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On the Locality of the No Hair Conjecture and the Measure of the Universe

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Abstract

We reanalyze the recently proposed proof by Jensen and Stein-Schabes [1] of the No Hair Theorem for inhomogeneous spacetimes, putting a special emphasis on the asymptotic behaviour of the shear and curvature. We conclude that the theorem only holds locally and estimate the minimum size a region should be in order for it to inflate. We discuss in some detail the assumptions used in the theorem. In the last section we speculate about the possible measure of the set of spacetimes that would undergo inflation.

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I. Introduction

In a recent publication Jensen and Stein-Schabes [JSS] [1] gave a proof of a somewhat modified version of the cosmological no hair conjecture valid for a large class of inhomogeneous spacetimes with negative or zero three-curvature. This was an attempt to generalize earlier work by Wald [2] on homogeneous models. Some other studies on the isotropization of homogeneous models and on the stability of de Sitter can be found in [3,4]. Despite claims that Wald's proof is not applicable to all homogeneous models [5,6], it is widely believed that if there is a non-zero cosmological constant present all anisotropic models will undergo inflation [7]. For the case of inhomogeneous models the theorem's predictions have been explicitly checked in at least two cases. One is the solution found by Barrow and Stein-Schabes [8] for a quasispherical Szekeres-type model and by Stein-Schabes for the case of a spherically symmetric model [9] (see also [10] for some other examples). In all these cases the universe starts in a highly inhomogeneous phase and evolves through an inflationary stage into a de Sitter like universe (at least locally).

The importance of such a conjecture is evident if we want to avoid or at least soften somehow the difficult task of determining the initial conditions for the Universe (these might only come when we have a complete theory of Quantum Gravity, or maybe Superstrings ?).

The theorem proved in [1] has several assumptions and restrictions of applicability that have led people [11] to speculate that even though the result is applicable to a large class of models this class may not be large enough, i.e. it might still be of zero measure in the set of all possible models. In this paper we would like to argue to the contrary and take the opportunity to discuss in more detail some of the assumptions used. In JSS it was briefly argued that the theorem might only hold locally, in this paper we will show that this is indeed the case by carefully analyzing the different assumptions used. We shall give an estimate of how large must a region be for the theorem to be applicable. In particular we will derive in a rigorous manner all the asymptotic limits on the metric and the energy momentum tensor, and point out some of the highs and lows in the result. For the notation and conventions we refer the reader to JSS.

Furthermore, we will give a heuristic argument to support our belief that the set of models that undergo inflation is not of zero measure (see [12] for some more discussion

on this point). We shall also comment on several recent attempts at establishing this result and how our point of view differs from those in the literature [13].

II. The No Hair Theorem

We will start by giving the conjecture used in JSS and give the principal steps for proving it. We will then discuss the assumptions and results.

Theorem : Any *expanding* universe whose spatial sections are *not positively curved* (${}^3R \equiv P \leq 0$), with a metric that can be written in a *synchronous form*, with a *positive cosmological constant* and an energy-momentum tensor satisfying the *Strong energy condition* and the *Dominant energy condition*, will approach asymptotically the de Sitter solution.

(i) The dominant energy condition states that $T_{\mu\nu}t^\mu t^\nu \geq 0$ and $T_{\mu\nu}t^\nu$ is non-spacelike for all non-spacelike t^ν;

(ii) The strong energy condition states that $(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)t^\mu t^\nu \geq 0$ for all non-spacelike t^μ (see [14]).[1]

In a synchronous reference system $g_{oo} = 1, g_{oa} = 0$ and one can define $h_{ab} = -g_{ab}$ and $s_{ab} = \dot{h}_{ab}$, where the dot denotes the time derivative. Then with $h = \det h_{ab}$ the volume expansion can be defined as $K = \frac{1}{2}(\dot{h}/h) = \frac{1}{2}s^a_a = \frac{1}{2}h^{ab}s_{ab}$. (Note that $\sqrt{h}$ can be interpreted as the three-volume V and so $K = (\dot{V}/V)$.)

If P_{ab} denotes the Ricci-tensor constructed from h_{ab} and we define P as its trace, introduce a traceless tensor σ_{ab} as $2\sigma_{ab} = s_{ab} - \frac{1}{3}s^c_c h_{ab}$, then we can recast the trace of the space-space and the time-time Einstein equations as:

$$\dot{K} = \Lambda - \frac{1}{3}K^2 - \sigma_{ab}\sigma^{ab} - (T^o_o - \frac{1}{2}T) \tag{3a}$$

$$\dot{K} + K^2 + P = -T^a_a + \frac{3}{2}T + 3\Lambda \tag{3b}$$

Note that here $T^a_b = -h^{ac}T_{cb}$, but $P^a_b = h^{ac}P_{cb}$! Eliminating $\dot{K}$ from this equations one gets:

$$\Lambda - \frac{1}{3}K^2 = -\frac{1}{2}\sigma_{ab}\sigma^{ab} - T^o_o + \frac{P}{2} \tag{4}$$

[1]The signature of $g_{\mu\nu}$ will be $(+,-,-,-)$; Greek indices run from 0 to 3, Latin ones from 1 to 3 and $T = T_{\mu\nu}g^{\mu\nu}$.)

From (i) we get that $T_{oo} = T^o_o \geq 0$, $\sigma_{ab}\sigma^{ab}$ is non-negative and using the assumption that the three curvature is not positive i.e. the models are spatially open or flat models, we get the following inequality

$$K^2 \geq 3\Lambda \tag{5}$$

From the strong energy condition with $t^\mu = \delta^\mu_o$ it follows that $T^o_o - \frac{1}{2}T \geq 0$ and hence from eqs. (3) and (4):

$$\dot{K} \leq \Lambda - \frac{1}{3}K^2 \leq 0 \tag{6}$$

From these two inequalities we can conclude that if $K > 0$ for some arbitrary time, it will be so for all time, and will approach $\sqrt{3\Lambda}$. Quantitatively, after integrating the first inequality we get

$$K \leq \sqrt{3\Lambda}\ \coth\left(\sqrt{\frac{\Lambda}{3}}(t + \tilde{t}(x^a))\right) \tag{7}$$

and so for large times K is forced to take values in the range

$$\sqrt{3\Lambda} \leq K \leq \sqrt{3\Lambda}(1 + f(x^a)e^{-2\sqrt{\frac{\Lambda}{3}}t}) \tag{8}$$

where $f(x^a)$ is some positive function, related to $\tilde{t}(x^a)$ in eq.(7).

From eqs. (4) and (8) we get:

$$0 \leq \left\{ \begin{array}{c} \sigma_{ab}\sigma^{ab} \\ -P \\ 2T^o_o \end{array} \right\} \leq 4f(x^a)\Lambda e^{-2\sqrt{\frac{\Lambda}{3}}t} \tag{9}$$

In JSS it was then concluded that $\sigma_{ab} = 0$ and $K = \sqrt{3\Lambda}$ asymptotically. Using the definition of σ_{ab} we can get in this limit that the three metric satisfies

$$\dot{h}_{ab} - 2\sqrt{\frac{\Lambda}{3}}h_{ab} = 0 \tag{10}$$

which has the general solution

$$h_{ab} = e^{2\sqrt{\frac{\Lambda}{3}}t}\tilde{h}_{ab}(x^c) \tag{11}$$

$$h^{ab} = e^{-2\sqrt{\frac{\Lambda}{3}}t}\tilde{h}^{ab}(x^c)$$

However, we also know that $\sigma^{ab} = h^{ac}h^{bd}\sigma_{cd}$, so

$$\sigma_{ab}\sigma^{ab} = \sigma_{ab}\sigma_{cd}h^{ac}h^{bd} \propto \left\{ \begin{array}{ll} e^{-4\sqrt{\frac{\Lambda}{3}}t}(\sigma_{ab})^2 & \text{from (11)} \\ e^{-2\sqrt{\frac{\Lambda}{3}}t} & \text{from (9)} \end{array} \right.$$

From these two conditions we can only conclude that $\sigma_{ab} \leq exp(\sqrt{\frac{\Lambda}{3}}t)$. In fact, if we now use a result proven by Ellis [15] stating that $\sigma_{ab}\sigma^{ab} = 0 \Longleftrightarrow \sigma_{ab} = 0$ it is clear that we cannot have the equality. The reason for this technical point is the fact that the above result only applies to the case where the shear tensor or its magnitude vanishes exactly and in our case this is only an asymptotic limit. What we shall show is that the result also holds, at least in first approximation. The locality of the result will become obvious when we complete the argument. Since we are looking for the asymptotic behaviour, this immediately suggests some sort of expansion for σ_{ab} of the form

$$\sigma_{ab}(x^a, t) = \sum_{\substack{n=0 \\ n\neq 2}}^{\infty} \sigma_{ab}^{(n)}(x^a) e^{-n\sqrt{\frac{\Lambda}{3}}t}. \tag{12}$$

This reminds us of the technique used by Starobinsky [4]. If we do this, it is straightforward to conclude from the definition of σ_{ab} that the metric has to have a similar expansion,

$$h_{ab}(x^a, t) = \tilde{h}_{ab}(x^a) e^{2\sqrt{\frac{\Lambda}{3}}t} + \sum_{n=0}^{\infty} h_{ab}^{(n)}(x^a) e^{-n\sqrt{\frac{\Lambda}{3}}t} \tag{13}$$

i.e. the leading term in the metric is really given by eq.(11).

It is convenient to introduce an orthonormal tetrad at this point. We shall denote with bars quantities expressed in this frame. The locality comes into play at this point as the existence of such a frame can only be guaranteed at one spacetime point, we shall denote it by $\overline{x}_o^{\mu}$. In this frame the dominant energy condition becomes $\overline{T}_{oo} \geq \mid \overline{T}_{ab} \mid$ [14]. This implies, for example, that if $\overline{T}_{oo} = \overline{T}^{oo} \propto \exp\,(-2\sqrt{\frac{\Lambda}{3}}\,\overline{t})$ then $\overline{T}^{ab} \propto \exp\,(-2\sqrt{\frac{\Lambda}{3}}\,\overline{t})$. In this frame the metric is diagonal and becomes de Sitter when the proper coordinate system is chosen, i.e

$$\overline{g}_{\mu\nu}(\overline{t}, \overline{x}_o{}^a) = \begin{pmatrix} 1 & 0 \\ 0 & -e^{-2\sqrt{\frac{\Lambda}{3}}(t_o - \overline{t})}\delta_{ab} \end{pmatrix} \tag{14}$$

At this spacetime point the components of the stress tensor become

$$\left.\begin{array}{l} \overline{T}^{oo} \\ \overline{T}^{oa} \\ \overline{T}^{ab} \end{array}\right\} \propto e^{-2\sqrt{\frac{\Lambda}{3}}\overline{t}} \tag{15}$$

and if we now go back to the unbarred synchronous system we get that $T_{oo} \propto \exp\,(-2\sqrt{\frac{\Lambda}{3}}\,t)$, $T_{oa} \propto \exp\,(-\sqrt{\frac{\Lambda}{3}}t)$, while the faith of T_{ab} is unclear.

Looking back at the rest of Einstein's equations we can now study what happens to the curvature of the manifold. From these we get that the trace of the energy-momentum tensor has to vanishes at least as exp $(-2\sqrt{\frac{\Lambda}{3}}t)$. Then to zeroth order the curvature is given by

$$P_{ab} = T_{ab} + \frac{1}{2}Th_{ab} \tag{16}$$

Since there is no information about the vanishing of the right hand side we cannot conclude that the universe becomes globally flat. However, we can analyse this quantity in the barred reference frame at time t_o and get

$$\overline{P}_{ab} = \overline{T}_{ab} + \frac{1}{2}\delta_{ab}\overline{T} \propto e^{-2\sqrt{\frac{\Lambda}{3}}t_o} \tag{17}$$

This implies that to first order the first and second derivatives of the spatial metric vanish. Therefore, there must exist a neighbourhood of the space-time point $\overline{x}^{\mu}_{o}$, (where the diagonalization was made) that will remain de Sitter like. The size of this region can be, in principle, estimated from eq.(16). It is given by $\ell = exp(\sqrt{\frac{\Lambda}{3}}(t - t_o))\ell_o$, with $\ell_o = \sqrt{\overline{x}_o{}^a\overline{x}_o{}^a}$.

Now it is quite natural to ask whether t_o and so ℓ do have any physical meaning, the choice of t_o being apparently arbitrary. The answer to this question is twofold. Firstly, if Λ were a genuine cosmological constant, we do not need any interpretation. In this case the theorem just states in mathematical form our expectation that after a period long enough (see eqs.(8) and (9) for the timescale $\sqrt{3/\Lambda}$) the cosmological constant will dominate in every region. The physical reason is quite simple: every quantity except of Λ will die out due to the expansion.

The situation is quite different if we think in the spirit of inflation. In this case Λ lives only a finite time τ, say from t_i to t_f. Then t_o should be in the interval $[t_i, t_f]$. Moreover, $t_o - t_i$ should be large compared to $\sqrt{3/\Lambda}$, the timescale for the asymptotic time evolution. Now choosing a t_o which satisfies the above requirements, it is conceivable that our presently observable Universe is within this region and assuming a graceful exit of the inflationary phase, the Universe would enter the standard radiation dominated phase. We can also see that the high degree of symmetry of our cosmic neighbourhood does not require the accurate specification of initial conditions (see the following section), but rather the general requirements (i) and (ii) for the energy-momentum tensor and $P \leq 0$ in a synchronous reference system. This last condition will also be discussed in Sect.III. We note also that ℓ_o is not independent of the choice of t_o, see eqs.(16) and (17). Qualitatively, the larger is t_o, the larger is ℓ_o.

The physical quantity ℓ does not depend on the choice of t_o (restricted to the interval given above): it has for every allowed choice the same order of magnitude.

Some comments are in order. The theorem does not imply that the very large structure of the Universe is highly symmetric. In fact there are probably large regions which are "differently smooth enough" joined smoothly to form a patchwork universe (See Wald, ref. 2). This structure is in general governed by eq. (16).

It is quite clear that the reason such a result can be obtained is the fact that the metric has some sort of isotropic scaling, i.e. the space- and time-dependent parts decoupled, basically leaving a universe whose features are frozen in. In some sense one has compatibility between an isotropic scaling and the dominant energy condition.

III.Initial Conditions and The Measure of Space

The problem of initial conditions can be thought of as that of specifying initial values for all matter and geometrical fields at one arbitrary initial (Cauchy) hypersurface and then using the dynamical equations to follow the evolution of such a configuration. As we have just mentioned all the fields present can be put into two distinct classes. On one hand we have the gravitational field and on the other the matter fields (like the standard scalar field).

We would like to make a remark about the distinction between our line of questioning and that of several papers that have appeared on the same subject, in particular those of Gibbons *et al* and Belinski *et al* [13]. In these papers questions related to the generality and/or measure of the inflationary models have been addressed. However, their emphasis has been on the initial conditions for the matter fields given a fixed gravitational background. Then the general strategy goes as follows: Take a Robertson-Walker spacetime, a real scalar field ϕ with potential $V(\phi)$, and ask: What is the measure of the set of initial conditions on the matter fields $(\phi(t_o), \partial_0\phi(t_o))$ and on the scale factor $R(t_o)$ that would lead to inflation. These papers have argued that *almost all* the RW models with a massive scalar field undergo inflation. We in contrast, take the matter fields as given, i.e. an inflaton field (some scalar field with a flat potential and the right initial values) and investigate the possible spacetimes or better said the possible gravitational degrees of freedom that would allow for inflation. Things like Bianchi models or some particular inhomogeneous models.

Of course, the most general investigation should enclose initial conditions for the matter fields and the gravitational degrees of freedom as well. This type of questioning was discussed in a qualitative way by Linde [16].

It has been shown in the previous section and in [1,7-10,12] that given a scalar field ϕ with the right potential, inflation is very common.[2] Furthermore, we shall argue that the set of models that undergo inflation is not of zero measure. To calculate the measure is not easy, the reason being that from the mathematical point of view there is no way of choosing the right measure, while the physical principles to guide us in the choice are somewhat obscure. There are many possible measures that satisfy all technical requirements, like being continuous, monotonically increasing, etc [18]. The one that sounds more natural to use is the invariant area of phase space, in particular we will use a normalized measure. In terms of this we can ask how probable are the initial conditions that lead to inflation. We will argue that even though we cannot fully answer this question, first because its answer probably lies outside of the classical regime that we are considering and second because we don't fully understand how to choose a measure, we can at least construct one where the set in question has non-zero probability. We also argue that although it might be possible to construct a very contrived measure where the solution is zero, this does not sound very reasonable. It has been argued by Penrose and others that using the time translational invariance of Einstein's equations we could in principle select the present time as the initial hypersurface on which to define a measure. Doing so, our universe would clearly be part of a set of zero measure, the set formed by the RW metric and some neighbourhood around it, and then by extrapolation take this measure back into the early epoch to get the "zero solution". This point of view has been criticized most eloquently by Turner [19]. His argument basically states that since the universe had an *"initial epoch"* then there would seem to be a more natural *"initial hypersurface"* than the one chosen by Penrose. We would like to say, that although the choice of a proper measure still escapes us, reasonable arguments can and have been constructed to at least conclude that the measure of the set of models that inflate is not zero.

We will consider two different settings. Let us think of the set of all possible spacetimes. It is clear that if the models are highly inhomogeneous the concept of a

[2] One should keep in mind that the inflaton field is also evolving, so one should ask if the vacuum dominated phase can last long enough. It has been shown in [17] that this does not present any major problem.

scalar curvature being of a given sign on the initial hypersurface all through space might not be the correct one. This quantity will, in general, be a function of the spatial coordinates and time.

It is clear that if the models have a scalar curvature of a given sign, even if it changes from point to point, the NHT can be used and so predict its inflation. If we restrict ourselves for a minute on these models it is not unreasonable to assume that there are as many positively as negatively curved models, and a set of zero measure (with respect to this subclass) which are flat. Then in a trivial way we can construct a function say $\mathcal{F}$ that assigns to every model the value of its mean curvature $\overline{P}$. This could be defined as the integral of $P(x^a)$ over the invariant three-volume divided by the invariant three-volume (in the case of a RW model it is just k). This is a well defined function since P has a well defined sign. One way of picturing this manifold would be to construct a surface with the average curvature and superimpose little hills and valleys according to the curvature distribution. It is clear then that the question of the measure has been reduced to asking, for example: What is the probability of choosing at random a real positive (negative or zero) number ? The most sensible and intuitive answer (but by no means the only one) is that there is a probability $p = \frac{1}{2}$ of this being positive or negative and zero of being zero. In this case, we have shown that there is a hypersurface where the measure of the set that has $\overline{P}$ of the right sign is not zero. Then, since we expect Einstein's equation to map nonzero measure sets into nonzero measure sets we can justify our earlier claim.

However, it is conceivable that this types of manifolds form themselves a set of zero measure in the space of all possible models. This constitutes our second scenario. If we now think of an arbitrary space where the sign of the curvature scalar is varying from point to point, then we can play a similar game as before. Let us subdivide this manifold into regions of order ℓ_o (as defined in the previous section), and then define a mean curvature of that small region in the same way as before. (This can be done since there is always a small neighbourhood of the diagonalizing point and we are choosing a nonsingular initial hypersurface). Then the conclusion is the same as before: unless the space is pathological (non-continuous, non-differentiable, not a simple topology, etc), the small region of space that satisfies the NHT is as probable as one that does not.

However, this does not necessarily mean that this region will inflate. The problem is that in order to make use of the NHT, we need the conditions - in particular the nonpositivity of P - to be fulfilled for all time of interest, not only at the initial

moment t_o. This may not be the case for all possible models. The sign of P can change from time to time in a given region, depending on the motion of the matter. Mathematically, we could write down the time evolution equation for P using eqs.(3) and (4) together with the conservation of the energy-momentum tensor $T^{\mu\nu}_{;\nu} = 0$ (see also [5]). But without additional input on the matter content it is little that we can say about the time evolution of P, or the average curvature $\overline{P}$. In principle every kind of behaviour is possible: if $\overline{P}$ is negative in a region at t_0, it can remain negative, can change the sign or even can oscillate between negative and positive values. No definitive prediction seems to be available. The only possibility is to have ℓ_o to be large enough in order to avoid the effects of the outer regions to become important (changing the sign of $\overline{P}$). A crude estimate can be $\ell_o > \tau$, which ensures that during the dominance of the vacuum energy (the effective cosmological constant) the properties of the region we are looking at do not change qualitatively. In this general case we cannot give any definitive claim about the measure of the inflating models. Qualitative claims could be made if we knew some details of the model, e.g. the lifetime of the effective cosmological constant.

We would like to express our belief again that in spite of all the above said the probability that a model with arbitrary initial conditions for the gravitational degrees of freedom inflates is not negligibly small but a reasonable number, say $\mathcal{O}(10^{-1})$.

We are aware of the subjective and almost esotherical nature of our arguments, unfortunately a full solution to this problem can only come from a Quantum Theory of Gravity. Nevertheless, it is interesting to note that, the classical description, when sensible processes are included, does not preclude inflation, but on the contrary seems to favor it.

Finally we note that the same conclusion, namely that we cannot reach any definitive answer whether inflation occurs or not in a general model, have recently been achieved by Raychaudhuri and Modak [5], although they have used a somewhat different approach to the question. We agree with them in saying that "a discussion of the probability of inflation starting from very general initial conditions ... cannot give any unequivocal conclusion - perhaps that is obvious." We can add to this statement only that many of the obvious results are far from obvious before stated explicitly.

IV. Conclusion

We have investigated in detailed some of the assumptions used by JSS [1] in proving the NHT. In particular we have done a more thorough asymptotic expansion of the shear tensor and concluded that indeed the metric approaches the de Sitter solution. However, this only happens locally. That is, what we have established is that the result can be applied to a small neighborhood of an inhomogeneous universe of a certain small size that can undergo inflation and eventually give rise to our Universe.

We have also discussed briefly the generality of inflation as a probabilistic process and argued that we can construct at least one measure where the probability of having the right initial conditions for the universe to undergo inflation is not zero or a very small number.

Of course, we do not claim this to be a solution of the problem of initial conditions as its applicability is restricted to classical universes described by Einstein's Theory, and we have made some questionable assumptions about the content of the Universe. However we feel it is encouraging the fact that classically, inflation is not the most improbable phenomenon in the early universe. We have also briefly discussed the difference in our approach where the basic degrees of freedom we are investigating are the gravitational ones while the matter content is arbitrarily fixed, and the work cited in [13] where the approach is the complement to ours. There the geometry of the spacetime is given and the initial conditions on the matter content are investigated. Clearly, we would need to study both simultaneously to give a more definite answer to this question [16].

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