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Axion Cosmology in Automatic $E_6 \times U(1)$ Models

by

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Abstract

An automatic $E_6 \times U(1)$ axion model is constructed which is cosmologically consistent, predicts three families of fermions at low energies, and admits cosmic strings capable of seeding galaxy formation.

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The existence of axions [1] would have exciting and far-reaching consequences for both particle physics and cosmology. Thus, the construction of models that implement the Peccei-Quinn mechanism in a phenomenologically and cosmologically consistent way is of great interest. The models become even more satisfying if the Peccei-Quinn (PQ) symmetry is automatic. What this means is that, given the fermionic and bosonic content of the theory, the most general renormalizable and gauge invariant Lagrangian with this content automatically has a global, color anomalous, $U(1)_{PQ}$ symmetry.

We would like to focus on the gauge group E_6 in this Letter. Automatic E_6 models have been previously constructed [2], but were phenomenologically unacceptable due to the antisymmetry of the fermion mass matrix, which leads to mass degenerate families and to a massless family if the number of families is odd. Furthermore, the cosmological constraints on axion models had not been satisfied in this model.

We now show that if we consider the <u>gauge</u> group $E_6 \times U(1)_F$, a consistent automatic model can be constructed and furthermore cosmic strings are created when $U(1)_F$ breaks.

Let us first turn to the fermion representation; we take the standard generations of fermions to live in 27's. The first question we may ask is whether there are any constraints that will limit the number of 27's in this theory. Such constraints do, in fact, exist. Primordial nucleosynthesis seems to limit the number of light neutrinos to be at most four [3], while b-decay phenomenology [4] seems to require at least three 27's. However, axion models provide further constraints. Sikivie [5] has shown that axion models are afflicted with a domain wall problem which is cosmologically unacceptable. This is because $U(1)_{PQ}$

has a residual discrete subgroup (RDS), $Z(N)_{PQ}$, which is left unbroken by QCD instanton effects, but is broken spontaneously. This problem may be solved in a particularly elegant way by means of the Lazarides-Shafi mechanism (LS) [6]. Before we explain how this works, let us say a few words about why we do not inflate the walls away [7]. In non-supersymmetric inflationary models, one must add gauge singlet scalar fields, coupled in a conformally invariant way (i.e. with no dimensionful couplings in the Higgs potential). This is clearly <u>not</u> consistent with automaticity. Furthermore, the reheating temperature must be fine-tuned to be less than the PQ breaking scale, v_{PQ}.

The LS mechanism chooses fermion representations and PQ charges so that the RDS can be identified with part or all of the center of the gauge group. This allows the discretely separated vacuua to be connected by continuous transformations, thus destabilizing the domain walls.

For E_6, with N_f 27's of PQ charge 1, the RDS is $Z(N_f)_{PQ}$ [8]. Since E_6 has center Z_3, in order for the LS mechanism to work we must choose $N_f = 3$. Note that $Z(N)_{PQ}$ cannot be embedded in $U(1)_F$ since $U(1)_F$ must have no color anomaly, while $U(1)_{PQ}$ <u>must</u> be color anomalous.

Let us now specify the fermionic and scalar representations of our model:

$$R_F = \underline{27}_0 \oplus \underline{27}_1 \oplus \underline{27}_{-1} \tag{1a}$$

$$R_H = \widetilde{\underline{27}}_{-2} \oplus \widetilde{\underline{351}}'_{-1} \oplus \widetilde{\underline{351}}'_1 \oplus \widetilde{\underline{351}}_0 \tag{1b}$$

Here, the subscripts denote the $U(1)_F$ charges, while the tildes denote the Higgs scalars. The Higgs representation is the simplest one consistent with automaticity and a reasonable fermion mass matrix. The Yukawa couplings allowed by the $E_6 \times U(1)_F$ symmetry are

$$27_0\ 27_{-1}\ \widetilde{351}'_1,\ 27_1\ 27_{-1}\ \widetilde{351}_0,\ 27_1\ 27_1\ \widetilde{27}_{-2},\ 27_0\ 27_1\ \widetilde{351}'_{-1} \tag{2}$$

There are no cubic Higgs couplings and the following quartic ones:

$$[\widetilde{27}_{-2}(\widetilde{27}_{-2})^+]^2,\ [\widetilde{351}_0\ \widetilde{351}_0^+]^2,\ [\widetilde{351}'_1\ \widetilde{351}'^{\,+}_{-1}]^2,\ [\widetilde{351}'_{-1}\ \widetilde{351}'_{+1}\ \widetilde{351}_0^+\ \widetilde{351}_0^+]$$

This gives rise to an automatic PQ symmetry where the fermion fields have charges q_0, $q_{\pm1}$ related to the bosonic charges $\tilde{q}_{01}$ $\tilde{q}_{+1}$, $\tilde{q}_{-2}$, $\tilde{q}_{-1}$ by:

$$\tilde{q}_0 = -(q_1 + q_{-1}),\ \tilde{q}_1 = -(q_0 + q_{-1}),\ \tilde{q}_{-2} = -2q_1,\ \tilde{q}_{-1} = -(q_0 + q_1),$$
$$\tilde{q}_{-1} + \tilde{q}_1 = -2(q_1 + q_{-1}). \tag{3}$$

The anomalous global U(1) has charges: $q_0 = q_{\pm1} = 1$ which then implies $\tilde{q}_0 = \tilde{q}_1 = \tilde{q}_{-2} = -2$ and also implies that the LS mechanism is operative at the E_6 level. The other solution of eqn. (3) gives the $U(1)_F$ charges.

We now turn to the question of the symmetry breaking pattern. The extra $U(1)_F$ gauge group was added on so that an automatic model with E_6 could be constructed. However, if the scale of $U(1)_F$ breaking is large enough (of order 10^{16} GeV depending on the value of Ω [9]), then cosmic strings [10] can be generated that can seed galaxy formation. Our symmetry breaking pattern is given by [f1]

$$
\begin{array}{l}
\{E_6 \otimes U(1)_F\} \otimes U(1)_{PQ} \dashrightarrow Z(3)_{PQ} \\
\quad \Big\downarrow \; \langle \widetilde{351'};\ 1,1\rangle \gtrsim M_X \\
\{\mathrm{spin}(10) \otimes U(1)_F'\} \otimes U(1)_{PQ}' \dashrightarrow Z(4)_{PQ}' \\
\quad \Big\downarrow \; \langle \widetilde{351'};\ 144,24\rangle\ \langle \widetilde{351};\ 144,24\rangle \sim M_X \\
(321) \otimes U(1)_{PQ}'' \dashrightarrow Z(5)_{PQ}'' \\
\quad \Big\downarrow \; \langle \widetilde{351'};\ 54,24\rangle,\ \langle \widetilde{351};\ 45,24\rangle \sim v_{PQ}'' \\
\qquad\qquad \langle \widetilde{27};\ 11,1\rangle \\
(321) \otimes Z(5)_{PQ}'' \\
\quad \Big\downarrow \; \langle \text{all } SU_L(2) \text{ doublets}\rangle \sim M_W \\
(31) \otimes Z(5)'''
\end{array}
\tag{4}
$$

Let us explain our notation in eq. (4). Firstly $\langle H;\ r,r'\rangle$ denotes the vacuum expectation value (VEV) of the scalar transforming as H under E_6, r under the Spin (10) subgroup of E_6 and r' under the SU(5) subgroup of this Spin (10). Secondly, the horizontal dotted lines indicate the RDS at each stage of symmetry breaking. Thirdly, $U(1)_{PQ}'$ is generated by $1/3(4Q-A)$, $U(1)_{PQ}''$ is generated by $Q'' = 1/4(5Q'-3B)$ and $U(1)_F'$ is generated by $F' \equiv 8F-A$, where A and B are the generators of the U(1)'s appearing in the decomposition $E_6 \supset \mathrm{Spin}\ (10) \otimes U(1)_A \supset \{SU(5) \otimes U(1)_B\} \otimes U(1)_A$. The last $Z(5)'''$ is generated by $Q''' \equiv 6Y-Q''$, where Y is the hypercharge. Finally (321) and (31) denote the groups $SU_c(3) \times SU_L(2) \times U_Y(1)$ and $SU_c(3) \times U(1)_{EM}$ respectively. This is basically example II of ref. 8. In that paper we show that even though there is a realignment of the PQ generator (i.e. mixing with broken gauge generators), the LS mechanism still holds at each step of the symmetry breaking pattern.

Note that when $U(1)''_{PQ}$ is broken by $\widetilde{\langle 351'; 144, 24\rangle}$, $\widetilde{\langle 27; 1, 1\rangle}$ and $\widetilde{\langle 351; 144, 24\rangle}$; no new <u>gauge</u> symmetries are broken. Thus v''_{PQ} is <u>not</u> constrained by renormalization group arguments so that it may be chosen to lie in the cosmologically allowed range: 10^9 GeV $\leq v''_{PQ} \leq 10^{12}$ GeV. This choice makes the model completely consistent with constraints on axion cosmologies.

We now turn to our final topic, the fermion mass matrices. Let us first decompose the 27 of E_6 with respect to SU(5):

$$\underline{27} = \underline{1} \oplus \{\underline{\bar{5}} \oplus \underline{5}\} \oplus \{\underline{1} \oplus \underline{\bar{5}} \oplus \underline{16}\} \tag{5}$$

The curly brackets indicate that the enclosed representations all belong to a single SO(10) representation. Thus we find that in addition to the standard $\underline{\bar{5}} \oplus 10$ SU(5) representation containing the known quarks and leptons, we have an additional $\underline{5} \oplus \underline{\bar{5}}$ and two singlets. The two singlets correspond to neutrino states, while the $\underline{5}$ and $\underline{\bar{5}}$ contain a down quark ($SU_L(2)$ singlet), a new $SU_L(2)$ lepton doublet (with charges -1 and 0) and their antiparticles respectively. The final particle count is then: one up quark (and antiquark, two down quarks (and antiquarks), two charge -1 leptons (and their antiparticles) and five neutral leptons. Let us first examine the up-quark mass matrix. It takes the form:

$$\begin{bmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & -b & c \end{bmatrix} \tag{6}$$

where the entries come from the SO(10) terms $16_0 16_{-1} \overline{126}_1$,

$16_1 16_{-1}(10+120)_0\ 16_1 16_1 10_{-2}$. The eigenvalues are (in the limit that $a \ll b \ll c$):

$$m_u \cong a^2c/b^2,\ m_c \cong b^2/c,\ m_t = c \tag{7}$$

Thus, we have the relations $m_u = (a/b)^2 m_t$, $m_c = (b/c)^2 m_t$. Next we consider the down quark mass matrix. In this case, we must also consider mixings among the new down quarks (D) and the standard down quarks (d). This matrix is a 6 × 6 and takes the form:

$$(\underline{d}^c,\ \underline{D}^c)_L \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \left(\frac{\underline{d}}{\underline{D}_L}\right) \tag{8}$$

where we have defined

$$\underline{d} = (d_0,\ d_{-1},\ d_1),\ \underline{D} = (D_0,\ D_{-1},\ D_1) \tag{9a}$$

$$A = \begin{bmatrix} 0 & a' & 0 \\ a' & 0 & b' \\ 0 & -b' & c' \end{bmatrix} \tag{9b}$$

$$B = \begin{bmatrix} 0 & d' & 0 \\ d' & 0 & e' \\ 0 & -e' & f \end{bmatrix} \tag{9c}$$

$$C = \begin{bmatrix} 0 & A' & 0 \\ A' & 0 & B' \\ 0 & -B' & C' \end{bmatrix} \tag{9d}$$

The entries in A come from the same entries corresponding in the up quark mass matrix. The entries in C come from $10_0 10_{-1} 54_0$, $10_1 10_{-1} 45_0$, $10_1 10_1 1_{-2}$, while those in B come from $10_0 16_{-1}\ 144_0$, $10_1 10_{-1}(16+144)_0$, $10_1 16_1 16_{-2}$. The VEV's contributing to C are all $\Delta I_W = 0$ (in fact, of order v''_{PQ}) while all others are $\Delta I_W = 1/2$. The charge −1 mass matrix follows the same pattern. Finally, the neutral mass matrix is a 15 × 15 matrix so we do not exhibit it. However, there are enough parameters so that all but three or four neutrinos are made heavy.

In conclusion, we have constructed an <u>automatic</u> $E_6 \times U(1)_f$ that satisfies all known cosmological constraints on axion models. In order to solve the axion domain wall problem only three 27's of E_6 are allowed. This corresponds to three low energy families, in agreement with experiment. Even with the addition of $U(1)_F$ to ensure automaticity, it is non-trivial to find a Higgs system that gives rise to a reasonable mass matrix. It is also interesting to note that even though F can be taken to be the diagonal generator of an $SU(2)_F$ symmetry, it can be shown that one cannot obtain reasonable fermion mass matrices even radiatively in this scheme while still solving all the cosmological axion problems. Thus, in some sense, our model is the simplest realistic automatic axion model containing E_6 as a gauge group.

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Footnotes

[f1] For simplicity, we do not allow $351'_{-1}$ to acquire any VEV's.

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