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Chiral Gauge Theories in the 1/N Expansion*

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ABSTRACT

We study the large N limit of various SU(N) gauge theories with chiral fermion content. Assuming that the leading $N \to \infty$ behavior is given by a sum of planar diagrams, we find that the gauge interaction must fail to confine color in some models. Other models, assuming both a planar diagram limit and confinement, must contain massless composite fermions.

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1. Introduction

Significant progress has recently been made toward establishing how global flavor symmetries are realized in vector-like gauge theories, like QCD. For example, if quark confinement is assumed, then we may invoke 't Hooft's anomaly argument [1] and inequalities similar to those derived by Weingarten [2] and Vafa and Witten [3], along with some mild technical assumptions, to show that the $SU(n)_L \times SU(n)_R \times U(1)_V$ chiral symmetry of QCD with n massless quark flavors is spontaneously broken to the maximal nonchiral subgroup $SU(n)_V \times U(1)_V$. This demonstration is an important achievement for QCD, since such a realization of global chiral symmetry is actually observed in Nature.

It is also of considerable interest to determine how global flavor symmetries are realized in theories which are not vector-like, but chiral; theories in which the gauge symmetry forbids masses for at least some of the elementary fermions. It has been suggested that many such "chiral gauge theories" contain massless composite fermions [4,5]; physics well below the Planck scale might conceivably be described by an effective chiral gauge theory in which quarks and leptons are composite and their masses are calculable. Unfortunately, very little is known about the behavior of chiral gauge theories beyond perturbation theory.

The central theoretical problem concerning chiral gauge theories which we wish to address can be formulated as follows: Consider an asymptotically free gauge theory with gauge group G and with massless left-handed Weyl fermions transforming as some complex representation of G. If the fermion representation is reducible, then this theory respects a group G_f of global flavor symmetries. We wish to know how the G_f symmetry is realized. This question has two parts:

(1) What subgroup H_f of G_f escapes spontaneous symmetry breakdown?

(2) What is the representation content under H_f of the massless fermions in the spectrum of the theory?

The massless fermions may be either composite or elementary, for the G gauge interaction may or may not be exactly confining; if, for example, the gauge group G is spontaneously broken to a subgroup H, massless elementary fermions may appear in the spectrum which are H singlets but not G singlets. Indeed, finding the realization of the gauge symmetry is itself a very important dynamical problem.

To determine the realization of the gauge symmetry and global flavor symmetry, we must have some knowledge about the intrinsically nonperturbative physics involved. An important step was taken by 't Hooft [1], who argued that the massless fermions in the spectrum of the theory must obey a remarkable algebraic condition; they must produce the same triangle anomalies for the unbroken flavor group H_f as the elementary fermions. This reasoning established that massless fermions are a necessary consequence of unbroken chiral symmetry. If there is a generator t of the unbroken flavor group H_f such that $tr\, t^3 \neq 0$, then there must be physical massless fermions that couple to the associated current. Furthermore, the anomaly condition places constraints on the H_f representation content of the massless fermions that, in a confining theory, are highly nontrivial, because composite fermions must typically be in different representations of H_f than the elementary fermions.

The anomaly condition alone, however, does not uniquely determine the realization of the global G_f symmetry, even if exact confinement is assumed. Further information is needed. In vector-like theories like QCD, this information can be obtained in various ways. As noted earlier, the realization of the chiral symmetry of QCD can be determined, if confinement is assumed, by appealing to the QCD inequalities [2, 3]. Unfortunately, analogous inequalities have not been derived for chiral gauge theories. Nonperturbative information about QCD can also be obtained by doing numerical calculations in lattice gauge theories. But there are technical obstacles which make it difficult even to formulate chiral gauge theories on the lattice. (A possible means of surmounting these obstacles is discussed by two of us in Ref. [6].)

Another approach to studying the nonperturbative behavior of QCD is the expansion in $1/N$, where N is the number of colors [7]. If quark confinement is assumed to apply in the $N \to \infty$ limit, then a surprising number of qualitative features of meson phenomenology can be derived in this approach [7-10]. The $1/N$ expansion can also be applied to the problem of finding the realization of global flavor symmetry. Given confinement and some technical assumptions it can be shown, without invoking QCD inequalities, that the chiral symmetry of QCD with n massless quarks is spontaneously broken to $SU(n)_V \times U(1)_V$ in the limit $N \to \infty$ [11].

Our aim in this paper is to determine, as far as possible, the realization of the global flavor symmetries of various chiral gauge theories in the limit $N \to \infty$. We restrict our attention, of course, to theories which remain asymptotically free as $N \to \infty$; thus, all fermions lie in one of the irreducible representations S, A, or F (or their complex conjugates) of the gauge group $SU(N)$. Here S denotes the symmetric two-index tensor, A the antisymmetric two-index tensor, and F the fundamental representation of the group $SU(N)$. (In principle, the adjoint representation is also allowed, even though it is real, if we define a chiral gauge theory as one in which masses for <u>some,</u> but not necessarily <u>all,</u> of the elementary fermions are forbidden by the gauge symmetry.)

We have reached two main conclusions. First, we have found that, if we assume the existence of an $N \to \infty$ limit given by a sum of planar diagrams, and also assume confinement (all physical states are SU(N) singlets), then there are models which necessarily respect unbroken chiral symmetries, and contain massless composite fermions, in the $N \to \infty$ limit. This is the strongest argument we know that massless composite fermions can really exist. The key point is that there will be massless composite fermions unless all global flavor symmetries with anomalies are spontaneously broken. Associated with each spontaneously broken symmetry is a Goldstone boson state which must be able to appear as an intermediate state in cut planar diagrams. But for some chiral symmetries with anomalies, we find that there is no candidate gauge-singlet Goldstone boson available in the $N \to \infty$ limit. Thus, the symmetry is unbroken, and there are massless composite fermions.

Our second main conclusion is that there are models in which the two assumptions underlying our first main conclusion are incompatible; either confinement, or the existence of an $N \to \infty$ limit dominated by planar diagrams, or both, must fail. In these models, there are chiral symmetries with anomalies for which neither a candidate gauge-singlet Goldstone boson nor a candidate multiplet of massless gauge-singlet composite fermions occurs in the cut planar diagrams. Thus, no possible realization of these symmetries is consistent with our assumptions. In previous papers [12, 13], we have considered realizations of the global flavor symmetries of such models that are consistent with confinement, at the expense of the existence of a conventional large N limit. It actually seems more plausible to us, though, that confinement is the incorrect assumption, and that the gauge symmetry is realized in a Higgs or Coulomb mode.

Although our second main conclusion may appear at first to undercut the assumptions which were the basis of our first main conclusion, such is not really the case (as we will argue in the concluding section). Rather, our second conclusion provides the strongest argument known to us that there are some chiral gauge theories which break their own gauge symmetry. And our first conclusion provides highly persuasive evidence that certain other chiral gauge theories really do contain massless composite fermions.

In Section 2, the $N \to \infty$ limit of QCD is reviewed. Chiral gauge theories which are likely to contain massless composite fermions are analyzed in Sections 3 and 4, and a nonconfining chiral gauge theory is discussed in Section 5. Section 6 contains our conclusions.

2. The Vector-Like Case

Before turning to examples of chiral gauge theories, let us briefly review the analysis of the large N behavior of QCD, a vector-like theory.

QCD is an SU(N) gauge theory coupled to left-handed Weyl fermions (quarks) that, in the notation introduced earlier, transform under the SU(N) (color) group as

$$n(F + \bar{F}); \tag{2.1}$$

here F signifies the fundamental representation of $SU(N)$ and $\bar{F}$ its conjugate. The $1/N$ expansion is carried out with $g^2 N$ fixed and taken to be order one, where g is the conventionally normalized gauge coupling.

Counting the powers of N associated with a given Feynman diagram is facilitated by 't Hooft's double line notation [7], in which the SU(N) gauge theory is approximated by a U(N) gauge theory, and gluons are represented by two lines of opposite orientation, each carrying an index which runs from 1 to N. (See Figure 1). In this notation, each closed "index loop" of a diagram indicates a factor of N.

With every connected vacuum bubble diagram, we may associate a two-dimensional surface, regarding each index loop as a face of the surface, each gluon propagator as an edge where two faces meet, and each quark propagator as an edge where only one face ends, part of the boundary of the surface. A graph with F faces, E edges, and V vertices carries F powers of N and $L - 1 = E - V$ powers of g^2, where L is the number of loops contained in the diagram (that is, the number of independent momentum integrals, not the number of index loops). Thus, this graph scales like

$$\text{Graph} \sim N^{F-E+V} (g^2 N)^{E-V}. \tag{2.2}$$

With $g^2 N$ considered of order one, we see that the number of powers of N carried by the graph turns out to be a topological invariant of the surface on which the graph can be drawn, the Euler characteristic $\chi = F - E + V$. By a well-known theorem of topology, we obtain $\chi = 2 - 2H - B$, and

$$\text{Graph} \sim N^{2-2H-B}, \tag{2.3}$$

where H is the number of "handles" on the surface and B is the number of "holes" or boundaries, that is, the number of quark loops. Evidently, in the $N \to \infty$ limit with $g^2 N$ fixed, the "planar" vacuum diagrams dominate; these are the diagrams containing no quark loops which can be drawn on a sphere, without any crossing of gluon lines. The contribution due to graphs with one quark loop is suppressed by one power of $1/N$; "nonplanar" gluon exchanges are suppressed by $1/N^2$.

It has not yet proved possible to sum the planar diagrams and determine the leading contribution in the $1/N$ expansion. But a surprising number of qualitative properties of the $N \to \infty$ limit can be extracted if we make the plausible assumption that color is confined in this limit; that is, that all physical states are color singlets [7-10]. The physics of the theory can then be probed with Green's functions of gauge-invariant composite operators. For example, consider

$$M_{ab} = \frac{1}{\sqrt{N}} \sum_i \bar{F}_{ia} F^i_b, \tag{2.4}$$

a generic color singlet meson operator, appropriately normalized to couple with strength of order one to a color-singlet state. (The index i is a color index running from 1 to N, a and b are flavor indices, and spin indices have been contracted in an unspecified way.) The leading diagrams as $N \to \infty$ contributing to the connected k-point function for M have one quark loop which is the boundary of the graph, and are of order $N^{1-k/2}$. It follows that, in the leading order of the $1/N$ expansion, the operator M couples only to one meson states, and that the mesons are noninteracting [7, 10]. The leading k-meson scattering amplitude, of order $N^{1-k/2}$, is a sum of pole terms (tree graphs). This analysis is readily extended to incorporate glueball states (the k-meson, l-glueball amplitude is of order $N^{1-k/2-l}$ for $k \neq 0$, N^{2-l} for $k = 0$) and one concludes that QCD becomes, in the $N \to \infty$ limit, a theory of an infinite number of noninteracting zero-width mesons and glueballs. (The number of states must be infinite, in order for the Green's functions to behave at large momentum as predicted by renormalization-group-improved perturbation theory.)

While the conclusion that M couples only to one-particle states follows from just the scaling properties as $N \to \infty$ of the M Green's functions, it is instructive to note that this conclusion is reinforced by an argument based on the structure of the planar diagrams [8, 9]. Whenever a planar diagram is cut, the intermediate state that occurs contains a quark-antiquark pair and some number of gluons with color indices contracted as

$$\bar{F}_{i_1} A^{i_1}{}_{i_2} A^{i_2}{}_{i_3} \cdots A^{i_{m-1}}{}_{i_m} F^{i_m}. \tag{2.5}$$

(See Fig. 2.) This state consists of a single system of color-singlet particles; it cannot be split into two or more color-singlet states. Thus, if color is confined, the intermediate state must be a perturbative approximation to a one-particle state, not a multiparticle state. One concludes again that M couples only to one-particle states in the $N \to \infty$ limit.

In the $N \to \infty$ limit, QCD with n massless quark flavors has a $U(n)_L \times U(n)_R$ global flavor symmetry. (The effects of the axial anomaly are not felt until the next-to-leading order in the $1/N$ expansion.) If confinement is assumed, it immediately follows that this symmetry must be spontaneously broken to an anomaly-free subgroup [11]. The 't Hooft anomaly condition requires that, if there is a flavor symmetry generator that has an anomaly and is not spontaneously broken, then the corresponding current must couple to a pair of massless fermions. But, in the $N \to \infty$ limit, assuming confinement, all quark bilinears couple only to one-meson states.

That the unbroken symmetry is actually the diagonal $U(n)_V$ can be inferred from a few additional mild assumptions (and without invoking QCD inequalities). The pattern of symmetry breakdown is characterized by a nonzero value for some gauge-invariant parameter, or "condensate." Since all mesons are quark-antiquark states, any Goldstone bosons are such states, and the condensate, to which Goldstone bosons must couple, may be regarded as a quark bilinear. The value of the condensate is determined by minimizing some potential, obtained by summing an infinite number of connected planar diagrams. But each diagram contains a single quark loop, so the potential can be written as a sum of terms, each involving a single quark flavor. The potential is minimized by minimizing it for each quark flavor separately, and therefore the minimum retains the $U(n)_V$ flavor symmetry. We conclude that the maximal nonchiral flavor symmetry, $U(n)_V$, is unbroken [11].

A similar conclusion can be obtained by appealing to QCD inequalities, provided, again, that confinement is assumed. According to 't Hooft's anomaly argument [1], there must be massless fermions coupling to the axial flavor currents if the associated flavor symmetries are not spontaneously broken. If all physical states are color-singlets, then these massless fermions cannot be quarks; they are baryons. But rigorous inequalities similar to those derived by Weingarten [2] and Vafa and Witten [3] show that the lightest pseudoscalar meson (the pion) is no heavier than the lightest baryon. Thus, the pion must be massless, and, barring the possibility that f_π "accidentally" vanishes, it is a Goldstone boson. The $SU(n)_L \times SU(n)_R \times U(1)_V$ chiral symmetry is therefore spontaneously broken, and QCD inequalities can be invoked to determine that the unbroken symmetry must be $SU(n)_V \times U(1)_V$ [3]. This argument is readily generalized to show that, in vector-like gauge theories, global flavor symmetries are always spontaneously broken to maximal nonchiral subgroups.

Although the argument based on QCD inequalities applies for any N, we find the argument based on the $1/N$ expansion more useful for our purposes, because it is more easily generalized to chiral gauge theories.

If the $N \to \infty$ limit is taken with the number n of quark flavors held fixed, then QCD becomes a theory of an infinite number of noninteracting zero-width mesons and glueballs. But it is also interesting to consider the $N \to \infty$ limit with n/N held fixed; that is, with the number of quark flavors of order N [8]. In fact, we will see later that the meson phenomenology of some chiral gauge theories resembles that of QCD with order N flavors.

For n/N of order one, a typical diagram contributing in leading order to the connected k-point Green's function for the meson operator M is planar, but contains many fermion loops; all the M insertions occur on a single loop, which may be taken to be the edge of the diagram. (See Figure 3.) The color factor

$1/N$ associated with a fermion loop is compensated by the n-fold flavor degeneracy of the loop.

The same counting as before shows that the k-meson amplitude is of order $N^{1-k/2}$. But the mesons nonetheless acquire finite widths. The rate for the decay of a meson resonance into $k-1$ mesons is of order one, because the order N^{k-2} flavor degeneracy of the final meson state compensates for the smallness of the amplitude. (See Fig. 4a.) Only the massless mesons, the Goldstone bosons of the spontaneously broken flavor symmetry, are exactly stable, and these mesons are noninteracting. (The two-meson total cross section is of order $1/N$.) If n/N is small but non-zero [8], then there is an infinite tower of meson resonances with typical widths of order n/N, and a tower of glueball states with typical widths of order $(n/N)^2$ (Fig. 4b).

In the $N \to \infty$ limit of QCD with n/N of order one, the flavor symmetry is $SU(n)_L \times SU(n)_R \times U(1)_V$; the effects of the axial anomaly cannot be ignored. Although quark loops are unsuppressed for n/N of order one, it is still true that quark bilinears do not couple to baryon-antibaryon pairs in the $N \to \infty$ limit; they couple only to multi-meson states. We may therefore conclude as before, if confinement is assumed, that the global flavor symmetry must be spontaneously broken to an anomaly-free subgroup. However, because connected graphs with many fermion loops occur, we cannot argue as before, without invoking QCD inequalities, that the diagonal subgroup $SU(n)_V \times U(1)_V$ must be unbroken.

3. A Chiral Gauge Theory with Massless Composite Fermions

As a first example of a chiral gauge theory, we consider a model with gauge group $SU(N)$ and left-handed Weyl fermions transforming under $SU(N)$ as the representation

$$S + (N + 4)\,\bar{F}. \tag{3.1}$$

a symmetric tensor and $(N+4)$ $\bar{N}$'s. The number $N+4$ is chosen to cancel the $SU(N)$ gauge anomaly.

This model respects a global flavor symmetry

$$G_f = \mathrm{SU}(N+4) \times \mathrm{U}(1); \tag{3.2}$$

the $\mathrm{SU}(N+4)$ mixes the different $\bar{F}$ flavors, and the U(1) is that combination of S number and $\bar{F}$ number, with charges

$$Q_S = N+4 \qquad Q_{\bar{F}} = -(N+2), \tag{3.3}$$

which survives in the presence of $\mathrm{SU}(N)$ instantons. We would like to determine how this symmetry is realized in the $N \to \infty$ limit, assuming exact confinement.

The $N \to \infty$ limit is dominated by planar diagrams, but both S loops and $\bar{F}$ loops are unsuppressed. The S propagator, like the gluon propagator, carries two color indices, and the color factor $1/N$ associated with an $\bar{F}$ loop is compensated by the $(N+4)$-fold flavor degeneracy. Thus, the meson phenomenology of this model resembles that of QCD with order N flavors, discussed in the last section. There are glueball resonances and meson resonances coupling to the bilinears $\bar{F}^\dagger\bar{F}$ and $S^\dagger S$, but all but the lightest states can decay into light $\bar{F}^\dagger\bar{F}$ mesons at a finite rate. All meson and glueball scattering cross sections vanish in the $N \to \infty$ limit.

In its "baryon" phenomenology, however, this model departs greatly from the behavior of QCD. There are composite fermions with the quantum numbers of

$$B_{ab} = \bar{F}_{ia}\, S^{ij}\, \bar{F}_{jb} \tag{3.4}$$

which are not decoupled from the meson physics. (Here i, j are color indices running from 1 to N, and a, b are flavor indices running from 1 to $N+4$.) The intermediate states $BB^\dagger$ can occur in cut planar diagrams (Fig. 5). Thus, meson and glueball resonances can decay into the $BB^\dagger$ channel with rates of order one. If there are massless baryons (and there are - see below) we expect all mesons and glueballs with nonzero mass to have finite widths for $N \to \infty$; the only stable particles in the spectrum of the theory are the massless baryons, and the massless mesons, if any. Excited baryon resonances can decay to the massless baryons by emission of $\overline{F}^\dagger\overline{F}$ mesons. And the baryons are noninteracting; all scattering amplitudes vanish in the $N \to \infty$ limit.

To see that the spectrum of this theory really contains massless composite fermions (assuming confinement), we now consider the realization of the global G_f symmetry. If the G_f symmetry is completely unbroken, then there are four flavor anomaly conditions which must be satisfied – there are $\mathrm{SU}(N+4)^3$, $\mathrm{SU}(N+4)^2\mathrm{U}(1)$, $\mathrm{U}(1)^3$, and U(1) ("gravitational" [14]) anomalies. All four conditions impose nontrivial constraints on the flavor quantum numbers of the massless composite fermions, but they admit a remarkably simple solution [5]. A state coupling to the operator B_{ab}, antisymmetrized in its flavor indices, has U(1) charge $Q_B = -N$, and is readily found to produce flavor anomalies which match those of the elementary fermions S and $\overline{F}$.

This solution to the anomaly conditions is so natural and appealing that it is very tempting to conjecture that the G_f symmetry is actually realized in this way, that G_f is completely unbroken and that there are $\frac{1}{2}(N+4)(N+3)$ massless fermions transforming as the antisymmetric tensor representation of flavor $\mathrm{SU}(N+4)$, with U(1) charge $Q = -N$. In fact, if we assume confinement and the existence of an $N \to \infty$ limit dominated by planar diagrams, we can reach a conclusion nearly this strong. The point is that, in the $N \to \infty$ limit, the global $\mathrm{U}(1)_Q$ symmetry cannot be spontaneously broken, and the only composite fermions in the spectrum have U(1) charge $Q = -N$. Since the $\mathrm{U}(1)^3$ and U(1) anomalies are $-\frac{1}{2}N^3(N+4)(N+3)$ and $-\frac{1}{2}N(N+4)(N+3)$ respectively, we know that $\frac{1}{2}(N+4)(N+3)$ of the fermions must be massless.

To argue that the $U(1)_Q$ symmetry remains unbroken, we consider candidate order parameters which could signal the spontaneous breakdown of this symmetry. If a gauge-invariant operator with nontrivial flavor quantum numbers condenses, then there must be a Goldstone boson which couples to the condensate, and appears in the spectrum of the theory. (That is the Goldstone boson couples to the "imaginary" part of the operator whose "real" part condenses.) Thus, if an operator is to condense in the $N \to \infty$ limit, we demand that a state with the quantum numbers of that operator appears in the cut planar diagrams. This requirement, along with confinement (the operator which condenses must be a gauge-singlet), greatly restricts the possible realizations of the flavor symmetry.

The key feature of the cut planar diagrams is that, along the cut, an index line pointing out through the cut is always followed by one pointing in. It is thus easy to see that the only color singlet states with $U(1)_Q$ charge that can occur in the cut planar diagrams are the baryon state B, the antibaryon $B^\dagger$, and multibaryon states constructed from them. Furthermore, while a baryon-antibaryon state $BB^\dagger$ can occur, a two-baryon state BB cannot (Fig. 6). (We should not expect BB to condense anyway, because the baryons are noninteracting.) We conclude that, if the only allowed condensates are Lorentz-singlet gauge-singlet operators which appear as intermediate states in cut planar diagrams, then there is no candidate condensate to break the $U(1)_Q$ symmetry; this symmetry must be unbroken.

As we have noted, the unbroken $U(1)_Q$ symmetry ensures the existence of $\frac{1}{2}(N+4)(N+3)$ massless composite fermions. But we cannot argue that the $SU(N+4)$ flavor symmetry is unbroken, however plausible this may seem, without making further assumptions. Confinement and the existence of an $N \to \infty$ limit dominated by planar diagrams do not, in principle, exclude the possibility that an operator with the flavor quantum numbers of $\bar{F}^{\dagger a}\bar{F}_b$ condenses. (A Lorentz-singlet operator with this structure is, for example, $\bar{F}^\dagger \gamma_\mu \bar{F} D_\nu G^{\mu\nu}$, where G is the gluon field.) It is even possible for several such condensates, misaligned, to break the $SU(N+4)$ symmetry completely.

Arguments identical to those above can be used to show, assuming confinement, that an $SU(N)$ gauge theory with fermions transforming as

$$A + (N-4)\,\bar{F}$$

contains $\frac{1}{2}(N-4)(N-3)$ massless composite fermions in the limit $N \to \infty$.

4. More Examples

In this section, we investigate the behavior of several more chiral gauge theories in the $N \to \infty$ limit. We assume, as before, that the gauge interaction is exactly confining.

(i) The Bars-Yankielowicz Model

Bars and Yankielowicz [15] proposed a generalization of the model discussed in Section 3. Their model is an $SU(N)$ gauge theory with left-handed Weyl fermions transforming under $SU(N)$ as the representation

$$S + (N+4)\bar{F} + n(\bar{F} + F); \tag{4.1}$$

that is, the same representation as before, except for a vector-like piece appended on.

This model respects a group of flavor symmetries

$$G_f = \mathrm{SU}(N+4+n) \times \mathrm{SU}(n) \times \mathrm{U}(1) \times \mathrm{U}(1). \tag{4.2}$$

In Ref. [15], a remarkably beautiful solution to the anomaly conditions was found. All G_f anomaly conditions are satisfied by the set of color-singlet composite fermions

$$\bar{F}S\bar{F} + \bar{F}^\dagger S^\dagger F + F^\dagger S F^\dagger, \tag{4.3}$$

transforming as the representation

$$\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}, 1\right) + (\bar{\square}, \square) + (1, \square\!\square)\ . \tag{4.4}$$

under the $\mathrm{SU}(N+4+n) \times \mathrm{SU}(n)$ flavor symmetry.

Unfortunately, in the $N \to \infty$ limit with n held fixed, this realization of the G_f symmetry is not possible. Because an F loop costs a factor of $1/N$, the $F^\dagger S F^\dagger$ fermions do not appear in the leading cut diagrams, and the G_f symmetry must break. An argument similar to that applied earlier to the case of QCD shows that the $\mathrm{SU}(n)$ symmetry of the F's should remain unbroken. Therefore, the n F's must condense with n of the $\bar{F}$'s, leaving the unbroken symmetry

$$H_f = \mathrm{SU}(N+4) \times \mathrm{SU}(n) \times \mathrm{U}(1) \times \mathrm{U}(1), \tag{4.5}$$

or perhaps a subgroup of H_f. Only the $\bar{F}S\bar{F}$ fermions are massless.

If the $N \to \infty$ limit is taken with n/N of order one, however, then F loops are unsuppressed, and the realization of the G_f symmetry advocated in Ref. [15] is not excluded by our arguments.

<u>(ii) Georgi's Models</u>

Georgi [16] has recently emphasized that there are chiral generalizations of QCD with nonsimple gauge groups which are quite likely to contain massless composite fermions. Consider, for example, a model with gauge group SU(N) × SU(M) and left-handed Weyl fermions that transform under the gauge group as the representation

$$M(N,1) + (\overline{N},M) + N(1,\overline{M}) \,. \tag{4.5}$$

(This representation is free of gauge anomalies.) We will denote these elementary fermions as

$$F_a^{(N,1)}, \quad F^{(\overline{N},M)}, \quad F_b^{(1,\overline{M})} \,. \tag{4.6}$$

Here a is a flavor index running from 1 to M and b is a flavor index running from 1 to N; color indices are suppressed. This model respects the group of global flavor symmetries

$$G_f = \mathrm{SU}(M) \times \mathrm{SU}(N) \times \mathrm{U}(1)_Q \,,$$

where the U(1)$_Q$ charge assignments are

$$Q(F_a^{(N,1)}) = 1 \,, \quad Q(F^{(N,\overline{M})}) = -1 \,, \quad Q(F_b^{(1,\overline{M})}) = 1 \,. \tag{4.7}$$

It is suggested in Ref. [16] that the G_f symmetry is unbroken, with the G_f anomaly conditions satisfied by the massless composite fermion multiplet

$$B_{ab} = F_a^{(N,1)} F^{(\overline{N},M)} F_b^{(1,\overline{M})} \,, \tag{4.8}$$

which transforms as the $(M,N)^1$ representation of G_f.

In the $N \to \infty$ limit, with M/N fixed, the only gauge-singlet states with nonzero U(1)$_Q$ charge which appear as intermediate states in the leading diagrams are the baryon B and the antibaryon $B^\dagger$ (Fig. 7). Since there is no candidate gauge-invariant Lorentz-invariant condensate to break the U(1)$_Q$ symmetry, this symmetry must be unbroken. The U(1)$_Q$ symmetry has an anomaly,

$\mathrm{tr}\, Q^3 = NM$. Thus, all NM of the B_{ab}'s must be massless, in agreement with Ref. [16]. Some breakdown of the SU(M) × SU(N) symmetry cannot be excluded by our arguments, however.

Next, consider a model with gauge group SU(N) × SU(M) × SU(N), and left-handed Weyl fermions transforming under the gauge group as the representation

$$M(N,1,1)^1 + (\overline{N},M,1)^{-1} + (1,\overline{M},N)^1 + M(1,1,\overline{N})^{-1} . \tag{4.9}$$

This model respects the flavor symmetry group

$$G_f = \mathrm{SU}(M)_L \times \mathrm{SU}(M)_R \times \mathrm{U}(1)_V , \tag{4.10}$$

where the $\mathrm{U}(1)_V$ charge assignments are indicated as superscripts in (4.9).

In the $N \to \infty$ limit, with M/N fixed, there are no gauge-singlet fermion states which appear as intermediate states in the leading diagrams. Thus, G_f must be spontaneously broken to an anomaly-free subgroup. Since $\mathrm{U}(1)_V$ has no anomaly, the obvious choice for the unbroken symmetry is

$$H_f = \mathrm{SU}(M)_V \times \mathrm{U}(1)_V , \tag{4.11}$$

as suggested in Ref. [16]. Indeed, the candidate condensate

$$F_a^{(N,1,1)} F^{(\overline{N},M,1)} F^{(1,\overline{M},N)} F_b^{(1,1,\overline{N})} , \tag{4.12}$$

capable of breaking G_f to H_f, occurs as an intermediate state in the leading diagrams. But our arguments do not exclude the possibility that the unbroken symmetry is some other anomaly-free subgroup of G_f.

It is obvious that this analysis can be extended to models with any number of simple gauge groups, supporting the conclusion of Ref. [16] that "even linear Mooses" like (4.5) contain massless composite fermions, while "odd linear

Mooses" like (4.9) do not.

(iii) A Model with both a Symmetric and an Antisymmetric Tensor

Our next example is an SU(N) gauge theory with left-handed Weyl fermions transforming under SU(N) as the representation

$$S + A + 2N(\bar{F}) \,. \tag{4.13}$$

The number of $\bar{F}$'s is chosen to ensure cancellation of the SU(N) gauge anomaly. This model respects the group of flavor symmetries

$$G_f = \mathrm{SU}(2N) \times \mathrm{U}(1)_Q \times \mathrm{U}(1)_R \,. \tag{4.14}$$

Under the U(1) symmetries the fermions carry the charges

$$\begin{array}{lll} Q_S = 1 & Q_A = 1 & Q_{\bar{F}} = -1, \\ R_S = (N-2) & R_A = -(N+2) & R_{\bar{F}} = 0 \,. \end{array} \tag{4.15}$$

The only color-singlet states carrying nonzero U(1)$_Q$ charge which appear in the cut planar diagrams of this model are the baryon states

$$B_{Sab} = \bar{F}_a S \bar{F}_b \,, \quad B_{Aab} = \bar{F}_a A \bar{F}_b \,. \tag{4.16}$$

and the corresponding antibaryons. Since there is no candidate order parameter, the U(1)$_Q$ symmetry cannot be spontaneously broken. And because the U(1)$_Q$ symmetry has an anomaly, $tr\ Q^3 = -N^2$, and the baryons all carry $Q = -1$, we know that N^2 of the baryons must be massless. It follows that the SU($2N$) symmetry must be spontaneously broken, because B_S and B_A transform as tensor representations of SU($2N$), with more than N^2 components.

Our arguments cannot determine precisely how the G_f symmetry is realized, so we will be content with presenting one possibility. The U(1)$_R$ symmetry can be spontaneously broken by a condensate with the structure

$$\bar{F}^{i} S_{ij} A^{jk} \bar{F}_{k} \; ; \tag{4.17}$$

such a state appears in the cut planar diagrams. Further condensates with the flavor quantum numbers of $F^{+}F$ can break the SU($2N$) flavor symmetry down to the SU(N) subgroup such that $2N$ transforms as $N + N$. The anomaly conditions for

$$H_f = \mathrm{SU}(N) \times \mathrm{U}(1)_Q \tag{4.18}$$

are then satisfied by massless baryons transforming under H_f as

$$\square\!\square^{\,1} + \begin{smallmatrix}\square\\ \square\end{smallmatrix}^{\,1} . \tag{4.19}$$

These massless baryons are presumably a dynamically determined mixture of the states B_S and B_A.

5. A Chiral Gauge Theory with Spontaneously Broken Gauge Symmetry

Our analysis of chiral gauge theories in the $1/N$ expansion has made essential use of the assumption of color confinement. This might be regarded as a weakness of the analysis, for one can plausibly argue that chiral gauge theories should not be expected to be confining in general [4]. One might expect instead that gauge-nonsinglet bilinear condensates form which break the gauge symmetry. Such a scenario is especially credible in view of recent Monte Carlo calculations [17] that indicate that condensation of fermion pairs can occur at surprisingly short distances. Our next example emphasizes that it is not always sensible to assume that the gauge interaction is confining.

The example [12, 13] is an SU(N) gauge theory with left-handed Weyl fermions transforming under the gauge group as the representation

$$S + \bar{A} + 8\,\bar{F} ; \tag{5.1}$$

The number of F's is chosen to ensure cancellation of the SU(N) gauge anomaly. This model respects a group

$$G_f = \mathrm{SU}(8) \times \mathrm{U}(1)_Q \times \mathrm{U}(1)_R \tag{5.2}$$

of global flavor symmetries. The charges of the fermions under the U(1) symmetries, which are preserved in the presence of instantons, may be chosen to be

$$\begin{aligned} &Q_S = 2 \qquad Q_{\bar{A}} = -2 \qquad\qquad Q_{\bar{F}} = -1, \\ &R_S = N - 2 \quad R_{\bar{A}} = -(N+2) \quad R_{\bar{F}} = 0. \end{aligned} \tag{5.3}$$

This model has a potentially interesting feature [12, 13]; one can construct a sequence of "baryon" operators of the form

$$\bar{F}_{i_1,a}\, S^{i_1,i_2} \left[\bar{A}_{i_2 i_3}\, S^{i_3,i_4} \ldots \bar{A}_{i_{n-2} i_{n-1}}\, S^{i_{n-1},i_n}\right] \bar{F}_{i_n,b}. \tag{5.4}$$

The operators in this sequence all have the same flavor SU(8) quantum numbers, but carry different $\mathrm{U}(1)_R$ charges. Thus, one might expect the massless fermions in the spectrum of this model to exhibit a sort of generation structure. Indeed, solutions to the anomaly conditions can be found for chiral subgroups of G_f such that the same representation of the unbroken nonabelian flavor symmetry occurs repeatedly, tagged by a U(1) quantum number which serves to distinguish the "generations" [12, 13].

Unfortunately, such a realization of the flavor symmetry is not allowed in the $N \to \infty$ limit. Although these baryons can occur as intermediate states in cut planar diagrams, the diagrams require an $\bar{F}$ loop, which costs a factor of $1/N$ relative to the leading diagrams. In fact, the large N limit of this model, assuming confinement, is a theory of an infinite number of noninteracting zero-width mesons and glueballs, containing no baryons at all, just like the large N limit of QCD.

Perhaps we should be willing to modify slightly the rules for counting powers of $1/N$, as applied to this model. For if there are order N massless baryon species (as required to satisfy the anomaly condition if a chiral subgroup of G_f is unbroken), all selected from the sequence (5.4), then this dynamically generated degeneracy could compensate for the formal $1/N$ suppression of the $\bar{F}$ loop, and allow baryon-antibaryon thresholds to occur in the $N \to \infty$ limit. But even with this modification, the analysis of the model runs into trouble.

To appreciate the trouble, we must consider the realization of the $U(1)_Q$ symmetry. The $U(1)_Q$ charge of a state simply counts the difference between the number of upstairs color indices and downstairs color indices carried by the state. Thus the $U(1)_Q$ charge vanishes for any gauge-singlet state which can be constructed without the aid of an N-index ε symbol. Since states involving the ε symbol cannot occur in cut planar diagrams (an upstairs index is always followed by a downstairs index), all gauge-singlet states which appear in cut planar diagrams have vanishing $U(1)_Q$ charge.

The $U(1)_Q$ charge has no $U(1)_Q^3$ anomaly, but it does have a $U(1)_Q$ ("gravitational") anomaly: we have $trQ = -6N$ for the elementary fermions. Since no candidate gauge-singlet condensate with a $U(1)_Q$ charge occurs in the cut planar diagrams, the $U(1)_Q$ symmetry cannot be spontaneously broken in the $N \to \infty$ limit, if confinement is assumed. On the other hand, the $U(1)_Q$ symmetry cannot be unbroken, because there are no gauge-singlet baryon states with $U(1)_Q$ charge in the $N \to \infty$ limit with which to satisfy the $U(1)_Q$ anomaly condition. We have reached a contradiction.

The simplest way out of this difficulty is to give up the assumption of color confinement; if the theory is in a Higgs phase or Coulomb phase, then there can be physical states which are not $SU(N)$ singlets. It is, in fact, not at all implausible that the gauge symmetry of this model should be spontaneously broken. In perturbation theory, there is an attractive interaction in the adjoint channel between S and $\bar{A}$. If the adjoint order parameter

$$S^{ij}\,\bar{A}_{jk} \tag{5.5}$$

condenses, it can be expected to break the gauge symmetry in a manner which admits no "complementary" confinement interpretation [5, 18]. (And indeed, in the $N \to \infty$ limit, this channel is as attractive in perturbation theory as the channel $S^{ij}F_j$; it ties for the distinction of being the <u>most</u> attractive channel.) For example, the adjoint order parameter could break SU(N) down to the (nonconfining) abelian subgroup $U(1)^{N-1}$; the theory is then in a Coulomb phase. This choice might be supported by a "tumbling" picture [4]: the adjoint order parameter might first break SU(N) to SU($N-1$) × U(1). Then SU($N-1$), getting strong at a slightly lower scale might become broken to SU($N-2$) × U(1). And so on.

But we cannot say, on the basis of our large N arguments, exactly how the gauge symmetry is realized in the $N \to \infty$ limit. We can say only that the SU(N) gauge interaction must be nonconfining; there are physical states which are not SU(N) singlets.

6. Conclusions

In Section 5, we described a chiral gauge theory in which color fails to be confined in the $N \to \infty$ limit. That model differs from those considered earlier in this paper in that there is an attractive channel in perturbation theory in which a bilinear fermion condensate can form that transforms as the adjoint representation of the gauge group. If such a condensate actually forms, it can be expected to break the gauge symmetry in a manner which admits no complementary confinement interpretation [5,18]. (The condensate preserves a Z_N

symmetry by which states can be classified.) Thus, by appealing to the $1/N$ expansion, we have merely verified by a nonperturbative argument what might have been naively expected on the basis of perturbation theory.

In the models considered in Sections 3 and 4, however, it is not possible to construct any scalar fermion bilinear which transforms trivially under the center Z_N of the gauge group. The condensation of a fermion bilinear in one of these models should admit a complementary confinement interpretation [5, 18]. Thus, our assumption that all physical states are color singlets seems sensible, and the conclusion that these models contain massless composite fermions carries some force.

Much remains to be learned about the nonperturbative dynamics of chiral gauge theories. Present attempts to determine how these theories behave involve some guesswork. But we have found that formal arguments based on the $1/N$ expansion provide reinforcement for what might otherwise be expected merely on the grounds of simplicity, and put these expectations on a firmer basis.

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Figures

1. Propagators and vertices in the "double-line" notation.

2. A typical cut through a planar diagram that contributes to the meson two-point function, in the "double-line" notation.

3. A typical planar diagram that contributes to a many-meson amplitude if the number of quark flavors is of order N. Solid lines are quarks and wavy lines are gluons.

4. Typical diagrams contributing to the decay amplitudes of (a) mesons and (b) glueballs. The indices a, b, c denote quark flavors.

5. A cut planar diagram in which a "baryon-antibaryon" intermediate state appears.

6. A cut diagram in which the intermediate state $B\,B\,B^\dagger B^\dagger$ appears. This diagram is nonplanar and is suppressed by $1/N^2$.

7. A cut planar diagram of the SU(N) × SU(M) model in which the $B\,B^\dagger$ intermediate state appears. Fermions are denoted by solid lines, SU(N) gluons by wavy lines, and SU(M) gluons by broken lines.

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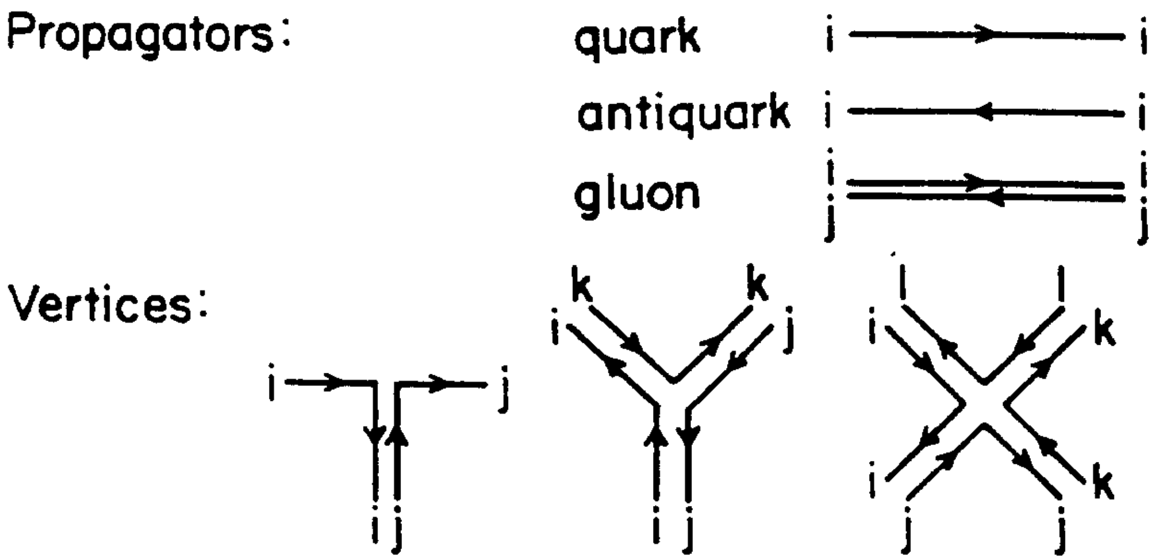

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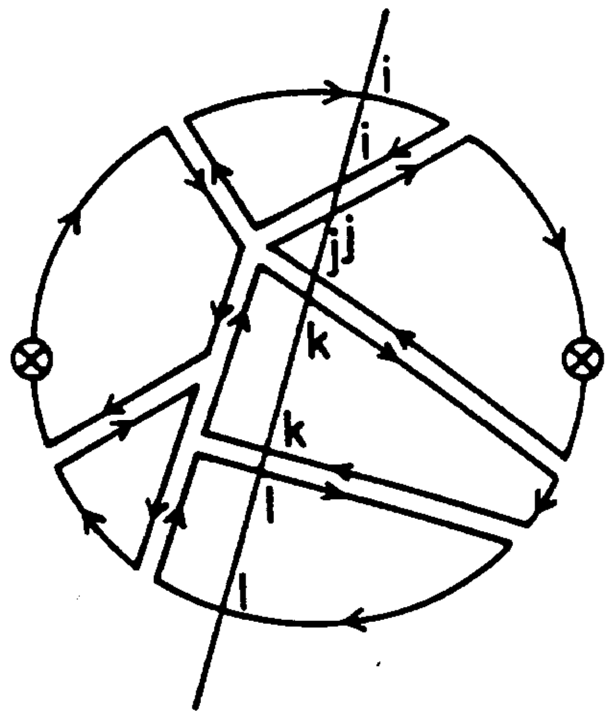

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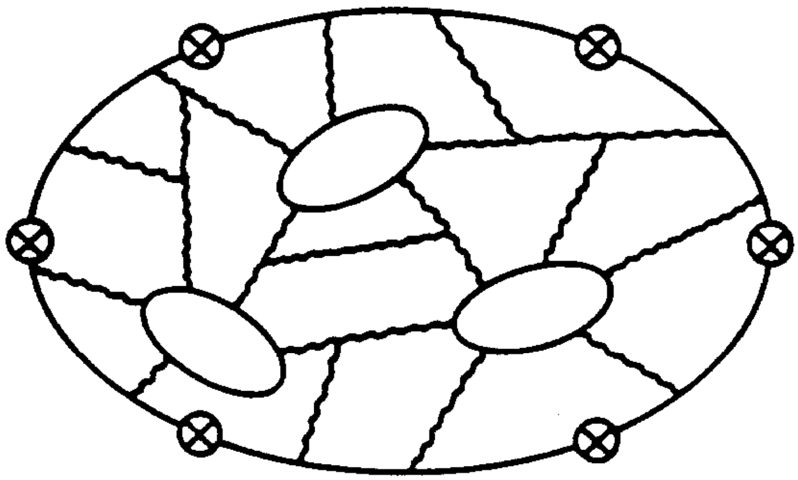

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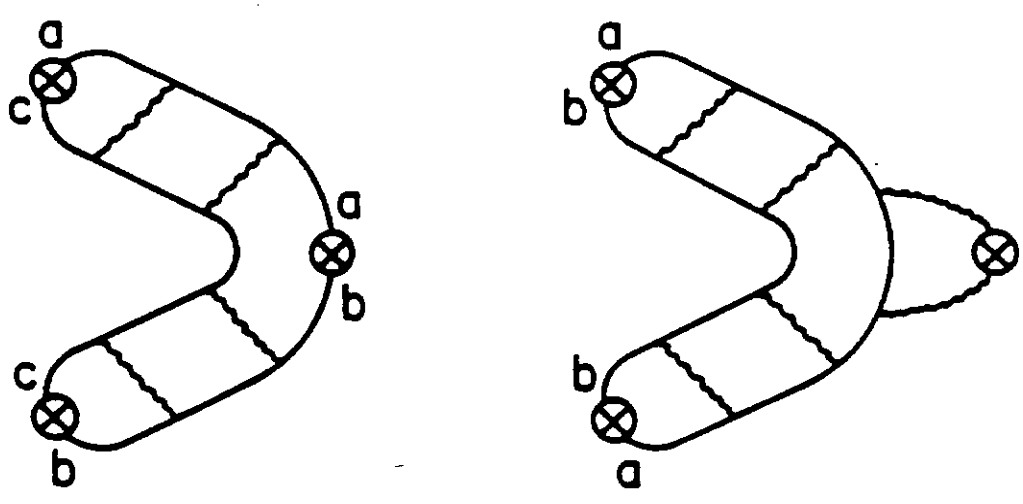
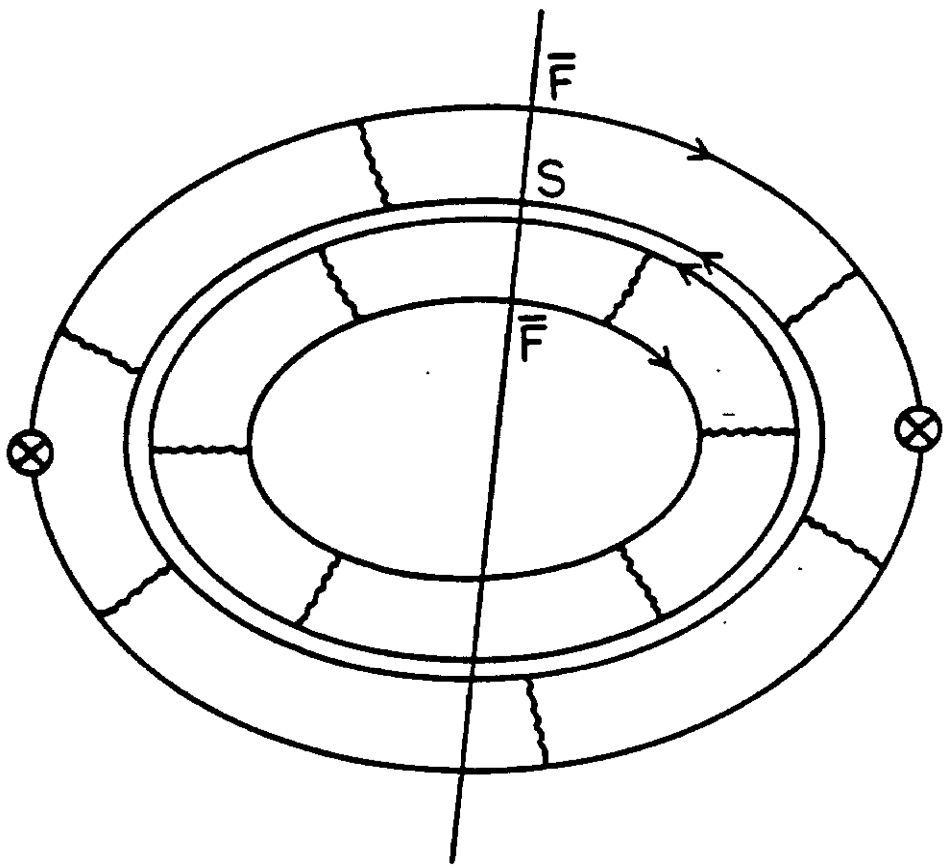


(a) (b)

(5)



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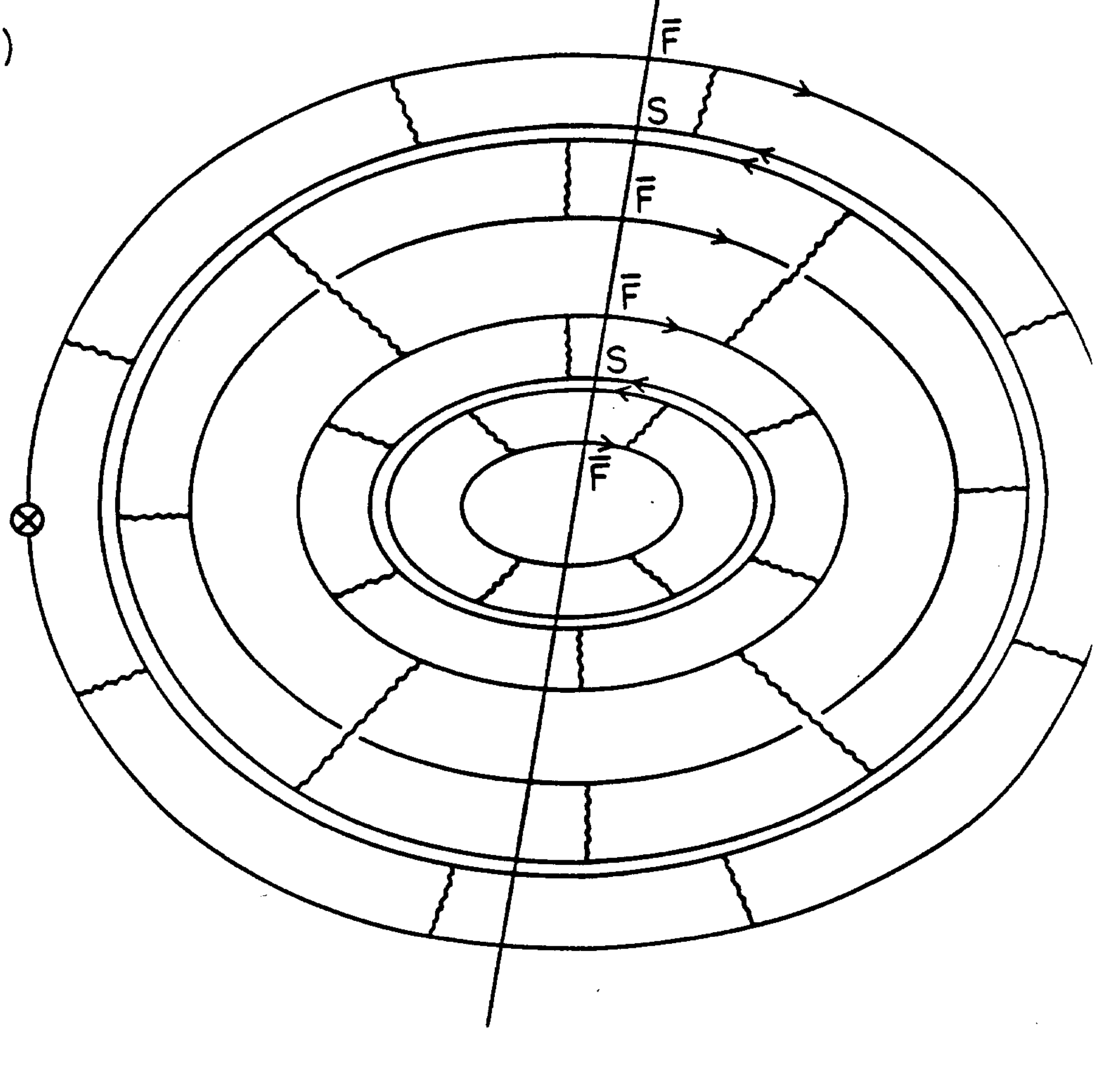

(7)