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Geometry of a Group of Area Preserving Diffeomorphisms

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Abstract

SDiff$\mathcal{M}$, the group of area preserving reparametrizations of two–dimensional surfaces $\mathcal{M}$, play an important role in the theory of relativistic membranes. We calculate the riemannian curvature of this group and analyze the stability of their geodesics. Since the geodesics are motions of an ideal fluid, this gives information on the instability of ideal fluid flow on $\mathcal{M}$.

In the light–cone gauge a residual symmetry of the relativistic membrane consists of a group of area preserving diffeomorphisms of a two–dimensional surface $\mathcal{M}$[1–3]. The role of this group is close to that of the group of conformal transformations in string theory.

In the fundamental paper of Arnold [4] it was shown that this group can also be considered as a configuration space for an ideal fluid filling the domain $\mathcal{M}$. Such a flow is described by a curve g_t in the group $SDiff\mathcal{M}$ and the kinetic energy of the moving fluid defines a right–invariant riemannian metric on this group.

We calculate the riemannian curvature of this group $SDiffS^2$. This formula allows us to find the curvature in any two–dimensional direction and estimate the stability of geodesics. Since the geodesics are motions of an ideal fluid (incompressible and non–viscous), this gives information the the instability of ideal fluid flow on S^2.

The Lie algebra $SdiffS^2$ corresponding to the group $SDiffS^2$ consists of all vector fields with divergence 0 on S^2. Since the first cohomology group of S^2 is trivial – $H^1(S^2,R) = 0$, every element ν of $SdiffS^2$ can be represented in a unique way through the functions ϵ_ν on S^2,*i.e.*

$$\nu^a = \epsilon^{ba}\partial_b\epsilon_\nu, \quad a,b = 1,2 \quad , \tag{1}$$

$\epsilon^{12} = 1$. This relation defines a homomorphism of the Lie algebra $SdiffS^2$ onto the Poisson algebra $PdiffS^2$

$$[\epsilon_\nu, \epsilon_\mu]_{\mathcal{P}} = \epsilon_{[\nu,\mu]_L} \tag{2}$$

where

$$[\epsilon, \eta]_{\mathcal{P}} = \partial_1\epsilon\partial_2\eta - \partial_2\epsilon\partial_1\eta \quad , \tag{3}$$

and

$$[\nu, \mu]_L = \nu^b\partial_b\mu^a - \mu^b\partial_b\nu^a \quad . \tag{4}$$

The kernel of this homomorphism consist of the constant functions.

Kinetic energy, *i.e.*, a positive definite quadratic form on the Lie algebra *SdiffS*^{2} is

$$\langle \mu, \nu \rangle = \int_{S^2} \mu \cdot \nu d^2\sigma \tag{5}$$

where $\sigma_1 = -\cos\theta, \sigma_2 = \phi$, $\mu \cdot \nu = g_{ab}\mu^a \nu^b$ is the scalar product giving the riemannian metric on S^2. The scalar product (5) in *SdiffS*^{2} can be represented as

$$\langle \mu, \nu \rangle = -\int d^2\sigma \epsilon_\mu \Delta \epsilon_\nu \equiv -(\epsilon_\mu, \ \Delta\epsilon_\nu) \tag{6}$$

where $\mu^a = \epsilon^{ba}\partial_b \epsilon_\mu$, $\nu^a = \epsilon^{ba}\partial_b\epsilon_\nu$, and Δ is the Laplace operator on S^2. We also have that

$$\langle [\mu, \nu]_L, \omega \rangle = -\left(\epsilon_{[\mu,\nu]_L}, \epsilon_\omega\right) = -\left([\epsilon_\mu, \epsilon_\nu]_{\mathcal{P}}, \Delta\epsilon_\omega\right) \tag{7}$$

Let us introduce orthogonal basis in the *PdiffS*^{2} by means of the functions $Y_{lm}(\sigma)$

$$\begin{aligned} Y_{lm}(\sigma) &= C_{lm} e^{im\sigma_2} \mathcal{P}_e^{|m|}(-\sigma_1) \\ C_{lm} &= (-1)^m \sqrt{\frac{2l+1}{4\pi} \frac{(l- \mid m \mid)!}{(l+ \mid m \mid)!}} \end{aligned} \tag{8}$$

and define structure constants as

$$[Y_{l_1 m_1}, Y_{l_2 m_2}]_{\mathcal{P}} = G^{l_3 m_3}_{l_1 m_1, l_2 m_2} Y_{l_3 m_3} \tag{9}$$

(see Appendix). Basis e_{lm} in the *SdiffS*^{2} algebra can be constructed by using homomorphism (1)

$$e^a_{lm} = \epsilon^{ba}\partial_b Y_{lm} \quad . \tag{10}$$

For e_{lm} we have that

$$\begin{aligned} \langle e_{l_1 m_1} \quad , \quad e_{l_2 m_2} \rangle &= -(Y_{l_1 m_1}, \Delta Y_{l_2 m_2}) = \\ &= (-1)^{m_1} l_1 (l_1 + 1) \delta_{l_1 - l_2} \delta_{m_1 + m_2} \quad . \end{aligned} \tag{11}$$

Structure constants of the *SdiffS*^{2} algebra coincide with the structure constants of the *PdiffS*^{2} algebra (see (9) and (7))

$$[e_{l_1 m_2}, e_{l_2 m_2}]_L = G^{l_3 m_3}_{l_1 m_1 l_2 m_2} e_{l_3 m_3} \quad . \tag{12}$$

Bernoulli equations in hydrodynamics can be represented as the Euler equation for the geodesics of the group G=*SdiffS*^{2}[4]

$$\dot{\nu} = -B(\nu, \nu) \quad , \tag{13}$$

where the bilinear operator on algebra TG_e, $B : TG_e \times TG_e \rightarrow TG_e$ is defined by the formula

$$\langle [\mu, \nu]_L, \omega \rangle \equiv \langle B(\omega, \mu), \nu \rangle \tag{14}$$

and can be expressed in terms of the vector fields ω and μ of the Lie algebra $B(\omega, \nu) =$ curl $\omega \wedge \mu$ + grad a. Using (14) and (12) it is easy to compute B in terms of the structure constant (12)

$$\begin{aligned} B(e_{l_1 m_1}, e_{l_2 m_2}) &= b^{l_3 m_3}_{l_1 m_1 l_2 m_2} e_{l_3 m_3} \\ b^{l_3 m_3}_{l_1 m_1 l_2 m_2} &= \frac{l_1(l_1+1)}{l_3(l_3+1)} G^{l_3 m_3}_{l_1 m_1 l_2 m_2} \quad . \end{aligned} \tag{15}$$

Riemannian metric (5) on the group *SDiffS*^{2} is right–invariant, therefore

$$\langle \nabla_{e_i} e_j, e_k \rangle + \langle e_j, \nabla_{e_i} e_k \rangle = 0 \quad , \tag{16}$$

where $i = (l, m)$, and $\nabla_{e_i} e_j$ is the covariant derivative. Since

$$[e_i, e_j]_L = \nabla_{e_i} e_j - \nabla_{e_j} e_i \tag{17}$$

and

$$\nabla_{e_i} e_j + \nabla_{e_j} e_i = -B(e_i, e_j) - B(e_j, e_i) \quad , \tag{18}$$

one can represent the connection as:

$$\nabla_{e_i} e_j = \Gamma^k_{ij} e_k = \frac{1}{2}\Big([e_i, e_j]_L - B(e_i, e_j) - B(e_j, e_i)\Big) \tag{19}$$

In the basis (10) Γ^k_{ij} is equal to

$$\Gamma^{l_3 m_3}_{l_1 m_1 l_2 m_2} = \frac{1}{2}\frac{l_3(l_3+1) - l_1(l_1+1) + l_2(l_2+1)}{l_3(l_3+1)} G^{l_3 m_3}_{l_1 m_1 l_2 m_2} \quad . \tag{20}$$

Riemannian curvature of the group G with right–invariant metric at any point g is determined by the curvature at the identity. Therefore, it is sufficient to calculate the curvature in different directions in the Lie algebra TG_e. The components of the curvature tensor Ω

$$\Omega(e_i, e_j) = \Big[\nabla_{e_i}, \nabla_{e_j}\Big] - \nabla_{[e_i, e_j]} \tag{21}$$

have the form

$$\begin{aligned} R_{ijkm} &= \langle \Omega\,(e_i, e_j)\, e_k, e_m \rangle \\ &= \langle \nabla_{e_i} e_k, \nabla_{e_j} e_m \rangle - \langle \nabla_{e_j} e_k, \nabla_{e_i} e_m \rangle \\ &+ \frac{1}{2}\langle [e_i, e_j]\,, [e_k, e_m] - B\,(e_k, e_m) + B\,(e_m, e_k)\rangle \end{aligned} \tag{22}$$

and for our case is equal to

$$\begin{aligned} R_{l_1 m_1 l_2 m_2 l_3 m_3 l_4 m_4} = \sum_m (-1)^m l(l+1) \Big[& d^l_{l_1 l_3} \cdot d^l_{l_2 l_4} \cdot G^{lm}_{l_1 m_1 l_3 m_3} \cdot G^{l-m}_{l_2 m_2 l_4 m_4} \\ - d^l_{l_1 l_4} \cdot d^l_{l_2 l_3} \cdot G^{lm}_{l_1 m_1 l_4 m_4} \cdot G^{l-m}_{l_2 m_2 l_3 m_3} & + k^l_{l_3 l_4} \cdot G^{lm}_{l_1 m_1 l_2 m_2} \cdot G^{l-m}_{l_3 m_3 l_4 m_4} \Big] \end{aligned} \tag{23}$$

where

$$\begin{aligned} d^l_{l_1 l_2} &= \frac{1}{2}\frac{l(l+1) - l_1(l_1+1) + l_2(l_2+1)}{l(l+1)} \;; d^l_{l_1 l_2} + d^l_{l_2 l_1} = 1 \quad , \\ k^l_{l_1 l_2} &= \frac{1}{2}\frac{l(l+1) - l_1(l_1+1) - l_2(l_2+1)}{l(l+1)} \quad . \end{aligned} \tag{24}$$

It is possible to rewrite (23) in a more symmetric way

$$
\begin{aligned}
2 \cdot R_{l_1 1 m_1 l_2 m_2 l_3 m_3 l_4 m_4} &= \sum_m (-1)^m \cdot l(l+1) \\
&+ \left[d^l_{l_1 l_3} \cdot d^l_{l_2 l_4} + k^l_{l_1 l_2} + k^l_{l_3 l_4} + d^l_{l_3 l_1} \cdot d^l_{l_4 l_2} \right] \cdot G^{lm}_{l_1 m_1 l_3 m_3} \cdot G^{l-m}_{l_2 m_2 l_4 m_4} \\
&- \left[d^l_{l_1 l_4} \cdot d^l_{l_2 l_3} + k^l_{l_1 l_2} + k^l_{l_3 l_4} + d^l_{l_4 l_1} \cdot d^l_{l_3 l_2} \right] \cdot G^{lm}_{l_1 m_1 l_4 m_4} \cdot G^{l-m}_{l_2 m_2 l_3 m_3}
\end{aligned}
\tag{25}
$$

where we use Jacobi identity (A.17).

As was mentioned above, the riemannian curvature of group manifold is closely connected with the behavior of its geodesics [4]. The vector field of geodesic variation $\delta\nu$ satisfies the Jacobi equation. This equation for the normal component $\delta\nu_\perp$ (normal to the velocity vector ν) can be written in the "Newton's " form with potential which is proportional to the curvature K in the direction $(\delta\nu_\perp, \nu)$

$$
\begin{aligned}
K_{(\nu,\delta\nu_\perp)} &\sim \langle \Omega(\nu, \delta\nu_\perp)\nu, \delta\nu_\perp \rangle \\
&= R_{l_1 m_1 l_2 m_2 l_3 m_3 l_4 m_4} \nu^{l_1 m_1} \delta\nu_\perp^{l_2 m_2} \nu^{l_3 m_3} \delta\nu_\perp^{l_3 m_4}
\end{aligned}
\tag{26}
$$

where

$$
\nu = \nu^{lm} e_m \quad , \delta\nu = \delta\nu^{lm} e_{lm} \tag{27}
$$

So that the formula (25) allow us to estimate stability of the geodesics of the group *SdiffS*^{2} (*i.e.* the stability of Earth atmosphere!)

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APPENDIX A:

It is much more convenient to calculate real structure constants $g^{l_3m_3}_{l_11m_1l_2m_3}$

$$G^{l_3m_3}_{l_1m_1l_2m_2} = -i(-1)^{m_3} g^{l_3-m_3}_{l_1m_1l_2m_2}$$

From the definition (9) one can see that

$$g^{l_3m_3}_{l_1m_1l_2m_3} = i\int_{S^2} Y_{l_3m_3}\,[Y_{l_1m_1}, Y_{l_2m_2}]_P\, d\mu \quad , \tag{A1}$$

For the functions on S^2 we have

$$\int_{S^2} \epsilon\partial_2\eta d\mu = -\int_{S^2} \eta\partial_2\epsilon d\mu \quad , \tag{A2}$$

$$\int_{S^2} \epsilon\partial_1\eta d\mu = \int \epsilon\eta\Big|_{-1}^{+1} d\sigma_2 - \int_{S^2} \partial_1\epsilon\cdot\eta d\mu \tag{A3}$$

Note that if ϵ or η contain derivatives in σ_2, then the first term in (A.3) is zero; so we obtain

$$\begin{aligned} \int_{S^2} \epsilon\,[\eta,\chi]\, d\mu &= \int_{s^2} \chi\,[\epsilon,\eta]\, d\mu = \int_{s^2} \eta\,[\chi,\epsilon]\, d\mu \quad , \\ g^{l_3m_3}_{l_1m_1l_2m_2} &= g^{l_2m_2}_{l_3m_3,l_1m_1} = g^{l_1m_1}_{l_2m_2,l_3m_3} \end{aligned} \tag{A4}$$

Structure constants g are real, and since $Y^*_{lm} = (-1)^m Y_{l,-m}$, then

$$g^{l_3m_3}_{l_1m_1,l_2m_2} = -g^{l_3,-m_3}_{l_1,-m_1,l_3,-m_3} \tag{A5}$$

Further from $Y_{lm}(\pi-\theta,\varphi+\pi) = (-1)^l Y_{lm}$ we have

$$g^{l_3m_3}_{l_1m_1,l_2m_2} = 0 \tag{A6}$$

if $l_1 + l_2 + l_3$ is even number, and, finally, since $g^{l_3m_3}_{l_1m_1,l_2m_2}$ is proportional to

$\int \exp i\,(m_1+m_2+m_3)\,\sigma_2 d\sigma_2$, then

$$g^{l_3 m_3}_{l_1,m_1,l_2m_2} = 0 \quad , m_1+m_2+m_3 \neq 0 \tag{A7}$$

To calculate the bracket (3) in the definition of structure constant (9) we shall use the formula

$$\begin{aligned} \partial_1 P_l^{|m|} &= -(1-\sigma_1^2)^{-\frac{1}{2}} P_l^{|m|+1} - \sigma_1(1-\sigma_1^2)^{-1}|m| \cdot P_l^{|m|} \quad , \\ \partial_2 Y_{lm} &= imY_{lm} \quad . \end{aligned} \tag{A8}$$

Then

$$\begin{aligned} &\{Y_{l_1m_1}, Y_{l_2m_2}\}_L = iC_{l_1m_1}C_{l_2m_2}e^{i(m_1+m_2)\sigma_2} \quad . \\ &\left[\left(1-\sigma_1^2\right)^{-\frac{1}{2}}\left(m_1 P_{l_1}^{|m_1|}P_{l_2}^{|m_2|+1} - m_2 P_{l_1}^{|m_1|+1}P_{l_2}^{|m_2|}\right) + \right. \\ &\left. +\sigma_1(1-\sigma_1^2)^{-1}\left(m_1|m_2| - m_2|m_1|\right) P_{l_1}^{|m_1|} \cdot P_{l_2}^{|m_2|}\right] \quad . \end{aligned} \tag{A9}$$

In order to reduce the upper index $|m|+1$ in Legendre polynomial, we'll use a recursion relation:

$$\left(1-\sigma_1^2\right)^{-\frac{1}{2}} P_l^{|m|+1} = (2l-1)P_{l-1}^{|m|} + (1-\sigma_1^2)^{-\frac{1}{2}} P_{l-2}^{|m|+1} = \sum_{k=0}^{[l-|m|-\frac{1}{2}]} [2(l-2k-1)+1] P_{l-2k-1}^{|m|} \quad , \tag{A10}$$

Substituting (A10) into (A9) we obtain

$$i\,[Y_{l_1m_1}, Y_{l_2m_2}]_P = \tag{A11}$$

$$\begin{aligned} &m_2 \sum_{k_1} \sqrt{2(l_1-2k_1-1)+1)(2l_1+1)} \sqrt{\tfrac{(l_1-|m_1|)\cdots(l_1-|m_1|-2k_1)}{(l_1+|m_1|)\cdots(l_1+|m_1|-2k_1)}} Y_{l_1-2k_1-1,m} Y_{l_2m_2} \\ -&m_1 \sum_{k_2} \sqrt{(2(l_2-2k_2-1)+1)(2l_2+1)} \sqrt{\tfrac{(l_2-|m_2|)\cdots(l_2-|m_2|-2k_2)}{(l_2+|m_2|)\cdots(l_2+|m_2|-2k_2)}} Y_{l_1m_1} Y_{l_2-2k_2-1,m_2} \end{aligned}$$

The integral in (A1) now can be calculated via the Wigner formula:

$$\int Y_{l_1m_1}Y_{l_2m_2}Y_{l_3m_3}d\mu = \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}}\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}\begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \tag{A12}$$

where $3j$ -symbol is nonzero if l_1, l_2 and l_3 produce a vector triangle, *i.e.* none of l_i is larger than a sum of two others; $3j$ -symbol turns to zero if m_i is larger than l_i. Finally we'll obtain that at $m_1 \gg 0, m_2 \gg 0$ or $m_1 \ll 0, m_2 \ll 0$ the expression for structure constants has a form

$$\left(\frac{4\pi}{(2l_1+1)(2l_2+1)(2l_3+1)}\right)^{\frac{1}{2}} g^{l_3m_3}_{l_1m_1l_2m_2} = \tag{A13}$$

$$m_2\sum_{k_1}(2(l_1-2k_1-1)+1)\sqrt{\frac{(l_1-|m_1|)\cdots(l_1-|m_1|-2k_1)}{(l_1+|m_1|)\cdots((l_1+|m_1|-2k_1)}}$$
$$\begin{pmatrix} l_1-2k_1-1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad \begin{pmatrix} l_1-2k_1-1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$
$$-m_1\sum_{k_2}(2(l_2-2k_2-1)+1)\sqrt{\frac{(l_2-|m_2|)\cdots(l_2-|m_2|-2k_2)}{(l_2+|m_2|)\cdots(l_2+|m_2|-2k_2)}}$$
$$\begin{pmatrix} l_1 & l_2-2k_2-1 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \quad \begin{pmatrix} l_1 & l_2-2k_2-1 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$

When m_1 and m_2 have different signs, $g^{lm}_{l_1m_1,l_2m_2}$ can be calculated by means of (A4). First coefficients for S^2 are

$$g^{10}_{l_1m_1l_2m_2} = (-1)^{m_1}m_1\sqrt{\frac{3}{4\pi}}\delta_{l_1l_2}\delta_{m_1+m_2} \quad , \tag{A14}$$

$$g^{11}_{l_1m_1l_2m_2} = (-1)^{m_1}\sqrt{(l_1-m_1)(l_1+m_1+1)}\sqrt{\frac{3}{8\pi}}\delta_{l_1l_2}\delta_{m_1+m_2+1}$$

$$g^{1-1}_{l_1m_1l_2m_2} = -(-1)^{m_1}\sqrt{(l_1+m_1)(l_1-m_1+1)}\sqrt{\frac{3}{8\pi}}\delta_{l_1l_2}\delta_{m_1+m_2-1}$$

from which we can se that Y_{10}, Y_{11} and Y_{1-1} form a finite subalgebra $SO(3)$:

$$\begin{aligned} [Y_{11}, Y_{1-1}] &= i\sqrt{\frac{3}{4\pi}} Y_{10} \ , \\ [Y_{11}, Y_{10}] &= i\sqrt{\frac{3}{4\pi}} Y_{11} \ , \\ [Y_{1-1}, Y_{10}] &= -i\sqrt{\frac{3}{4\pi}} Y_{1-1} \ , \end{aligned} \tag{A15}$$

which has the following commutation relations with the other generators:

$$\begin{aligned} [Y_{10}, Y_{lm}] &= -im\sqrt{\frac{3}{4\pi}} Y_{lm} \ , \\ [Y_{11}, Y_{lm}] &= i\sqrt{(l-m)(l+m+1)}\sqrt{\frac{3}{8\pi}} \cdot Y_{lm+1}, \\ [Y_{1-1}, Y_{lm}] &= i\sqrt{(l+m)(l-m+1)}\sqrt{\frac{3}{8\pi}} Y_{lm-1} \ . \end{aligned} \tag{A16}$$

Using Jacobi identity for the Poisson bracket one can find that

$$\begin{aligned} -(-1)^m g^{lm}_{l_1m_1l_2m_2} \cdot g^{l-m}_{l_3m_3l_4m_4} &+ (-1)^m g^{lm}_{l_1m_1l_3l_3} \cdot g^{l-m}_{l_2m_2l_4m_4} \\ -(-1)^m g^{lm}_{l_1m_1l_4m_4} &\cdot g^{l-m}_{l_2m_2l_3m_3} = 0 \end{aligned} \tag{A17}$$

Let us substitute in (A17) (l, m) equal to $(1,1)$ and $(1,-1)$. Then using (A17) for $g^{11}_{l_1m_1l_2m_2}$ and $g^{1-1}_{l_1m_1l_2m_2}$ we come to the relations

$$\begin{aligned} \sqrt{(l-m)(l+m+1)}g^{l\,m+1}_{l_1m_1,l_2m_2} &+ \sqrt{(l_1-m_1)(l_1+m_1+1)}g^{lm}_{l_1m_1+1,l_2m_2} \\ &+ \sqrt{(l_2-m_2)(l_2+m_2+1)}g^{lm}_{l_1m_1,l_2m_2+1} = 0 \ , \end{aligned} \tag{A18}$$

$$\begin{aligned} \sqrt{(l+m)(l-m+1)}g^{l\,m-1}_{l_1m_1,l_2m_2} &+ \sqrt{(l_1+m_1)(l_1-m_1+1)}g^{lm}_{l_1m_1-1,l_2m_2} \\ &+ \sqrt{(l_2+m_2)(l_2-m_2+1)}g^{lm}_{l_1m_1,l_2m_2-1} = 0 \ . \end{aligned} \tag{A19}$$

Since $g^{l0}_{l_1 m_1 l_2 m_2} = g^{l_1 m_1}_{l_2 m_2 l0}$ it is easy to calculate this coefficient using the general representation (A 13). For example

$$g^{20}_{l_1 m_1 l_2 m_2} = \tag{A20}$$

$$3m_1(-1)^{m_1}\delta_{m_1+m_2,0}\left[\sqrt{\frac{5(l_1-m_1)(l_1+m_1)}{4\pi(2l_1-1)(2l_2+1)}}\delta_{l_1,l_2+1}+\sqrt{\frac{5(l_1+1-m_1)(l_1+1+m_1)}{4\pi(2l_1+1)(2l_1+3)}}\delta_{l_1,l_2-1}\right]$$

and so on. Then (A18) and (A19) allow us to calculate all structure constants.

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