly, the vertex operator in the theory of the preceeding section carries charge. The effective action for scales $\sim L$ is not symmetric under the global $U(1)$ symmetry, the amount of breaking being proportional to α_ℓ. But it is precisely the conservation of this charge that stabilizes Lee's wormhole. Heuristically, the theory may favor the exchange of N wormholes of charge $Q = 1$ than that of one wormhole of charge $Q = N$. At first sight, the summation over large wormholes will be α_ℓ dependent. But this complication can be avoided knowing, *a posteriori*, that the probability distribution $\mathcal{P}(\alpha)$ is infinitely peaked about definite values of the wormhole parameters. There is no reason to expect this value of α_ℓ to be vanishingly small. Thus, it is perhaps plausible that the large wormholes don't even really exist in this theory. Since we cannot prove this assertion we will make it an assumption[5].

The calculation of the vertex operator $K(x)$ is in principle understood but hard to carry out in practice. It was pioneered by Hawking[10]. We want to consider a transition amplitude $< \psi_2 \mid \psi_1 >_w$ between some states $\psi(h,\phi)$ when one wormhole end is inserted into the smooth background fields that interpolate between the initial and final states ψ. We then construct $K(x)$ by requiring that the matrix element of this operator between the states ψ and without the wormhole insertion reproduces the transition amplitude $< \psi_2 \mid \psi_1 >_w$. In what follows we will have in mind the particular wormhole solutions described in the preceeding section, but the results might be more generally applicable to other theories with charge stabilized wormholes.

The path integral expression for the transition amplitude is

$$< \psi_2 \mid \psi_1 >_w = \int_{b.c.} [dg][d\phi] e^{-I} \tag{79}$$

The integral includes only smooth configurations. The boundary conditions are

[5]Coleman[2] assumes the effect of large wormholes can be neglected. It has been argued that the effects of large wormholes are disastrous[20]. We have just presented an argument why this worry may be unjustified, but we needed very specific assumptions: (i)Only 'real' wormholes (*i.e.*, saddle points of the action) are relevant, and (ii)It is global charge that stabilizes the wormholes. Preskill has presented[17] an argument independent of these assumptions which indicates that the effects of small wormholes dominate over those of large ones.

that the manifold have two boundaries where the fields match on to the values of h and ϕ specified by the states ψ_i, and a third boundary where the configuration matches on to that of a wormhole end. It is this last one boundary condition that we are concerned with here. It is convenient to think of the transition amplitude as going to a state $< \psi_1^* \times \psi_2 \mid$ from the wormhole end 'state' $\mid w >$. The transition amplitude between these states is still given by the path integral in (79). Inserting a complete set of states we have

$$< \psi_1^* \times \psi_2 \mid w >= \int [dh][d\phi] < \psi_1^* \times \psi_2 \mid h, \phi >< h, \phi \mid w > \tag{80}$$

We intend to use here the one aspect of the state $\mid w >$ that was emphasized in the preceeding section, namely that it is a state of definite charge Q. Therefore we can write

$$\mid w >= \int [dh'][d\rho'][d\tilde{\theta}'] \Psi_Q(h', \rho', \tilde{\theta}') \mid h', \rho', \tilde{\theta}', Q > \tag{81}$$

so that

$$< h, \rho, \theta \mid w >= \Psi_Q(h, \rho, \tilde{\theta}) e^{iQ\Theta}. \tag{82}$$

We can now use eq. (82) in eq. (80). Since we have assumed that the fields we are integrating over are smooth on the scale of the wormhole, the wormhole insertion can be thought of as being localized at a point x. Also, smoothness guarantees that on the scale of the wormhole $\Theta \simeq \theta$. Finally, the leftover integral over the fields at x removes the corresponding boundary condition. Thus, we have

$$K_Q(x) = \Psi_Q(h(x), |\phi(x)|, 0) e^{iQ\theta(x)} \tag{83}$$

This is the result announced earlier. The vertex operator carries charge. This was not unexpected[21], but we think it is nice to see a simple derivation of the result. It also tells us that to go any further we have to understand the 'wormhole wavefunction' Ψ_Q. All we can offer here is the general statement that it must be a linear combination of $U(1)$-symmetric scalar (with respect to coordinate transformations) operators.

It is also apparent from the above derivation that $\Psi_{-Q} = \Psi_Q^*$, for inverting the 'time' direction simply corresponds to charge conjugation[22]. Correspondingly we have $K_{-Q} = K_Q^*$ and in the wormhole summation formula one should use $\beta = |\int d^4x\, K_Q|^2$. The corresponding wormhole parameter α_Q is now a complex variable[21], and the lagrangean is shifted by $\alpha_Q K_Q + \text{h.c.}$

4 The Sliding Newton's Constant Problem

Coleman noticed that if one is interested in vacuum to vacuum amplitudes, the expression for $Z(\alpha)$ in eq. (77) exponentiates:

$$Z(\alpha) = \exp\left(\int_{conn}[dg][d\phi]e^{-I_{eff}(\alpha)}\right) \tag{84}$$

The path integral is now only over connected graphs with no boundaries, *i.e.*'Hartle-Hawking boundary conditions'[22]. The integral cannot be done, but if one concentrates on manifolds with the topology of the sphere, one can formally write

$$\ln Z(\alpha) = \exp(-\Gamma_\alpha) \tag{85}$$

Here Γ_α is the value of the effective action[6], Γ_{eff}, at its lowest local minimum. If the only massless particle in the theory is the graviton, then Γ_{eff} has an expansion in local operators built out of the metric,

$$\Gamma_{eff} = \int d^4x\sqrt{g}\left(\Lambda - \frac{1}{16\pi G}R + \cdots\right) \tag{86}$$

where the ellipses indicate terms which are higher order in the curvature tensor. While we don't have the ability to express the coupling constants in this effective action as explicit functions of the fundamental couplings of the theory, this expansion is useful because it is written directly in terms of observable quantities. Λ is the cosmological constant which is observed to be essentially zero, and G is Newton's Constant. To see this simply imagine introducing observers. This could be done by performing the path integral with sources or in the presence of a background metric field. To the extent that Γ_{eff} is written only in terms of essential coupling constants[23], the observers could perform experiments to measure them (precisely in the same fashion we do).

[6]Here by 'effective action' we mean the one that results from integrating out the fluctuations of up to arbitrarily long wavelengths. It is the functional that generates 1PI Greens' functions.

The dependence on the wormhole parameters is totally hidden in the coupling constants in (86). Now, for positive Λ the lowest minimum of Γ_{eff} is known to be at the sphere of radius (squared) $r^2 = 3/8\pi G\Lambda$, giving

$$Z(\alpha) = \exp\exp\left(\frac{3}{8G^2\Lambda}\right) \tag{87}$$

This lead Coleman to conclude that the probability distribution $\mathcal{P}(\alpha)$ is infinitely strongly peaked at those values of α for which the cosmological constant vanish.

We are ready to review the sliding G problem[7]. It was noticed in ref [6] that $\mathcal{P}$ is also strongly peaked at values of α for which G vanishes! How do we compare two infinities? To define the theory properly one must introduce an infrared regulator. This is not easy to accomplish in general relativity in a covariant manner, since one essentially is restricting the integration over the conformal factor in the metric. But we will assume that somehow one can regulate on, say, the diameter. For the regulated theory one expects the effective action to stop growing with radius when the radius grows larger than the cutoff D. Therefore one has

$$-\Gamma_\alpha = \begin{cases} 3/8G^2\Lambda & \text{for } 1/G\Lambda \lesssim D^2 \\ < D^2/G & \text{for } 1/G\Lambda \gtrsim D^2 \end{cases} \tag{88}$$

This is a disaster. The probability distribution $\mathcal{P}(\alpha)$ is infinitely peaked at vanishing G *for any value* of Λ. If indeed the wormhole parameters allow $1/G$ to run off to infinity, then not only the cosmological constant problem is not solved, but the whole program is in flagrant contradiction with experiment!

The dependence of $1/G$ (or, for that matter, the dependence of any parameter in the effective action) on the wormhole parameters comes from two distinct contributions, which we refer to as 'tree level' and 'quantum corrections' respectively. The tree level contributions are simply the α-dependence in $I_{eff}(\alpha)$, the effective action just above the wormhole length scale. Let us refer to these α dependent parameters as the 'bare coupling constants'. When one integrates out the longer wavelengths in constructing the full effective action Γ_{eff}, the resulting $1/G$ will depend on other bare couplings. These are the quantum corrections.

[7]For a lucid and more complete explanation see ref. [17]

Preskill[17] has pointed out that one can worry about the sliding G problem separately for the tree level and the quantum correction pieces. Let us denote by G_0 the bare parameter. Then, as $G_0 \to 0$ gravity becomes free. The quantum corrections are non. Thus one must understand why, if at all, the wormhole parameter dependence in G_0 does not allow it to vanish. An interesting proposal to that effect was put forward in [17]. There it was pointed out that, if one goes beyond the dilute gas approximation, the α dependence in I_{eff} is no longer linear and the resulting function of α associated with $1/G_0$ could be bounded from above. Moreover, ref. [17] argues that whether this occurs depends on the nature (*i.e.*, attractive *vs.* repulsive) of the effective force between wormhole ends, as viewed from the smooth background.

Here we would like to propose an alternative solution. It will not require that we go beyond the dilute gas approximation, so, if correct, it supercedes Preskill's proposal. On the other hand, it applies only to charged wormholes of the kind we have been discussing so far. As we have had occasion to stress, we believe that only 'real' wormholes should make a contribution to the effective action, and the charged ones are the prototypical example of such objects. The restriction to charged wormholes, while a strong assumption, is, we believe, a reasonable one. As we will see, the result depends crucially on the relative sign of different terms in the effective action. While we have not calculated these signs, there is no reason to believe, *a priori*, that they would go one way or another. In a sense, the same is true of Preskill's proposal where we don't know, *a priori*, what the sign of wormhole end interactions is.

Let us focus on the effect of $Q = 1$ wormholes in the theory discussed in the above sections. The extension to the higher Q case is straightforward, and will be postponed. The effective action at the wormhole scale $M \sim L^{-1}$ will have α dependent contributions only in the combination $\alpha\phi$ times $U(1)$ symmetric operators. In particular, there is no term of the form $M^2\alpha R$, since it is forbidden by the symmetry. This is not to say that there are no α-dependent corrections to $1/G_0$. Indeed, one has a term of the form $cM\alpha\phi R$, with c a numerical constant. In general one may have also additional terms with higher powers of $|\phi|^2/M^2$, but we will discard them here for simplicity. We believe one may generalize our results in a straightforward manner when these higher dimension terms are included. Naïvely one then expects to have a shift in the bare Newton's Constant $\Delta(1/G_0) = c\alpha v M + \text{c.c.}$ where v stands for the vacuum expectation value (VEV) of ϕ. If this conclusion held, then by letting $|\alpha| \to \infty$ with $-\frac{\pi}{2} - \arg c < \arg \alpha < \frac{\pi}{2} - \arg c$ we would have $1/G_0 \to \infty$.

Fortunately, this conclusion doesn't need to hold. We have missed the fact that the VEV of ϕ depends on α. It obviously affects the magnitude of the VEV. But, more importantly, because the scalar field potential is no longer symmetric, *it picks the phase of* ϕ. This is the central idea. If we insist that $|\alpha| \to \infty$, it is conceivable that the phases of ϕ and α are correlated in such a way as to decrease $1/G_0$. Let's see how this works in a simple model. We take for the effective potential at the scale L

$$V_{eff}(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2 + bM^3\alpha\phi + \text{h.c.} \tag{89}$$

where b is a numerical constant. To streamline the argument, let us absorb the phase of α in ϕ, so that we can think of α as a real positive parameter. Minimizing V with respect to θ gives the conditions

$$\alpha|b|\rho \sin(\theta + \arg b) = 0 \tag{90}$$

$$\alpha|b|\rho \cos(\theta + \arg b) < 0 \tag{91}$$

Minimizing with respect to ρ yields a cubic equation the details of which we are not interested in. It simply gives the shifted value ρ_0 of the VEV. The point is that the VEV of θ is $\theta_0 = \pi - \arg b$. Now the shift in the bare Newton's Constant is

$$\begin{aligned}\Delta\left(\frac{1}{G_0}\right) &= \alpha|c|M\rho_0 \cos(\theta + \arg c) \\ &= -\alpha|c|M\rho_0 \cos(\arg c - \arg b)\end{aligned} \tag{92}$$

which is always nonpositive for $\cos(\arg c - \arg b) \geq 0$. This is the announced result. We must admit it is rather simplistic, but we find the simplicity, in fact, compelling. We can generalize the above argument to the case where the coefficients c and b are some unknown functions of $|\phi|^2/M^2$. Whereas it is hard to solve the case analytically, it is obvious that there is still a possibility for the two terms 'pulling' in opposite directions. One may simply replace $c \to c(\rho_0/M)$ and

$b \to b(\rho_0/M)$ in the above discussion. The case of more than one α parameter would correspondingly introduce new functions $b', b'', \ldots$ and $c', c'', \ldots$, and we could imagine that whatever physics determined the relative phase of b and c will determine the corresponding new relative phases to be the same. If the proposal works, then clearly the α parameters for which $1/G_0$ is maximized are numbers of order unity, and if the same holds true for the solution to the equation $\Lambda_0 = 0$, then there is no pressing need to improve the dilute gas approximation in the manner suggested in ref [17].

5 Quantum Corrections

Even if we succeed in finding a theory where the bare Newton's Constant is bounded away from zero, one must worry about the quantum corrections (as defined earlier). The dependence on the wormhole parameters that $1/G$ acquires through its dependence on other bare parameters is very interesting since it contains implicit information on the values of physical coupling constants.

There is another way in which coupling constants could be fixed. It was realized by Coleman[2], implemented in [6] and followed up in [24], that once the probability distribution $\mathcal{P}(\alpha)$ picks up the subspace of wormhole parameter space where $\Lambda = 0$, then $\mathcal{P}(\alpha)$ is still infinitely peaked at those values of coupling constants for which Γ_α is minimized. This could then determine, at least in principle, some or all of the low energy parameters of the theory. In ref. [6] it was shown, for example, that the hierarchy problem could be solved, provided not all of the wormhole parameters were determined by the vanishing of Λ and the maximization of $1/G$.

One important difference between these two ways of fixing coupling constants is that the former yields information on the bare couplings whereas the latter gives the result directly in terms of effective low energy parameters. A more important distinction is that the former must be considered before the latter, since the corresponding peaking of $\mathcal{P}(\alpha)$ is infinitely stronger.

We do not claim to understand the renormalization of $1/G$. Some features of this problem are understood at one loop level (see, *e.g.*, [23,25]). Let us briefly investigate the effect of a massive particle on $1/G$. By dimensional analysis, the shift that the massive particle produces on $1/G$ must be proportional to m^2, m being the mass of the particle. For example, from the one loop effective action for a real massive scalar propagating on a sphere[6],

$$\Delta\left(-\frac{1}{32\pi G}\right) = \frac{1}{16\pi^2}(\xi - \frac{1}{6})m^2 \ln(m^2/\mu^2) - \frac{19}{48}m^2 \qquad (93)$$

Here ξ is the coupling of the scalar to the curvature scalar R, normalized so that $\xi = \frac{1}{6}$ for a conformally coupled scalar. For $\xi = 0$ this agrees with the result in ref. [25] (which also gives the result for a free massive fermion, this being again eq. (93) with $\xi = 0$). We have included a finite renormalization term (*i.e.*, the no-log term proportional to m^2) since the counterterm is proportional to the combination $(\xi - \frac{1}{6})m^2$, as evident from the dependence on the renormalization point μ.

If we now consider both m^2 and ξ as functions of the wormhole parameters, we are faced with the problem that they may induce $G \to 0^+$. Of course, the functions $m^2(\alpha)$ and $\xi(\alpha)$ themselves may be bounded functions of the α's. Whether this is the case requires a more detailed analysis of the structure of the theory in question. After all, it was an understanding of the dependence on the wormhole parameters of the VEV of the charged scalar that led us to the the finiteness of $1/G_0$ in Section 4. For example, if the scalar (or, rather, pair of scalars) under consideration is precisely the charged scalar of Section 4, then $\xi(\alpha) = 0$ and $m^2(\alpha) = 2\lambda(3\rho_0^2(\alpha) - v^2)$ at tree level. So we may have $G \to 0^+$ by having $\rho_0^2(\alpha) \to \infty$. But we must be careful not to upset the result $G_0 \neq 0$. For if $G_0 \to 0^+$ as $\rho_0^2(\alpha) \to \infty$, gravity becomes free and the quantum correction is irrelevant. We must enquire whether $G_0 m^2 \ln m^2$ diverges as $\rho_0^2(\alpha) \to \infty$. In the simple $Q = 1$ sector studied earlier, we had $\rho_0(\alpha) \sim \alpha^{1/3}$ and $G_0 \sim (\alpha\rho_0(\alpha))^{-1} \sim \alpha^{-4/3}$, for large α, so that $G_0 m^2 \ln m^2 \sim \alpha^{-2/3} \ln \alpha$ and the theory is 'safe' (in the sense that $G \not\to 0$). We can go a bit further. For general Q, the leading α_Q-dependent term in the effective potential is $\sim \alpha_Q \rho^Q$. For $Q < 4$ and large α one has $\rho_0 \sim \alpha_Q^{1/(4-Q)}$, $G_0 \sim \alpha_Q^{4/(4-Q)}$ and $G_0\rho_0^2 \sim \alpha_Q^{-2/(4-Q)}$ and the theory is safe again. The situation is more complicated for $Q \geq 4$, for the potential becomes unbounded from below. Here we simply plead ignorance, and return to our main argument[8]. What we want to stress is that there is a need for understanding the

[8]Many aspects of the $Q \geq 4$ case must be understood to proceed further. First and

particular dependence of bare couplings on wormhole parameters to establish the behaviour of G, and with it the values of fundamental parameters.

It is instructive to compare the condition $G \to 0^+$ with the vanishing of Λ. In the latter case the wormhole parameters need to be carefully adjusted to be on the hypersurface $\Lambda = 0$, but this can be achieved with finite values of the wormhole parameters. On the other hand, the condition $G \to 0^+$ requires that either $\alpha \to \infty$ or that some coupling constants depend on α in a singular way[9].

Consider now the case of a massive fermion field. We see from eq. (93) (with $\xi = 0$) that $m^2(\alpha)$, if allowed, is pushed towards large values. Of course, we have argued that we need to know first whether $m^2(\alpha) \to \infty$ is indeed accessible. Let us assume it is and explore the consequences. We propose that this does not force upon us the conclusion $G \to 0^+$. In fact, it may strike the reader as inconsistent to include in the effective theory for long wavelength fluctuations a field of mass $m \gtrsim M$ (let alone $m \to \infty$!). We have a 'bootstrap' condition for $m^2(\alpha)$. If $m^2(\alpha) < M^2$ then $\mathcal{P}(\alpha)$ is negligibly small. But there is no way to consider the case $m^2(\alpha) \gtrsim M^2$ consistently other than by integrating out the fermion field before (or simultaneously with) the wormhole. This seems to imply that $m \sim M$ with wormhole parameter space restricted to the subspace $m^2(\alpha) \gtrsim M^2$. (We have restricted our attention here to fermions, since for scalars one may have $|\xi| \to \infty$ even for finite mass). This bootstrap mechanism seems to be quite general, and should apply beyond the 1-loop approximation. After all, it only seems reasonable to expect that physics at a scale M could induce a shift ΔG_0^{-1} of at most of the order of M^2.

To summarize, we have argued that, as was the case for G_0, to study the

foremost is the possibility, discussed in Section 3, that the small wormholes (say, those with $Q < 4$) destabilize the large ones ($Q \geq 4$). This would alleviate the problem of the unbounded potential in a drastic way (*i.e.*, by eliminating it altogether). Secondly, since one is forced to consider large values of the field ρ, it is imperative to include in the effective potential the full dependence on ρ. The function $f_Q(x)$ in the term $\alpha_Q \phi^Q f_Q(|\phi|) + \text{c.c.}$ in the potential may perhaps vanish, as $x \to \infty$, more rapidly than x^{-Q}. These logical possibilities, admittedly, are at the moment wild speculations.

[9]Singular behaviour for finite α may arise in a variety of ways. The crucial observation of [6] was that the coefficient of the $\mathcal{O}(R^2)$ term in Γ_{eff} has the singular behaviour $\sim \ln m^2(\alpha)$ as $m^2(\alpha) \to 0$. In the same limit, though, we know from eq. (93) that no singularity arises for $1/G$.

condition $G \to 0^+$ one needs to understand the α dependence of bare couplings. Moreover, our bootstrap condition indicates that particle masses are driven to the wormhole scale, giving only finite renormalizations of G^{-1}.

Before concluding this section we'd like to present a specific example which contains a small surprise. We may introduce a majorana fermion with mass m_0 into the theory of a charged scalar considered earlier. A Yukawa coupling, g, is forbidden by the $U(1)$ symmetry, but it may arise when the wormhole is integrated out, with $g \propto \alpha$. This is precisely the form of the coupling that gives rise to the shift in G_0^{-1}, so that $\Delta m_0 \sim \Delta G_0 \sim \alpha\rho_0(\alpha)$. Therefore $G_0 m_0^2 \sim \alpha\rho_0(\alpha)$ and we are in the situation described in the preceeding paragraph (the sign of this correction could 'push' together with that of the corresponding term in the potential, *cf.*, Section 4). We must choose $m_0(\alpha) \gtrsim M$ and then the fermion produces a finite shift in G_0^{-1} of order M^2. We then must maximize G_0^{-1} given the condition $\Delta m_0 \gtrsim M$. It is rather surprising, and perhaps amusing, that since the dependence of ΔG_0^{-1} on the α's is the same as that of Δm_0, the maximization of ΔG_0^{-1} given $\Delta m_0 \gtrsim M$ fixes only a 1-dimensional hypersurface in α-space. Of course, this conclusion is upset by any deviation from $\Delta m_0/\Delta G_0^{-1} =$ constant. But, if this persists, an analysis of the $\mathcal{O}(R^2)$ terms in Γ_{eff} would then be required to fix the remaining wormhole parameters[6,24].

In the absence of a miracle, such as the one described at the end of the preceeding paragraph, we must conclude then that the maximization of $1/G$ fixes all of the wormhole parameters and with them, implicitly, all of the low energy parameters. Moreover, it generically implies that if a particle's mass term is not protected by a symmetry (unbroken by wormhole effects), then the mass will be order the wormhole scale M. As in ref [17], though through a different argument, we seem to be forced into the conclusion that the mass scale of low energy physics must be generated dynamically.

6 Conclusions

Let us review what we have done. We will do so with an emphasis on the assumptions and conjectures that we have made.

We have found Lee's wormhole as a saddle point configuration for the effective action between states of definite charge. In deriving this result we assumed a path integral formulation for the transition amplitude in question and a particular prescription for the rotation into imaginary time. These assumptions are probably quite safe and therefore we believe this result, while uninspiring, is quite solid. It may well be the only point of lasting value in this work. From this it was straightforward to derive the quantization condition on the charge carried by the wormhole. The generic form of the vertex operator was then derived. The result is quite as expected, but nevertheless useful for the applications presented below.

We argued that only wormhole configurations which are saddle points of the action (and small fluctuations about them), as opposed to any topology changing configuration, should play a role in determining the effective action for long distance physics. As this is not on firm ground, it should be taken as an assumption for what follows. We then conjectured that small wormholes may destabilize larger ones. Henceforth, this should also be taken as an assumption.

With these assumptions, it was then argued that the model of Lee may not suffer from the sliding Newton's constant problem. We proposed a simple mechanism to explain this, whereby the scalar potential and the coupling of the scalar to curvature 'pull' in opposite directions. Whether this is indeed the case requires further investigation. We consider this the most important proposal in this work.

If this mechanism operates for the bare Newton's constant, then one must address the issue of quantum corrections to it. We analyzed the contributions from massive particles to $1/G$ at one loop. We concluded that, not surprisingly, a knowledge of the wormhole parameter dependence of the various bare coupling constants of the theory is needed to draw any conclusions. Even without access to this information, we formulated a 'bootstrap' condition which seems to indicate that the quantum corrections, rather than driving G to zero, will produce in it a shift of order of the wormhole scale and drive unprotected particle masses to that scale. We believe this condition, on logical grounds, to be of very general validity and therefore will apply even when some assumptions (*e.g.*, the 1-loop approximation) are dropped. Direct information on observable low energy parameters (other than the cosmological constant) seems to be generically unaccessible.

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References

[1] L. Abbott, *Sci. Am.* **258**,v.5,258(1988)

[2] S. Coleman, 'Why There is Nothing Rather Than Something: A Theory of the Cosmological Constant', Harvard Preprint HUTP-88/A022(1988), unpublished

[3] E. Baum, *Phys. Lett.* **133B**, 185(1983)

[4] S. W. Hawking, *Phys. Lett.* **134B**, 403(1984)

[5] J. Polchinski, Texas University preprint, UTTG-31-88(1988), unpublished

[6] B. Grinstein and M. B. Wise, *Phys. Lett.* **212B**, 407(1988)

[7] S. Coleman, *Nucl. Phys.* **B307**, 867(1988)

[8] S. B. Giddings and A. Strominger, *Nucl. Phys.* **B307**, 854(1988)

[9] K.Lee, 'Wormholes and Goldstone Bosons', Fermilab Preprint, FERMILAB-Pub-88/27-T(1988), unpublished

[10] S. W. Hawking, *Phys. Rev.* **D37**, 904(1988)

[11] S.B. Giddings and A. Strominger, *Nucl. Phys.* **B306**, 890(1988)

[12] R. Rajaraman, *Solitons and Instantons* North–Holland Publishing Company, Amsterdam, 1982

[13] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge 1982

[14] S. W. Hawking and G. F. R. Ellis, *The Large scale Structure of Spacetime*, Cambridge University Press, Cambridge 1973

[15] R. Arnowitt *et al*, in *Gravitation: An Introduction to Current Research*, ed. by L. Witten, Wiley, New York 1962

[16] S. Coleman, 'The Uses of Instantons' in *Aspects of Symmetry*, Cambridge University Press, Cambridge 1985

[17] J. Preskill, 'Wormholes in Spacetime and The Constants of Nature', California Institute of Technology preprint, CALT-68-1521(1988), unpublished

[18] I. Klebanov *et al*, 'Wormholes and The Cosmological Constant', SLAC preprint, SLAC-PUB-4705(1988), unpublished

[19] A. K. Gupta and M. B. Wise, 'Comment on Wormhole Correlations', California Institute of Technology preprint, CALT-68-1520-Rev(1988), unpublished

[20] W. Fischler and L. Susskind, 'A Wormhole Catastrophe', Texas University preprint, UTTG-26-88(1988), unpublished

[21] J. E. Kim and K. Lee, 'The Scale Problem in Wormhole Physics', Fermilab preprint, FERMILAB-PUB-88/95-T(1988), unpublished

[22] J. B. Hartle and S. W. Hawking, *Phys. Rev.* **D28**, 2960(1983)

[23] S. Weinberg, in *General Relativity: An Einstein Centennial Survey,* ed. by S. W. Hawking and W. Israel, Cambridge, 1979

[24] B. Grinstein and C. T. Hill, 'The Trace Anomaly and Low Energy Phenomenological Implications of Wormholes', Fermilab preprint, FERMILAB-PUB-88/190-T(1988), unpublished

[25] P. Candelas and D. J. Raine, *Phys. Rev.* **D12**, 965(1975)