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DEVELOPMENT OF QCD JETS EMITTED BY COLOR-SINGLET SOURCES*

R.K. Ellis
Fermilab
Batavia, IL 60510

J.F. Gunion
University of California
Davis, CA 95616
and
University of Oregon
Eugene, OR 97403

Jan Kalinowski†
University of California
Davis, CA 95616

B.R. Webber
Cavendish Laboratory
University of Cambridge
Cambridge, ENGLAND

Abstract

We compare the angular-ordering approximation to QCD jet development with full calculations to order α_s in the following cases: emission of quark jets by a color-singlet vector source (as in e^+e^- annihilation) and emission of gluon jets by a color-singlet scalar ($F^a_{\mu\upsilon} F^{a\mu\upsilon}$) source. In contrast to the case of a color-octet (gluon) source, we find that the approximation is good in those regions of phase space where the next-to-leading corrections to the amplitude are large.

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† On leave of absence from the University of Warsaw

As a by-product of work on the extension of the QCD jet calculus to include higher-order corrections,[1,2] two of us[3] have recently investigated the validity of the angular-ordering approximation[4] for the non-leading corrections to the exclusive process gluon → 3 gluons. The conclusion was that the approximation is good in the strictly strongly-ordered momentum regions $x_i \ll x_j \ll x_k$ but breaks down rapidly outside those regions.† In particular, for $x_i \ll x_j \sim x_k$ the next-to-leading correction is large and the discrepancy between the angular-ordering approximation and the exact result is of order 100%. This is illustrated in Fig. 1, which is reproduced from Ref. 3. These results suggest that the angular-ordering approximation might be inadequate to describe the soft gluon component of a pair of hard gluon jets produced by splitting of a single virtual gluon.

The discrepancies are even larger when one investigates the process gluon → quark + antiquark + gluon. If x_3 represents the gluon momentum fraction, the angular-ordering approximation to the non-leading correction is (in the notation of Refs. 2,3)

$$W_{g\to gq\bar{q}}(x_1,x_2,x_3) =$$

$$C_F\, H(x_2+x_3)\, F\left(\frac{x_2}{x_2+x_3}\right) \cdot \frac{1}{x_2+x_3} \log\left(\frac{x_2x_3}{x_1x_2+x_2x_3+x_3x_1}\right) + (1 \leftrightarrow 2)$$

$$+\, 2\, C_A\, P(x_1+x_2)\, H\left(\frac{x_1}{x_1+x_2}\right) \cdot \frac{1}{x_1+x_2} \cdot \log\left(\frac{x_1x_2}{x_1x_2+x_2x_3+x_3x_1}\right), \qquad (1)$$

where H, F and P@ are the $g \to q\bar{q}$, $q \to qg$ and $g \to gg$ Altarelli-Parisi

† Here x_i represents the light-cone momentum fraction of final state jet i with respect to the initial jet.

@ $H(x) = x^2 + (1-x)^2$; $F(x) = \frac{1+x^2}{1-x}$; $P(x) = \frac{1}{x} + \frac{1}{1-x} - 2 + x(1-x)$.

splitting functions. A comparison with the exact expression (2.16) of Ref. 2 is shown in Fig. 2. We see again that the approximation is valid in the strongly-ordered x_i regions (the corners of phase space) but becomes very poor, even when the gluon is soft, outside these regions.

The large discrepancies illustrated in Figs. 1 and 2 may be traced to contributions in which a soft gluon is emitted from the incoming virtual gluon. (The associated emission probability is infrared singular in this region and consequently large.) Thus it is possible that the angular ordering prescription is a much better approximation for color-singlet jet sources from which such an emission cannot occur. Whether this implies that the angular-ordering prescription is good for physical processes (which are always color-singlet initiated) will be left to later discussion.

We shall examine two different color-singlet source jets. In the first case we consider two quark jets emitted by a vector source, $\gamma \to gq\bar{q}$. One simply has to set $C_A = 0$ in the expression for $g \to gq\bar{q}$ - Eq. (1) for the angular-ordering approximation and Eq. (2.16) of Ref. 2 for the exact next-to-leading result. As shown in Fig. 3, the angular-ordering approximation then becomes good for all soft gluon momentum values and for a considerable range of hard values as well. The only places where the relative discrepancy is large are those in which the quark or antiquark is very soft, and the amplitude is very small.

We have also calculated three-gluon jet production from a color singlet source. In this case the most convenient type of source is a

scalar coupling $g' F^a_{\mu\nu} F^{a\mu\nu}$, which leads to the diagrams shown in Fig. 4. The Feynman rules for the source are given in Ref. 5. We find that the exact non-leading correction at order α_s is, (omitting a factor of $C_A \frac{1}{4} \frac{(g')^2 (g_s)^2}{64\,\pi^4}$),

$$
\begin{aligned}
& V_{s\to ggg}(x_1,x_2,x_3) \\
& \quad = \Big[2\,P\Big(\frac{x_1}{x_1+x_2}\Big)\frac{1}{(x_1+x_2)}\,\ell n\,(x_1+x_2) \\
& \quad + \frac{2}{x_1}\,\ell n\,\frac{x_1}{(x_1+x_2)(x_3+x_1)} + \frac{2}{x_2}\,\ell n\,\frac{x_2}{(x_1+x_2)(x_2+x_3)} \\
& \quad + \frac{13}{6} + \frac{1}{4}(x_1+x_2) + \frac{3}{2}\frac{(x_1-x_2)^2}{(x_1+x_2)^3} - \frac{1}{2}\frac{(x_1-x_2)^2}{(x_1+x_2)^2} \\
& \quad - \frac{1}{4}\frac{(x_1-x_2)^2}{(x_1+x_2)} - \frac{1}{2}\frac{1}{(x_1+x_2)}\Big] + [\text{cyclic}]
\end{aligned}
\tag{2}
$$

which is to be compared with the angular-ordering approximation

$$
W_{s\to ggg}\,(x_1,x_2,x_3) \;=\; 2\,P\Big(\frac{x_1}{x_1+x_2}\Big)\cdot\frac{1}{x_1+x_2}\cdot\log\Big(\frac{x_1x_2}{x_1x_2+x_2x_3+x_3x_1}\Big) + \text{cyclic}\,. \tag{3}
$$

The $x_i^{-1}\,\ell n\,x_i$ and x_i^{-1} singularities of eqs. (2) and (3) agree, so that the angular-ordering approximation is valid when any gluon is soft, independent of the energies of the other two. As shown in Fig. 5, the approximation is good throughout a substantial region near the edges of phase space, although at the symmetric point $x_1 = x_2 = x_3 = 1/3$ (where the absolute correction is very small) the relative error is maximal at about 60%.

Figure 6 shows sections through the surfaces displayed in the earlier figures along the line $x_1 = x_2$, to emphasize the difference between the regions of validity of the angular-ordering approximation in the color-octet and color-singlet cases. Obviously the angular-ordering approximation has a much larger range of validity in the color-singlet source cases.

The physical implications of these results for Monte Carlo simulations of jet evolution are not entirely obvious. The much more satisfactory agreement of the angular-ordering approximation with the exact next-to-leading calculation for color-singlet sources might suggest that the angular-ordering approximation could be reliable for color-singlet initiated reactions in general. This could occur if the infrared singular emissions from a given highly energetic colored jet are cancelled by destructive interference with soft emissions from other jets which are color-correlated with the first due the over all color-singlet nature of the physical reaction. This is inevitably the case when the light cone momentum fraction x_i of the soft emission is so small that this secondary emitted gluon is traveling at a large angle to the jet axis or, in general, is in a region of phase space where it can interfere with a wide-angle emission from the other jets. However the region where such interference between emission from energetic colored jets obviously occurs is confined to $x_i < Q_0/Q$, where Q_0 represents a typical confinement scale or transverse momentum and Q represents the energy of the primary jet. In the region $x_i > Q_0/Q$ the jet calculus expansion is based on the idea that

the emission from a colored jet occurs independently of the presence of other color-correlated jets in the order of α_s being considered. Thus for large Q there is a region of phase space where soft emission (in the sense that $Q_0/Q \ll x_i \ll 1$) from an energetic colored jet does not obviously interfere with radiation from other color-correlated jets. Whether or not there is a hidden destructive interference mechanism that restores the approximate validity of angular-ordering in this important region of phase space requires additional investigation. The best further test of this possibility is to examine $e^+e^- \to q\bar{q}gg$ using the known matrix elements, Ref. (6), although the comparison might have to be performed numerically if the integrations over transverse degrees of freedom prove too laborious to be done analytically.

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FIGURE CAPTIONS

FIGURE 1 The ratio (W-V)/V as a function of x_1 and x_2 for the process $g \to ggg$. V is the exact non-leading order α_s^2 correction and W is the angular-ordering approximation to it. Figure from Ref. 2. The light cone momentum fractions of the three gluons are x_1, x_2 and $x_3 = 1 - x_1 - x_2 > 0$.

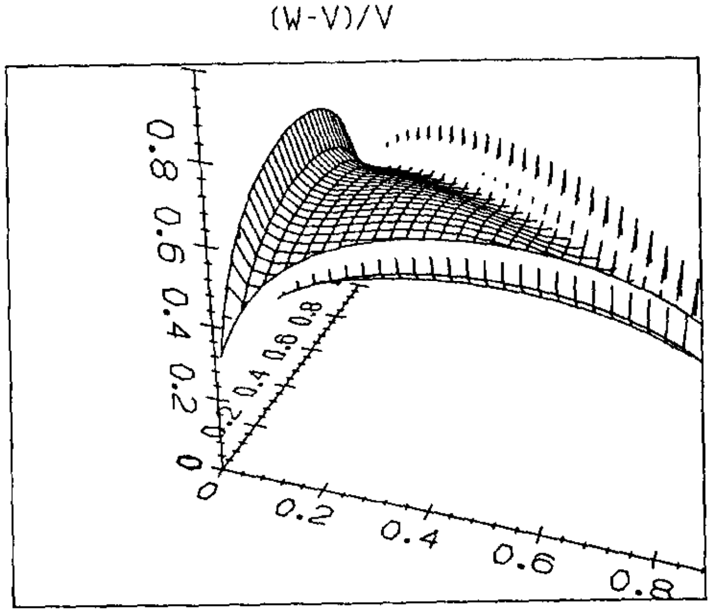
FIGURE 2 The ratio (W-V)/V as a function of x_1 and x_2 for the process $g \to gq\bar{q}$, with x_1 and x_2 representing the q and $\bar{q}$ light-cone momentum fractions.

FIGURE 3 The ratio (V-W)/V for the process $\gamma \to gq\bar{q}$, where γ represents a color-singlet vector source; x_1 and x_2 are the q and $\bar{q}$ light-cone momentum fractions respectively.

FIGURE 4 Feyman graphs for the process $s \to ggg$, where s is a color-singlet scalar source.

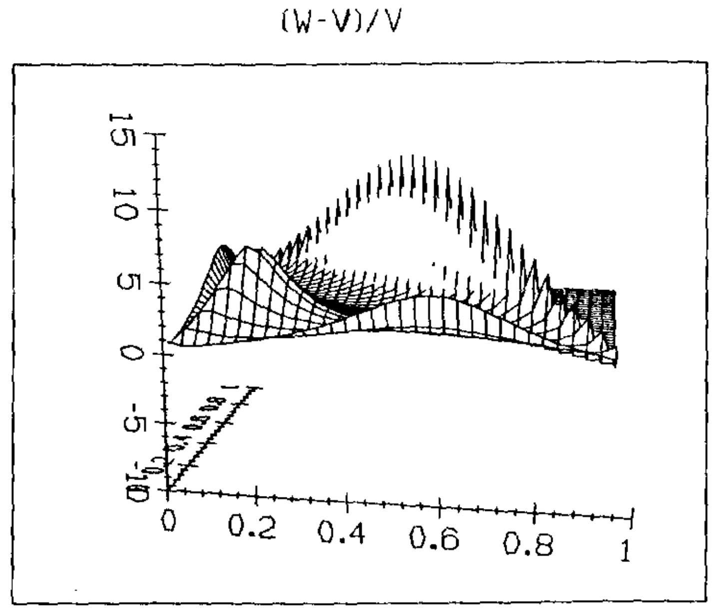
FIGURE 5 The ratio (W-V)/V as a function of x_1 and x_2 for the process $s \to ggg$.

FIGURE 6 Sections through Figs. 1, 2, 3, 5 along the line $x_1 = x_2$.



Ratio as a function of x_1 and x_2

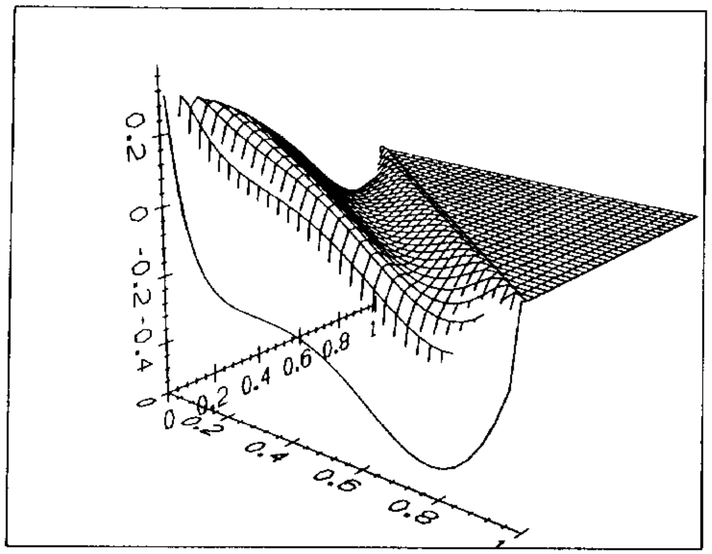
Figure 1



Ratio as a function of x1 and x2

Figure 2

(V-W)/V



Ratio as a function of x1 and x2

Figure 3

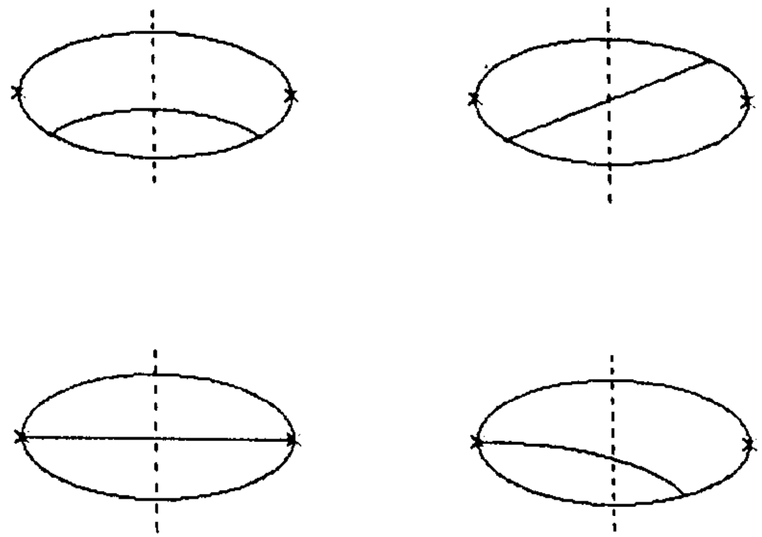
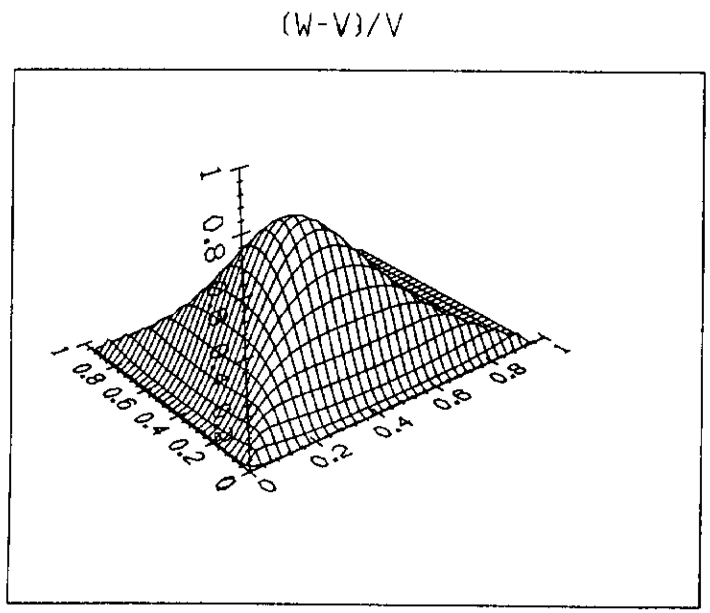


Figure 4



Ratio as a function of x1 and x2

Figure 5

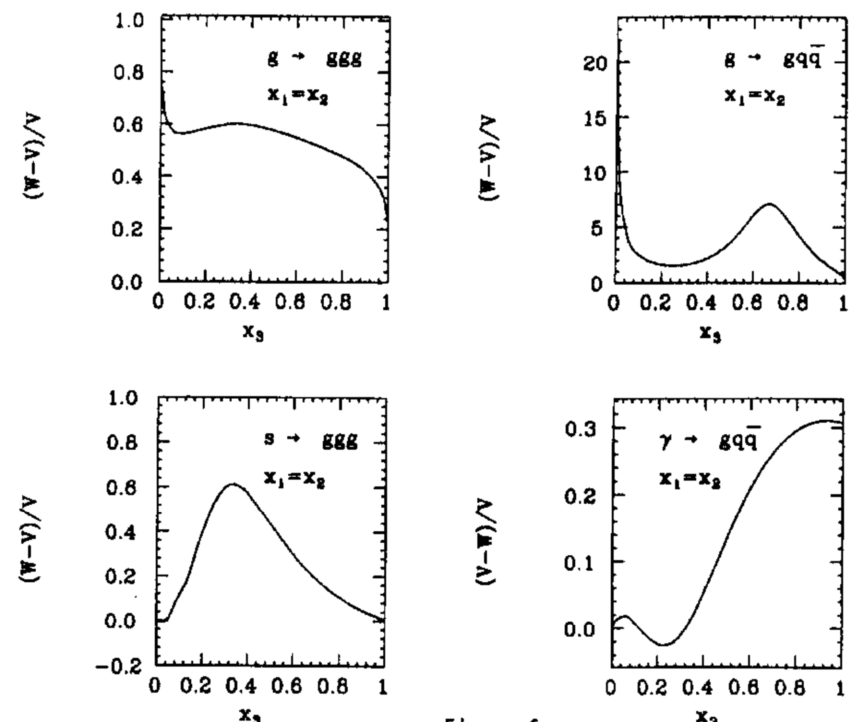


Figure 6