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ABSTRACT

A formalism for analyzing spin correlations in multibody decays of Λ_c is presented. It is a generalization of the α,β,γ formalism used for two-body hyperon decays. Application to semileptonic Λ_c decays, and to a few nonleptonic decays are also discussed.

1. INTRODUCTION

The recent successes in photoproduction and hadroproduction of sizeable samples of charmed hadrons herald a future in which the size of data sets will increase still further by orders of magnitude. Of special interest will be detailed properties of charmed baryons, states difficult to study in e^+e^- collisions because of a lack of statistics.

It is quite thinkable that these charmed baryons may be produced polarized. The process $pp \rightarrow \Lambda_c + X$ is quite similar to $pp \rightarrow \Lambda + X$. The observed polarization of Λ is believed to be due to polarization of the strange quark in its production.[1] This mechanism (albeit poorly understood) may well generalize from strange-quarks to charm-quarks.

Given that the possibility does exist, it is important to make the search. It seems to be the case that the theoretical phenomenology needed to back up such searches has not received too much attention.[2] The purpose of this note is to provide a formalism for doing this. It is meant to generalize the "α,β,γ" phenomenology used for nonleptonic 2-body decays of hyperons.[3] We were motivated mainly by 3-body decays of Λ_c such as $\Lambda_c \rightarrow pK^-\pi^+$ or $\Lambda_c \rightarrow \Lambda\pi^+\pi^0$, but it turns out that the formalism is flexible enough to handle multibody decays such as $\Lambda_c \rightarrow \Lambda\pi^+\pi^+\pi^-$ or semileptonic decays, e.g. $\Lambda_c \rightarrow \Lambda\mu^+\nu_\mu$, as well as reducing directly for 2-body decays to the extant α,β,γ formalism.

In the next section we lay out the formalism. We then apply it in Section 3 to semileptonic Λ_c decay. Various nonleptonic channels are discussed in Section 4. Section 5 contains concluding remarks.

2. BASIC FORMULAE

We consider the decay of Λ_c to a final baryon (which without loss of generality we call Λ) and spinless mesons. The matrix element for decay can be written

$$T = \bar{u}(p,s)\ \mathbb{M}\ u\ (P,S) \tag{2.1}$$

where p,s are the momentum and spin of the Λ and P,S are the corresponding quantities for the Λ_c. The amplitude $\mathbb{M}$ depends on γ-matrices and on the momenta of the remaining decay particles. Since the squared matrix element is

$$\begin{aligned} |T|^2 &= \mathrm{Tr}\left(\frac{1+\gamma_5\not{s}}{2}\right)\left(\frac{\not{p}+m}{2m}\right)\mathbb{M}\left(\frac{\not{P}+M}{2M}\right)\left(\frac{1+\gamma_5\not{S}}{2}\right)\bar{\mathbb{M}} \\ &\equiv |T|^2\ [1 - S_\mu\tilde{A}^\mu - s_\mu\tilde{a}^\mu + S_\mu s_\nu\ \tilde{B}^{\mu\nu}] \end{aligned} \tag{2.2}$$

the complexity of the spin angular distribution is limited to terms linear in s^μ and S^μ. The quantities $\tilde{A}_\mu$, $\tilde{a}_\mu$, and $\tilde{B}_{\mu\nu}$ are functions of the four-momenta of the parent Λ_c and its decay products.

It will be convenient to hold the configuration of decay momenta fixed in space while looking at the angular distribution of the spin degrees of freedom s^μ and S^μ relative to that configuration. This implies a choice of coordinate frame fixed with respect to the decay configuration. By convention we choose, in the rest frame of the Λ_c, the z axis along the Λ momentum vector. Without any real loss of generality, we also specialize in what follows to 3-body final states. The y axis is then chosen normal to the decay plane.

To understand more about the properties of the spin asymmetries, we proceed in 4 stages of increasing complexity:

1) <u>Λ_c polarized; Λ polarization not observed</u>:

This means that we average s_μ-dependent terms to zero. It then makes sense to go to the Λ_c rest frame, in which case we can write for the decay-width

$$d\Gamma = \Gamma_o \; (T_1,T_2) dT_1 dT_2 \; \frac{d\Omega_S}{4\pi} \;\; [\; 1 + \vec{S} \cdot \vec{\lambda} \; (T_1,T_2) \;] \qquad (2.3)$$

T_1 and T_2 are Dalitz-variables, the kinetic energies of two of the final particles in the Λ_c rest frame. The magnitude and orientation of the vector $\vec{\lambda}$ can be dependent on the Dalitz variables. $d\Omega_S$ is the solid angle element of the unit spin vector $\vec{S}$ of the Λ_c in its rest frame.

2) <u>Λ_c unpolarized, Λ polarization observed</u>

The formula is essentially the same, although one should consider the spin orientation $\vec{s}$ in the rest-frame of the Λ, not the Λ_c:

$$d\Gamma = \Gamma_o(T_1,T_2)dT_1dT_2 \frac{d\Omega_s}{4\pi} \left[1 + \vec{s} \bullet \vec{a}\ (T_1,T_2) \right] \qquad (2.4)$$

$\vec{a}$ and $\vec{A}$ are not the same vector. However we shall show in the next section that their magnitudes are in fact equal.

3) <u>Both Λ_c and Λ polarized; Λ extreme nonrelativistic in the Λ_c rest frame</u>

For this case we restrict our attention to the upper components of the Dirac spinors; what needs to be evaluated is

$$\mathrm{Tr} \left(\frac{1+\vec{\sigma}\bullet\vec{s}}{2} \right) \tilde{M} \left(\frac{1+\vec{\sigma}\bullet\vec{S}}{2} \right) \tilde{M}^{\dagger} \qquad (2.5)$$

with, in a convenient normalization,

$$\tilde{M} = 1 + \vec{\sigma} \bullet (\vec{R} + i\vec{I}) \qquad (2.6)$$

The (real) amplitudes $\vec{R}$ and $\vec{I}$ depend on location in the Dalitz plot. The differential width is

$$d\Gamma = \Gamma_o(T_1,T_2)dT_1dT_2 \frac{d\Omega_S}{4\pi} \frac{d\Omega_s}{4\pi} [1+\vec{S}\bullet\vec{A} + \vec{s}\bullet\vec{a} + \vec{B}\bullet\vec{s}x\vec{S} + s_iS_jC_{ij}] \quad (2.7)$$

Note that A_i, a_i, B_i and C_{ij} are not quite the same as the components of the previously defined quantities $\tilde{a}^{\mu},\tilde{A}^{\mu},\tilde{B}^{\mu\nu}$, but differ by a Lorentz boost. The quantities $\tilde{a}^{\mu},\tilde{A}^{\mu},\tilde{B}^{\mu\nu}$ will not enter into our phenomenology.

Straightforward evaluation of the trace yields

$$\vec{\Lambda} = \frac{2(\vec{R} - \vec{R} \times \vec{I})}{1 + R^2 + I^2}$$

$$\vec{a} = \frac{2(\vec{R} + \vec{R} \times \vec{I})}{1 + R^2 + I^2} \tag{2.8}$$

$$\vec{B} = \frac{2\vec{I}}{1 + R^2 + I^2}$$

$$C_{ij} = \frac{(1 - R^2 - I^2)\ \delta_{ij} + 2(R_i R_j + I_i I_j)}{1 + R^2 + I^2}$$

We now can see that $\vec{\Lambda}$ and $\vec{a}$ are equal in magnitude. The average orientation measures rather well the real part $\vec{R}$ of the spin-dependent amplitude; the difference in orientation is dependent upon $\vec{I}$, the imaginary part. However, $\vec{I}$ is probably better measured by the antisymmetric part of the double-spin correlation, proportional to $\vec{B}$.

Note that a small spin dependent amplitude $((R^2 + I^2) << 1)$ implies large spin transfer from Λ_c to Λ, i.e. $C_{ij} \approx \delta_{ij}$. This is itself an excellent signature.

4) <u>Both Λ_c and Λ polarized; relativistic kinematics</u>

The general case is essentially no more complicated than what has already been presented. We may write the decay amplitude, in the Λ_c rest frame, as

$$T \equiv \bar{u}(p,s)\ \mathbb{M}\ u(P,S) \tag{2.9}$$

where

$$u(P,S) = \begin{bmatrix} \chi_1(S) \\ \chi_2(S) \\ 0 \\ 0 \end{bmatrix} \tag{2.10}$$

and

$$u(p,s) = \left[\frac{\not{p}+m}{2m}\right] \begin{bmatrix} \chi_1(s) \\ \chi_2(s) \\ 0 \\ 0 \end{bmatrix} \tag{2.11}$$

where $(\not{p}+m)/2m$ is the Lorentz boost operator of the Λ rest-frame spinor to the Λ_c rest frame. Then

$$T = (\chi^\dagger(s)\ 0) \left[\left[\frac{\not{p}+m}{2m}\right] \mathbb{M}\right] \begin{bmatrix} \chi(S) \\ 0 \end{bmatrix} \tag{2.12}$$

and we can identify $\tilde{\mathbb{M}}$ as the upper left-hand 2x2 submatrix of $(\not{p} + m)\mathbb{M}/2m$, evaluated in the Λ_c rest frame:

$$\tilde{\mathbb{M}} = \left[\frac{1+\gamma_o}{2}\right] \left[\frac{\not{p}+m}{2m}\right] \mathbb{M} \left[\frac{1+\gamma_o}{2}\right] \tag{2.13}$$

The previous formula, Eqn. 2.7, for spin correlations stands unchanged; we must remember that $\vec{s}$ is the spin of the Λ as seen in the

Λ rest frame, while $\vec{S}$ is as seen in the Λ_c rest frame. All other variables are evaluated in the Λ_c rest frame.

We can apply these results to two-body decays such as $\Lambda_c \to \Lambda\pi^+$. In this case the formalism gracefully reduces to the standard α,β,γ phenomenology. Evidently $\vec{R}$ and $\vec{I}$ must be proportional to $\vec{p}$, the Λ momentum; the coefficient is the ratio of p-wave to s-wave decay amplitudes

$$\vec{R} + i\vec{I} = \hat{p}\left(\frac{P}{S}\right) \tag{2.14}$$

Choosing the z-axis along the Λ momentum vector, this implies, according to Eqn. (2.8),

$$\begin{aligned}
&A_x = A_y = a_x = a_y = 0 \\
&A_z = a_z = \frac{2\mathrm{Re}\ S^*P}{|S|^2 + |P|^2} = \alpha \\
&B_x = B_y = 0 \\
&B_z = \frac{2\ \mathrm{Im}\ S^*P}{|S|^2 + |P|^2} = \beta \\
&C_{xx} = C_{yy} = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2} = \gamma \\
&C_{xz} = C_{yz} = 0 \\
&C_{zz} = 1
\end{aligned} \tag{2.15}$$

3. BETA DECAY OF Λ_c

The formalism above actually applies to Λ_c semileptonic decays as well, since there is only one helicity state for the final leptons which contributes. We therefore can evaluate $\tilde{M}$ for that case as well. Furthermore, this exercise will lead to insight into the structure of nonleptonic decays to the extent that a "factorization" hypothesis applies to the dynamics of nonleptonic charm decays. We write for the decay amplitude, ignoring for the moment weak-magnetism corrections, etc:

$$T = \bar{u}(p,s) \quad [\, g_V \gamma_\mu + g_A \gamma_5 \gamma_\mu \,]\; u(P,S)\; J^\mu \tag{3.1}$$

where J^μ depends upon lepton momenta only. We now write this out explicitly in 2-component notation, and find

$$T = (\chi^\dagger(s)\;\; 0)\begin{pmatrix} 1+\gamma & -\gamma\vec{\sigma}\cdot\vec{\beta} \\ \gamma\vec{\sigma}\cdot\vec{\beta} & 1-\gamma \end{pmatrix}\left[\frac{g_V}{2}\begin{pmatrix} J_o & -\vec{\sigma}\cdot\vec{J} \\ \vec{\sigma}\cdot\vec{J} & -J_o \end{pmatrix} + \frac{g_A}{2}\begin{pmatrix} \vec{\sigma}\cdot\vec{J} & -J_o \\ J_o & -\vec{\sigma}\cdot\vec{J} \end{pmatrix}\right]\begin{pmatrix} \chi(S) \\ 0 \end{pmatrix} \tag{3.2}$$

Here $\vec{\beta} = \vec{P}_\Lambda / E_\Lambda$ and $\gamma = E_\Lambda / m_\Lambda$, as measured in the Λ_c rest frame. This allows, after multiplying the matrices, the identification with the spin-dependent quantities $\vec{R}$ and $\vec{I}$ previously introduced:

$$\vec{R} + i\vec{I} = \left[\, 1 - \frac{\gamma}{1+\gamma}\,\vec{\beta}\cdot\vec{V} \,\right]^{-1}\left[\left(\frac{g_A}{g_V}\right)\left(\vec{V} - \frac{\gamma\vec{\beta}}{1+\gamma}\right) - i\,\frac{\gamma}{1+\gamma}\left(\vec{\beta}\times\vec{V}\right)\right] \tag{3.3}$$

where $\vec{V} = \vec{J}/J_o$ depends on the kinematical variables of the leptons (or in general of the spectator system).

The quantity $\vec{V}$ in the case of semileptonic decay is easy to calculate. Neglect of lepton mass gives

$$\vec{V} = \frac{\bar{v}\,(q)\ \vec{\gamma}\ (1-\gamma_5)u(k)}{\bar{v}\,(q)\ \gamma_o(1-\gamma_5)u(k)} \tag{3.4}$$

Multiplying numerator and denominator by $u^\dagger k)v(q)$ allows $\vec{V}$ to be computed via a trace calculation:

$$\begin{aligned}\vec{V} &= \frac{\mathrm{Tr}\ \not{q}\ \vec{\gamma}\ (1-\gamma_5)\ \not{k}\ \gamma_o}{\mathrm{Tr}\ \not{q}\ \gamma_o(1-\gamma_5)\ \not{k}\ \gamma_o} \\ &= \frac{\hat{q} + \hat{k} + i\ (\hat{q} \times \hat{k})}{1 + \hat{q} \cdot \hat{k}}\end{aligned} \tag{3.5}$$

Here $\hat{k}$ and $\hat{q}$ are unit 3-vectors in the directions of neutrino and charged lepton momenta, respectively.

To get a feel for the physics, consider the extreme cases when the leptons are collinear:

(a) <u>Small dilepton mass: $\hat{q} \simeq \hat{k}$</u>

In this limit $\vec{V} \simeq \hat{q}$ is real. Notice that this lepton configuration has helicity zero and behaves as a spinless meson. The phenomenology is similar to $\Lambda_c \to \Lambda + \pi^+$, as evaluated in the "factorization" approximation. We shall review that case again in the next section. At a more general level, the phenomenology is similar to that for $\Lambda \to p\pi^-$. The quantity $\vec{R} + i\vec{I}$ (which evidently points in the direction $\hat{q}$) is analogous to the ratio of p-wave to s-wave Λ-decay amplitudes. In the present case we have

$$(\vec{R} + i\vec{I})_{\hat{q} \to \hat{k}} \to \hat{q} \left(\frac{g_A}{g_V}\right) = O(1) \tag{3.6}$$

This yields a large angular asymmetry in this limit. If measurable, this provides a good test measure of the V-A structure of the decay.

(b) Back-to-back leptons: $\hat{q} \approx - \hat{k}$

In this case the helicity of the leptons is unity and the quantity $\vec{V}$ has the (1,i,0) structure of a spin-1 circularly polarized quantum. Its magnitude however is large in the limit. Writing

$$\hat{q} + \hat{k} = \hat{n}\theta \tag{3.7}$$

with

$$\hat{n} \cdot \hat{q} = \hat{n} \cdot \hat{k} << 1$$

$$\hat{n}^2 = 1 \tag{3.8}$$

we have

$$\vec{V} = \frac{2}{\theta} \left[\hat{n} + i\, \hat{n} \times \hat{k} \right] \tag{3.9}$$

As $\theta \to 0$, the decay becomes collinear. In general the velocity $\vec{\beta}$ of the Λ does not vanish but it lies in the direction of the leptons. Hence $\vec{\beta}$ tends to be orthogonal to $\vec{V}$ in the limit and can be either along or against the direction $\hat{k}$. However $\vec{V}$ diverges. The net result is that $\vec{\beta} \cdot \vec{V}$ remains finite. However in the last term $\vec{\beta} \times \vec{V} = \pm i|\beta|\vec{V}$, so that

$$\vec{R} + i\vec{I} \xrightarrow[\hat{q} + \hat{k} \to 0]{} \frac{2}{\theta} \left[1 - \frac{\gamma}{1+\gamma} \vec{\beta}\cdot\vec{V} \right]^{-1} (\hat{n} + i\hat{n} \times \hat{k}) \left[\frac{g_A}{g_V} + \frac{\beta\gamma}{1+\gamma} \right] \tag{3.10}$$

What is important is that $\vec{R} + i\vec{I}$ in this limit blows up, and is proportional to the helicity-one vector $\hat{n} + i\hat{n} \times \hat{k}$. This implies that the angular-distribution parameters $\vec{A}$ and $\vec{a}$ are maximal and opposite in direction (due to the $\vec{R} \times \vec{I}$ term) as required by the necessity of spin flip to create the helicity of the lepton pair. Inspection of Σ_{ij} shows it is large as well, as expected.

4. NONLEPTONIC DECAYS

A. Two-Body Final States

The general formalism for these decays has already been presented at the end of Section 2. If the factorization ansatz is made for this decay amplitude, then one couples the weak transition current of Λ_c and Λ, as used in the semileptonic decay, to the pion's weak current:

$$J_\mu \propto F_\pi q^\mu \tag{4.1}$$

where q_μ is the 4-momentum of the pion. Because the pion will be extreme-relativistic, it follows that

$$\vec{V} = \frac{\vec{J}}{J_o} \cong - \hat{p} \tag{4.2}$$

From (3.3) we again obtain

$$\vec{R} + i\vec{I} = - \left(\frac{g_A}{g_V} \right) \hat{p} \tag{4.3}$$

If, as naively expected, $g_A/g_V \approx 1$, we would have $\alpha \approx -1$ and $\beta \approx 0$, $\gamma \approx 0$. This leads to both Λ_c and Λ spins preferentially oriented along the direction of the pion.

B. Factorization: $\Lambda_c \to \Lambda\pi^+\pi^o$ and $\Lambda_c \to \Lambda\pi^+\pi^+\pi^-$

Under the factorization hypothesis, we expect $\Lambda_c \to \Lambda + \rho^+$ and $\Lambda_c \to \Lambda + A_1^+$, to have large branching fractions. The coupling of virtual W to ρ is proportional

$$< 0 | j_\mu | \, p > \alpha \; (p_+ - p_-)_\mu \equiv \Sigma_\mu \tag{4.4}$$

This vector Σ_μ is pure spacelike in the rest frame of the ρ. When boosted to the Λ_c rest frame the new vector Σ'_μ is proportional to J_μ, as defined in Eqn. (3.1). Then it follows that

$$\vec{V} = \frac{\vec{\Sigma}'}{\Sigma'_o} \tag{4.5}$$

still has a magnitude large compared to unity in the rest-frame of the Λ_c, because the boost velocity is not very large. Furthermore $\vec{V}$ is real. We may again revert to the form of Eqn. (3.3), with $\vec{V}$ given by Eqn. (4.3). However, in this case the orientation of $\vec{V}$ and of $\vec{\beta}$ are not strongly correlated. The consequences for the observed asymmetries are not transparent, and are best determined by a simulation.

The case of $\Lambda_c \to \Lambda A_1 \to \Lambda\pi^+\pi^+\pi^-$ is similar. To the extent that A1 $\to \rho^o\pi^+$ via an s-wave decay, the polarization of the decay rho is strongly correlated to that of the A1. It may be again used in the above formalism for the $\Lambda_c \to \Lambda\rho^+$ decay mode.

C. The decay $\Lambda_c \to pK^-\pi^+$

This mode is an especially favorable mode to detect experimentally. From the theoretical standpoint it is a more difficult case; the factorization hypothesis appears less applicable. One should be guided by the interpretation of unpolarized data, e.g. whether there are strong resonance bands in the Dalitz plot.

Since the spin in the final state is unlikely to be observed, the issue boils down to whether $\vec{R}$ is likely to be large or small, and in which direction it is likely to point. Intuition might argue that, for configurations in which the K^- has a large final-state momentum, the kaon would follow the same pattern that a strange quark would under the same circumstances. This is the same as what a fast Λ would do in semileptonic decays. This is treated as case (a) in section 3, and is in turn the same as what happens in $\Lambda_c \to \Lambda + \pi$ (in factorization approximation). There we found that the Λ direction is strongly correlated with the parent-Λ_c spin, according to Eqns. (3.6) and (4.3).

5. CONCLUSIONS

Our main point has been to provide what is hopefully a reasonably simple and intelligible formalism for analyzing spin correlations in weak decays of Λ_c. The main result is given in Eqn. (2.7), where the general distribution is given in terms of three vectors $\vec{A}, \vec{a}$, $\vec{B}$ and a second rank symmetric tensor C_{ij}. These quantities depend upon the final-state momentum variables, e.g. the Dalitz variables in the case of a 3-body final state. For two-body decays their components, evaluated in a coordinate system fixed with respect to the final-state decay configuration, directly reduce to the α,β,γ parameters used in 2-body nonleptonic hyperon-decay phenomenology. For more than two particles in the final state, it is still appropriate to consider the quantities $\vec{A}, \vec{a}$, $\vec{B}$ and C_{ij} in a coordinate system fixed with respect to the final-state configuration. They depend upon the spin dependent decay amplitude $\vec{R} + i\vec{I}$ defined in Eqns. (2.9) to (2.13); the dependence is given explicitly in Eqn. (2.8).

Because the helicities of final state leptons are fixed, the formalism works for Λ_c semileptonic decays as well. The expressions for $\vec{R}$ and $\vec{I}$ in that case are given in Eqn. (3.3) in terms of yet another vector $\vec{V}$, the ratio of space components to time component of the leptonic weak current. This is computed and exhibited in Eqn. (3.5).

If nonleptonic Λ_c decays can be described in terms of the factorization hypothesis, the semileptonic formalism can be extended to them as well. We gave examples for the decays $\Lambda_c \to \Lambda\pi^+$, $\Lambda\rho^+$, and ΛA_1^+. However these are not solid predictions. Even less solid is our speculation that the correlation of fast K^- with Λ_c spin in $\Lambda_c \to pK^-\pi^+$ is similar to the correlation of fast Λ with Λ_c spin in $\Lambda_c \to \Lambda\mu^+\nu_\mu$ and/or $\Lambda_c \to \Lambda\pi^+$.

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