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An Order Parameter that Tests the Existence of Charged Vector Bosons in the Georgi-Glashow Model

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ABSTRACT

On the example of the Georgi-Glashow model, we define order parameters that test the existence of charged states in the case of partial breakdown of symmetry.

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The vacuum overlap order parameter (VOOP) proposed in [1] seems to indicate correctly the phase transitions in several gauge theories with matter fields [2]. A different approach, inspired from the Coulomb gauge treatment of electrodynamics, leads to similar conclusions [3]. Both parameters test whether a specific attempt to construct charged states is successful; the phase transition they indicate is tied to the occurence versus nonoccurence of these charged states.

It is in general very difficult to get information on all possible charged states in a given model. Sometimes there are obvious candidates for charged states. In gauge theories one often searches for states with a nontrivial total electric flux corresponding to the center of the gauge group G. If however all matter fields transform trivially under the center, we do not expect to find gauge invariant states with a nontrivial center-electric flux. On the other hand, there might be candidates for charged states whose charge corresponds to a subgroup H of G. Typical examples are in models with "partial breakdown of symmetry" (Higgs mechanism), which have been discussed in the literature in the framework of perturbation theory. In this framework one speaks of "gauge symmetry breaking" and "the unbroken group H".

In this paper we indicate how to construct the charged vector bosons in the Georgi-Glashow model [4]. On a hypercubic lattice this model has the action [5]:

$$S = -\beta \sum_{p} \tfrac{1}{2} \mathrm{Tr}\, U(p) - \kappa \sum_{x,\mu} (\varphi_x, D^1(U_{x,\mu})\, \varphi_{x+\hat{\mu}}) \tag{1}$$

where the lattice gauge field $U_{x,\mu}$ is an $SU(2)$ matrix in the fundamental representation that lives on the link starting from x and going in the μ-direction, $U(p)$ stands for the product of U's around the plaquette p, φ_x is a real 3-component field with $|\varphi_x| = 1$, D^1 denotes the adjoint representation and $\hat{\mu}$ denotes the unit vector in direction μ. The Boltzmann measure is:

$$e^{-S} \prod_{x} d\varphi_x \prod_{x,\mu} dU_{x,\mu} \tag{2}$$

where $d\varphi_x$ is the normalized rotationally invariant measure on the 2-sphere S^2 and $dU_{x,\mu}$ is the Haar measure on $SU(2)$. The phase diagram of this model is depicted in fig. 1. In the Higgs phase the "unbroken group" is $U(1)$.

Using the original VOOP one does not get information about states with a $U(1)$ charge, as seen e.g. from the numerical results of [6]. Theoretically this can be understood as follows. The original VOOP is the infinite distance limit of a gauge invariant two-point-function of the φ-field. This two-point-function is defined as the scalar product between the vacuum (hence the name vacuum overlap) and a candidate for a "dipole" of particles carrying the adjoint representation as their charge. The gauge invariant dipole state is constructed by taking a pair of static adjoint $SU(2)$-sources (lowest energy *eigenstates* of the Hamiltonian that transform

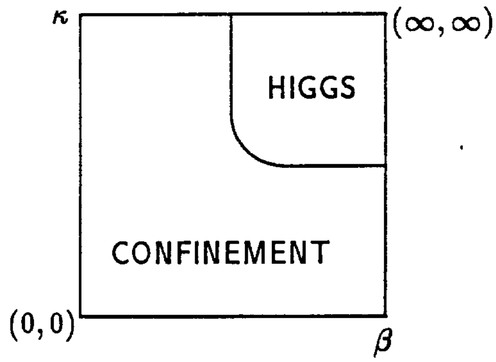


Figure 1: Phase diagram of the lattice Georgi-Glashow model with a fixed length Higgs field (from [5]).

as the adjoint representation of $SU(2)$ at two spatial sites $\underline{x}$ and $\underline{x}'$), and then replacing the sources by dynamical matter fields φ [1]. Now, in the Higgs phase the vacuum can be viewed as a *condensate* of the φ-fields [7], which *screens the adjoint sources.* Therefore the attempt to create a charged state by sending one of the partners in the dipole to infinity has to fail. The two-point-function approaches a constant, which can be interpreted as the gauge invariant counterpart to the usual "Higgs expectation value". On the other hand, in the confinement phase the φ-two-point-function also approaches a constant for large distances, albeit for a different reason: the dipole state is constructed to have a bounded energy [1]; if the distance between the two partners becomes too large, a pair of dynamical φ's pops out of the vacuum and we are left with two clouds of "hadrons" around $\underline{x}$ and $\underline{x}'$.

In order to construct a candidate for a charged state with a $U(1)$ charge using a similar procedure, we need as an intermediate step to construct static $U(1)$-sources. However, all static sources transform under an irreducible representation of $SU(2)$, and one expects them to be screened. In order to overcome this problem we *enlarge* the set of fields and the Hilbert space of the system in a natural way. The aim is to define a *dynamics in the new sectors of the Hilbert space* with respect to which the physically interesting *$U(1)$-sources become static.*

Exploiting the fact that the Higgs field φ lives in a homogeneous space of the gauge group $G = SU(2)$, we replace the field φ_x by the new field $V_x \in G$, which is related to φ_x by

$$\varphi_x = D^1(V_x^{-1})\,\phi \in S^2 \qquad (3)$$

where ϕ is the vector (0 0 1). By (3) S^2 is identified with the homogeneous space G/H, where $H \cong U(1)$ is the stability group of ϕ, $H = \{h \in G \mid D^1(h)\,\phi = \phi\}$. Since (3) is invariant under left multiplications of V by elements of H, the action expressed in terms of the V and the U is invariant under the gauge group $G \times H$

$$V_x \mapsto h_x V_x\, g_x^{-1}, \quad U_{x,\mu} \mapsto g_x U_{x,\mu}\, g_{x+\hat{\mu}}^{-1}, \qquad h_x \in H, \quad g_x \in G\,. \qquad (4)$$

The extension of the Boltzmann measure on the new configuration space is

$$e^{-S} \prod_x dV_x \prod_{x,\mu} dU_{x,\mu} \tag{5}$$

where dV_x is again the Haar measure.

To make the interpretation of the model as a gauge theory with gauge group H more transparent, we replace the variables $U_{x,\mu}$ by $W_{x,\mu}$ defined as

$$W_{x,\mu} = V_x U_{x,\mu} V_{x+\mu}^{-1} . \tag{6}$$

Under the gauge transformation (4)

$$W_{x,\mu} \mapsto h_x W_{x,\mu} h_{x+\hat{\mu}}^{-1} . \tag{7}$$

In analogy to the perturbative treatment of the Georgi-Glashow model, we decompose the field W into a "$U(1)$ gauge field" $W^{\parallel}$ and a "spin one" matter field $W^{\perp}$ by making the coset decomposition

$$W = W^{\perp} W^{\parallel} \tag{8}$$

where $W^{\perp}$ is a rotation around an axis in the x-y plane, the rotation angle being not larger than π, and $W^{\parallel} \in H$. Up to discontinuities, these conditions fix $W^{\perp}$ and $W^{\parallel}$ uniquely. Notice that the set of coset representatives in which $W^{\perp}$ takes its values is invariant both under charge conjugation and under gauge transformations. For the latter we have:

$$W_{x,\mu}^{\perp} \mapsto h_x W_{x,\mu}^{\perp} h_x^{-1}, \quad W_{x,\mu}^{\parallel} \mapsto h_x W_{x,\mu}^{\parallel} h_{x+\hat{\mu}}^{-1} , \qquad h_x \in H . \tag{9}$$

In order to discuss the structure of the state space of the theory we pass to the temporal gauge, $U_{x,0} = 1$ for all timelike links. For a finite spatial volume the state space is the Hilbert space of quadratically integrable functions on the direct product of copies of G for all sites and links. The transfer matrix $T = e^{-\hat{H}}$ ($\hat{H}$ is the Hamiltonian) has the form

$$T = e^{-\frac{1}{2}A}\, t\, e^{-\frac{1}{2}A} \tag{10}$$

where A is the operator of multiplication with the spatial action (underlining always refers to purely spatial objects; $j = 1, 2, 3$ denotes the spatial directions),

$$A(V, U) = -\beta \sum_{\underline{p}} \tfrac{1}{2} \mathrm{Tr}\, U(\underline{p}) - \kappa \sum_{\underline{x}, j} (\phi, D^1(V_{\underline{x}}\, U_{\underline{x},j} V_{\underline{x}+\hat{j}}^{-1})\, \phi) \tag{11}$$

and t has the kernel

$$t(V, U; V', U') = \exp \left\{ \beta \sum_{\underline{x}, j} \tfrac{1}{2} \mathrm{Tr}\, U_{\underline{x},j} U_{\underline{x},j}'^{-1} + \kappa \sum_{\underline{x}} (\phi, D^1(V_{\underline{x}}\, V_{\underline{x}}'^{-1})\, \phi) \right\} . \tag{12}$$

Notice that t and thus T vanishes on the orthogonal complement of the subspace of H-gauge invariant functions.

We now want to identify operators corresponding to H-electric flux and H-electric charge density. The H-electric field should be the canonical conjugate of the multiplication operator by $W^{||}$, so we define it to be

$$\big(E_{\underline{x},j}(h)\,f\big)(V,W) = f(V,W')\,, \quad \begin{cases} W'_{\underline{x},j} = W_{\underline{x},j}h \\ W'_{\underline{x}',j'} = W_{\underline{x}',j'}\,, \quad \underline{x}' \neq \underline{x} \text{ or } j' \neq j\,. \end{cases} \tag{13}$$

Notice that $E_{\underline{x},j}(h)$ is the exponential of the usual electric field, and therefore it satisfies a multiplicative Gauss' law

$$q_{\underline{x}}(h) = \rho_{\underline{x}}(h) \prod_j E_{\underline{x}-\hat{j},j}(h)\, E_{\underline{x},j}(h^{-1}) \tag{14}$$

where $q_{\underline{x}}(h)$ is the H-gauge transformation operator at $\underline{x}$ and the H-charge density operator $\rho_{\underline{x}}(h)$ is defined such that (14) is an identity

$$\big(\rho_{\underline{x}}(h)\,f\big)(V,W) = f(V',W')\,, \quad \begin{cases} V'_{\underline{x}} = h^{-1}V_{\underline{x}} \\ V'_{\underline{x}'} = V_{\underline{x}'}\,, & \underline{x}' \neq \underline{x} \\ W'_{\underline{x},j} = h^{-1}W_{\underline{x},j}h \\ W'_{\underline{x}',j} = W_{\underline{x}',j}\,, & \underline{x}' \neq \underline{x}\,. \end{cases} \tag{15}$$

The next step is to find operators that create a definite H-electric flux and an H-electric charge. Let u be a function on G with the property

$$u(hW) = u(Wh) = \chi(h)\,u(W) \tag{16}$$

for some nontrivial character χ of H. Examples of such functions are the diagonal matrix elements $D^j_{mm}(W)$ of the spin-j representation of $SU(2)$ in the basis of eigenvectors of the rotations around the 3-axis. We have the following commutation relations for the operator of multiplication by $u(W_{\underline{x},j})$:

$$E_{\underline{x},j}(h)\,u(W_{\underline{x},j}) = \chi(h)\,u(W_{\underline{x},j})\,E_{\underline{x},j}(h)\,, \quad \rho_{\underline{x}}(h)\,u(W_{\underline{x},j}) = u(W_{\underline{x},j})\,\rho_{\underline{x}}(h)\,. \tag{17}$$

Thus $u(W_{\underline{x},j})$ creates an electric flux equal to χ on the link $\underline{x}, j$. Let on the other hand w be a function on G with the property

$$w(hW) = \chi(h)\,w(W)\,, \quad w(Wh) = w(W)\,. \tag{18}$$

Examples of such functions are $D^j_{m0}(W)$ with integer m. Because of the commutation relation

$$\rho_{\underline{x}}(h)\,w(W_{\underline{x},j}) = \overline{\chi(h)}\,w(W_{\underline{x},j})\,\rho_{\underline{x}}(h)\,, \quad E_{\underline{x},j}(h)\,w(W_{\underline{x},j}) = w(W_{\underline{x},j})\,E_{\underline{x},j}(h) \tag{19}$$

the operator of multiplication by $w(W_{\underline{x},j})$ creates an H-electric charge equal to $\overline{\chi}$ at the point $\underline{x}$.

Consider now a spatial line $\underline{L}$ starting at $\underline{x}$ and ending at $\underline{x}'$, and let us denote by $u(\underline{L})$ the product of $u(W_{\underline{x},j})$'s along $\underline{L}$. By acting with $u(\underline{L})$ on the vacuum we obtain a state with an H-electric flux along $\underline{L}$ and a pair of external H-charges (H-sources) at $\underline{x}$ and $\underline{x}'$. Following the discussion of the VOOP [1], we want to modify this state in such a way that its energy remains bounded as $\underline{x}' \to \infty$. In other cases [1,2], this was achieved by acting on it with the Euclidean evolution operator. However, as mentioned before, the transfer matrix (10-12) gives infinite energy to all states with nontrivial external H-charge. Actually this is not a surprise. When extending the model, we did not give any dynamics to the new degrees of freedom $\sigma_x := (V_x^{-1})^{\|}$, which were the only carriers of the H-charge (we use here the coset decomposition (8) for V_x^{-1}). Let us introduce a dynamics for the σ's by adding an extra term to the action

$$S \mapsto \tilde{S} = S - \tilde{\kappa} \sum_{x,\mu} \mathrm{Re}\, u(W_{x,\mu}) \tag{20}$$

which couples the new degrees of freedom in a G-gauge invariant way. For $\tilde{\kappa} \neq 0$, H is only a global symmetry. In the limit $\tilde{\kappa} \to 0$, H becomes local and the σ's become infinitely heavy. By subtracting the self-energy of the σ-particle we shall obtain a new transfer matrix that is finite on the H-sources. This will correspond to a new Boltzmann measure with only a time independent local H-gauge symmetry, which is a situation similar to the temporal gauge in the standard framework.

The transfer matrix corresponding to (20) is

$$\tilde{T}_{\tilde{\kappa}} = e^{-\frac{1}{2}\tilde{A}}\, \tilde{t}_{\tilde{\kappa}}\, e^{-\frac{1}{2}\tilde{A}} \tag{21}$$

where $\tilde{A}$ is the spatial part of the action $\tilde{S}$ and

$$\tilde{t}_{\tilde{\kappa}}(V,U;V',U') = t(V,U;V',U')\ \exp\left\{ \tilde{\kappa} \sum_{\underline{x}} \mathrm{Re}\, u(V_{\underline{x}} V_{\underline{x}}'^{-1}) \right\} . \tag{22}$$

$\tilde{T}_{\tilde{\kappa}}$ is positive if u is a function of positive type, i.e. for all functions f on $SU(2)$

$$\int dV \int dV'\, \overline{f(V)}\, u(VV'^{-1})\, f(V') \geq 0\,. \tag{23}$$

This condition is fulfilled for all $u = \sum_j c_j D^j_{mm}$ with positive coefficients c_j, where the integer or half-integer m specifies the character χ of (16).

For finite $\tilde{\kappa}$, $\tilde{t}_{\tilde{\kappa}}$ is already locally H-gauge invariant, so we may decompose it into a sum of positive operators

$$\tilde{t}_{\tilde{\kappa}} = \sum_n P_n \tilde{t}_{\tilde{\kappa}} P_n \tag{24}$$

where P_n projects onto the subspace of functions with external H-charge $m\,n(\underline{x})$ at the point $\underline{x}$ ($n(\underline{x})$ is an integer, n is a multiindex). Then, denoting by I_k the k^{th} modified Bessel function, and by $\omega_{\underline{x}}$ the expression $|u(V_{\underline{x}}V'^{-1}_{\underline{x}})|$,

$$(P_n \tilde{t}_{\tilde{\kappa}} P_n)(V,U;V',U') = t(V,U;V',U') \prod_{\underline{x}} I_{n(\underline{x})}(\tilde{\kappa}\,|\omega_{\underline{x}}|) \left(\frac{\omega_{\underline{x}}}{|\omega_{\underline{x}}|}\right)^{n(\underline{x})} . \tag{25}$$

We now use the fact that $I_k(\lambda)\,\lambda^{-|k|} \rightarrow (|k|!)^{-1}$ for $\lambda \searrow 0$ and define a renormalized operator $\tilde{t}_{ren}$ by

$$\tilde{t}_{ren,\tilde{\kappa}} = \sum_n \left(\prod_{\underline{x}} \frac{|n(\underline{x})|!}{\tilde{\kappa}^{|n(\underline{x})|}}\right) P_n \tilde{t}_{\tilde{\kappa}} P_n . \tag{26}$$

In the limit $\tilde{\kappa} \rightarrow 0$ we obtain

$$\tilde{t}_{ren} = \lim_{\tilde{\kappa}\rightarrow 0} \tilde{t}_{ren,\tilde{\kappa}} \tag{27}$$

with the kernel

$$\tilde{t}_{ren}(V,U;V',U') = t(V,U;V',U') \prod_{\underline{x}} \left\{1 + 2\sum_{n=1}^{\infty} \mathrm{Re}\left[u(V_{\underline{x}}V'^{-1}_{\underline{x}})\right]^n\right\} . \tag{28}$$

Clearly $A = \lim_{\tilde{\kappa}\rightarrow 0} \tilde{A}$. The new transfer matrix

$$\tilde{T} = e^{-\frac{1}{2}A}\,\tilde{t}_{ren}\,e^{-\frac{1}{2}A} \tag{29}$$

coincides with the old one on the H-gauge invariant subspace $\mathcal{H}$ of the Hilbert space $\tilde{\mathcal{H}}$ of our extended model, that is on functions depending only on U and φ. $\mathcal{H}$ is precisely the Hilbert space of the original Georgi-Glashow model.

We are now ready to define our candidate for a $U(1)$-charged vector boson (W-state). Let us denote the vacuum by Ω. For simplicity let us assume that $\underline{L}$ is a straight line. Following [1] we first consider an *energy regularized* and *normalized* state $\tilde{\Psi}^{(n)}_{\underline{x}\underline{x}'}$ that contains a pair of $U(1)$ sources at $\underline{x}$ and $\underline{x}'$,

$$\tilde{\Psi}^{(n)}_{\underline{x}\underline{x}'} = \|\tilde{T}^n\,u(\underline{L})\,\Omega\|^{-1}\,\tilde{T}^n\,u(\underline{L})\,\Omega \tag{30}$$

The gauge invariant candidate for a dipole of charged W's is the state

$$\Phi^{(n)}_{\underline{x}\underline{x}',jj'} = \overline{w(W_{\underline{x},j})}\,w(W_{\underline{x}',j'})\,\tilde{\Psi}^{(n)}_{\underline{x}\underline{x}'} . \tag{31}$$

Using the methods of [1] we can prove that the energy of $\Phi^{(n)}_{\underline{x}\underline{x}',jj'}$ stays bounded as $\underline{x}' \rightarrow \infty$, provided that the energy regulating parameter n grows *at least linearly* with the distance $|\underline{x} - \underline{x}'|$. Essentially the proof only relies on the positivity of $\tilde{T}$ and on the perimeter law for the u-Wilson loop. The latter follows from reflection positivity with respect to oblique lattice planes [8] in the *original* model.

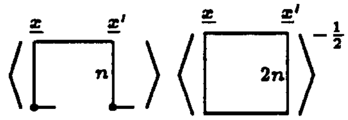
In the Higgs phase we expect that in the limit of infinite separation $\Phi^{(n)}_{\underline{x}\,\underline{x}',jj'}$ approaches, in the sense of expectation values, a W-state with $U(1)$-charge 1. On the other hand, in the confinement phase we expect to obtain in the same limit a local excitation of the vacuum. Hence the modified VOOP

$$\lim_{\underline{x}'\to 0}\ \lim_{n\to 0}\ (\Omega, \Phi^{(n)}_{\underline{x}\,\underline{x}',jj'}) \tag{32}$$

tests the existence versus nonexistence of the charged W by being zero or nonzero respectively [1].

In terms of Euclidean expectation values we obtain the following expression for this order parameter:

$$(\Omega, \Phi^{(n)}_{\underline{x}\,\underline{x}',jj'}) = \tag{33}$$



where the strings stand for products of $u(W_{x,\mu})$'s (the vertical direction is time, the horizontal direction is space), and the $w(W_{\underline{x},j})$'s at time zero are represented by a dot (at $\underline{x}$ or $\underline{x}'$) and a short line (in direction j or j').

The state $\Psi^{(n)}_{\underline{x}\,\underline{x}'}$ defined by

$$\Psi^{(n)}_{\underline{x}\,\underline{x}'} = \sigma_{\underline{x}}\, \overline{\sigma_{\underline{x}'}}\, \tilde{\Psi}^{(n)}_{\underline{x}\,\underline{x}'} \tag{34}$$

is in $\mathcal{H}$ (but it is not G-gauge invariant). All operators in the field algebra of the original problem (the algebra generated by the U's, the φ's, and their respective canonical conjugates) have identical expectation values in the states $\Psi^{(n)}_{\underline{x}\,\underline{x}'}$ and $\tilde{\Psi}^{(n)}_{\underline{x}\,\underline{x}'}$. In particular this is true for all observables (G- and H-gauge invariant operators). We can now see that it would have been possible to construct the physical W-state without ever enlarging the problem. We could have used some other procedure to construct a state with similar properties to $\Psi^{(n)}_{\underline{x}\,\underline{x}'}$ (in the Higgs phase, after $\underline{x}'\to\infty$ as described above, this state contains a nontrivial $U(1)$-electric flux at infinity); then we would have replaced the sources with dynamical matter fields by using $\sigma_{\underline{x}}\, w(W_{\underline{x},j})$ instead of the $w(W_{\underline{x},j})$ in a formula analogous to (31). The extended formalism just provides us with a specific (and simple) method to construct $\Psi^{(n)}_{\underline{x}\,\underline{x}'}$ and to prove the finiteness of the energy of $\Phi^{(n)}_{\underline{x}\,\underline{x}',jj'}$.

The $U(1)$ sources and therefore the W-state too can be introduced in another way, using gauge invariant charged fields which explicitely create the W particle accompanied by its Coulombic electric field. Using the ideas of [3] one has to find a *physical gauge*, which in our case would be the unitary gauge and for the remaining $U(1)$ gauge symmetry the Coulomb gauge, and then study the W-two-point-function in this gauge. If

$$\left\langle \overline{w(W_{\underline{x},j})}\, w(W_{\underline{x}',j'}) \right\rangle \to 0 \tag{35}$$

as $\underline{x}' \to \infty$, there are no translationally invariant states in the sector of the W, which is therefore charged. This is the analogue of the vanishing of the generalized VOOP. If however the r.h.s. of (35) approaches a nonzero value as $\underline{x}' \to \infty$, then this indicates screening of the electric charge. For a tetrahedron-discretized $SU(2)$, this approach has been pursued in [9]. Actually in a confining phase one has to be a little more careful [3]. It is possible that the spacelike physical gauge W- two-point-function behaves as in (35), but that the charged state whose existence this behaviour indicates is unphysical because it has infinite energy. Such a conclusion would be drawn from the fact that as soon as the separation between the two points becomes nonzero in Euclidean time too, the two-point-function is *identically* zero, for any spacelike separation [3].

In the Higgs phase of the Georgi-Glashow model we expect to be able to compute (33) and (35) perturbatively and thus confirm the existence of the $U(1)$-charged W. In the confinement phase however, where we do not expect to have charged W's, perturbation theory can be only used at short distances. In order to compute the order parameters and two-point-functions in this case, other techniques like small-β and hopping parameter expansions can be used [10]. Computer simulations may also turn out to be useful.

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