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The Lattice $\lambda\phi^4$ Model With Yukawa Couplings to Staggered Fermions*

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Abstract

An investigation of the $\lambda\phi^4$ theory coupled via Yukawa couplings to fermions has been initiated on the lattice. Several algorithms for dynamical fermions have been tested on this model including hybrid molecular dynamic and hybrid Monte Carlo algorithms. Some preliminary results for renormalized Yukawa couplings are presented.

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In this talk, I would like to describe investigations that have been initiated on the lattice to study the single component $\lambda\phi^4$ theory with Yukawa couplings to fermions. The motivation for undertaking these studies is that one has here a simple model in which to examine, nonperturbatively, a coupling that plays an important role in the electroweak theory. According to the Standard Model, Yukawa couplings are responsible for all lepton masses and for current quark masses. The conceptual issues that we ultimately hope to resolve, include:

- What happens to the renormalized coupling y_R as the cutoff $\to \infty$?
- Can one place bounds on y_R and hence also on fermion masses?
- What is the feedback of fermions onto the scalar sector? How do fermions affect recent lattice bounds on Higgs masses?[1]

In order to be able to address these questions, several technical challenges must be met. To list just a few, one must gain control over:

1. dynamical fermions,
2. ways to extract renormalized couplings from finite lattice simulations, and
3. how to handle resonance and decay phenomena on finite lattices.

I would like to present a progress report on how these challenges have been met to date. In brief, we believe challenges 1 and 2 are under control. We do not have much to say about challenge 3.[2] Whether this lack of understanding will cause severe problems for our analysis of the model remains to be seen.

The action under investigation is,

$$S = S_B + S_F + S_Y \tag{1}$$

where,

$$S_B = \sum_n \left\{ 4\phi^2(n) - \sum_\mu \phi(n)\,\phi(n+\mu) \right\} + \frac{m^2}{2} \sum_n \phi^2(n) + \frac{\lambda}{4} \sum_n \phi^4(n) \tag{2}$$

$$S_F = \frac{1}{2} \sum_n \sum_\mu \sum_{f=1}^{2} \bar{\chi}_f(n)\eta_\mu(n)\left[\chi_f(n+\mu) - \chi_f(n-\mu)\right] \tag{3}$$

In (3), $\eta_\mu(n)$ are the usual staggered fermion phases and we work with two sets of staggered fermions in order to ensure a non–negative fermion determinant.

For the Yukawa term in the action, S_Y, we have experimented with two versions,

$$S_Y^{(1)} = y \sum_n \phi(n) \sum_f \bar{\chi}_f(n)\chi_f(n) \tag{4}$$

and

$$S_Y^{(2)} = y \sum_n \phi(n) \sum_f \left(\frac{1}{16} \sum_{hypercube} \bar{\chi}_f \chi_f \right)_n \tag{5}$$

We will call $S_Y^{(1)}$ the "local" and $S_Y^{(2)}$ the "hypercubic" coupling. Although we find that the lattice phase diagram depends sensitively on whether (4) or (5) is used, we believe any result which is relevant to continuum physics should follow from either version of the lattice action. Most of our investigations employ action $S_Y^{(2)}$. However, other groups are now investigating $S_Y^{(1)}$ as well, and it will be interesting to compare their results with ours.[3]

It is informative to consider action (1) in several limiting situations. For instance, in the $y \to \infty$ limit, one finds, after rescaling the χ–fields and carrying out the fermion integration exactly, the following bosonic effective action

$$S_{eff}^{(1)} = S_B - \sum_n \ell n \left(\phi(n)\right)^2 \tag{6}$$

or

$$S_{eff}^{(2)} = S_B - \sum_n \ell n \left(\Phi(n)\right)^2 \tag{7}$$

$$\left(\Phi \equiv \frac{1}{16} \sum_{hyperc.} \phi(n) \right)$$

Another interesting limit is:

$$y \to \infty, \;\; m^2 \to \infty \;\; \text{with} \;\; y/m = \;\text{fixed.}$$

Then, after rescaling $\phi \to \frac{1}{m}\phi$, one finds

$$S \to \frac{1}{2} \sum_n \phi^2(n) + \frac{y}{m} \sum_n \phi(n) \left(\frac{1}{16} \sum_{hyperc.} \bar{\chi}_f \chi_f \right) + S_F \tag{8}$$

Upon integrating out the ϕ–fields (8) reduces to a Nambu–Jona Lasinio four fermion action. This is a well–known model, which in the continuum version can exhibit both a chirally broken ($< \bar{\chi}\chi > \neq 0$) and a chirally symmetric phase.

During the past year, we have developed and tested code for the action (1) using several hybrid molecular dynamic algorithms.[4] We now have a rough phase diagram in the (m^2, y) plane. These results are summarized in reference [5], to which the reader is referred to for further details and some plots. Recently, we have also implemented a hybrid Monte Carlo algorithm[6] for this model and find this to be a fairly satisfactory algorithm. Using this algorithm we have carried out some feasibility studies of extracting renormalized Yukawa couplings y_R. So I would like to end my talk with a few comments on our y_R studies.

There are many ways to define a "renormalized" coupling. For instance

$$y_R^{(1)} \equiv \frac{aM_F}{< \phi_R >} = \frac{aM_F}{< \phi >}\sqrt{Z_\phi} \tag{9}$$

One can also define "off–shell Euclidean" couplings, similar to those discussed in ref. [7]

$$y_R^{(2)} \equiv \frac{-\Gamma^{(3)}(0)}{\Gamma_F^{(2)}(0)\sqrt{\Gamma_B^{(2)}(0)}}\,(aM_F)\,(aM_B) \tag{10}$$

where,

$$\Gamma^{(3)}_{(p)} = \frac{1}{L^3T}\sum_{x_1 x_2 y} e^{ip(x_1-x_2)} < \chi(x_1)\bar{\chi}(x_2)\phi(y) >_{conn.}$$

$$\Gamma_F^{(2)}(p) = \frac{1}{L^3T}\sum_{x_1 x_2} e^{ip(x_1-x_2)} < \chi(x_1)\bar{\chi}(x_2) >$$

$$\Gamma_B^{(2)}(0) = \frac{1}{L^3T}\sum_{y_1 y_2} < \phi(y_1)\phi(y_2) >_{conn.}$$

For antiperiodic boundary conditions in time for the fermionic degrees of freedom, one cannot set $P_0 = 0$ on a finite lattice. The minimal momentum is $P_{min} = \left(\frac{\pi}{T}, \vec{0}\right)$. For this case, one can define the renormalized coupling as,

$$y_R^{(3)} \equiv \frac{|\Gamma^{(3)}(P_{min})|}{|\Gamma_F^{(2)}(P_{min})|\sqrt{\Gamma_B^{(2)}(0)}}\left(a\tilde{M}_F\right)(aM_B) \tag{11}$$

with

$$a\tilde{M}_F = \sqrt{(aM_F)^2 + \sin^2\frac{\pi}{T}}$$

We have evaluated y_R on 8^4 lattices (which is too small, at least in the time direction) at two points in the phase diagram using the hybrid Monte Carlo algorithm. We find

$\underline{\lambda = 1, y = 1, m^2 = 0.5}$ ($< \phi >\sim .437$)

$$y_R^{(3)} = .85 \pm .10$$

$$y_R^{(2)} = .87 \pm .21$$

$$y_R^{(1)} = .87 \pm .15$$

$\underline{\lambda = 1, y = 5, m^2 = 20}$ ($< \phi >\sim .158$)

$$y_R^{(3)} = 1.71 \pm .31$$

$$y_R^{(2)} = 1.37 \pm .43$$

$$y_R^{(1)} = 2. \pm 1.$$

For $y_R^{(3)}$ and $y_R^{(1)}$, we carried out 200,000 molecular dynamic time steps and performed a Metropolis accept/reject step every fifth time step. For $y_R^{(2)}$, we had only half the statistics. The largest uncertainty comes from (aM_B) and in the case of definition $y_R^{(1)}$ from Z_ϕ. Clearly these calculations need to be repeated on larger lattices and then one should try moving closer to the critical point and determine the behavior of y_R as a function of $(aM_F)^{-1}$.

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