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The Cosmology/Particle Physics Interface

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ABSTRACT

We review the Cosmology/Particle physics interface focusing on two of its most active areas, inflation and dark matter.

1. INTRODUCTION

In less than a decade, the interface between elementary particle physics and cosmology has grown from oblivion to become a central arena for physics and astronomy. To attempt to review the whole interface would now require at least a full book[1] rather than a Comments article. Thus, this article will concentrate on two of the sub-topics which have shown great activity in the past year, namely, inflation and the dark matter problem. These topics are slightly related, since inflation requires a dark matter solution which is somewhat different than the traditional astronomical dark halo problem. However, before going into detail on these two problems, it is useful to cite other sub-topics where activity has occurred and give references to appropriate reviews on those topics.

The whole field opened up as the hot Big Bang model became well established with quantitative observational verifications of the 3 °K background[2] and the light element abundances.[3] In particular, the current state-of-the-art predictions of Big Bang nucleosynthesis on the abundances of ^{3}He, ^{4}He, D and ^{7}Li, range over 9 orders of magnitude and yet are not in disagreement with observed primoridial abundances. These predictions also extend to the number of neutrino flavors[4] with the current prediction of 3 with an upper bound[3] of 4. This cosmological prediction about fundamental physics will be checked[5] with measurements of the width of the Z^o in colliders. Such verification will mark the first, but hopefully not the last, time that cosmology has led to fundamental particle physics and changes astrophysics from being a parasite that uses other areas of physics without producing any new knowledge about fundamental processes.

In addition to the constraints on neutrino flavors, cosmology also limits other properties including masses, lifetimes, abundances and coupling constants for various proposed elementary particles. For a detailed review of the procedures used, see ref. 6. An area where particle physics and condensed matter physics have led to a better understanding of the early universe has been in the study of phase transitions. For a summary of the current state-of-the-art in this area, see ref 7 on phase transitions in the early universe.

Such phase transitions include electroweak symmetry breaking, super-symmetry (SUSY) breaking and/or grand unified (GUT) symmetry breaking. This latter phase transition was originally thought to be tied together with inflation, although it now seems that they must be distinct. The GUT transition is also the transition where a net baryon number is produced in the early universe[8]. SUSY breaking has become an area of intense interest recently with many theories connecting SUSY or supergravity (SUGR) to higher numbers of dimensions. Thus, even the question of how our universe achieved its present 3 spatial plus 1 timelike dimension is beginning to be studied.

The one area where particle physics and astrophysics have had a long history of interfacing is in the study of cosmic rays. Particle physicists have gone to the cosmic rays whenever energies were required in excess of what current accelerators could produce (the big bang goes even higher, but it is not accessible). The new data coming from Utah's Flys Eye are beginning to signficantly improve our understanding of the very high energy cosmic rays[9]. The interaction of these particles with the 3 °K background[10] also ties these studies into cosmological questions. In particular, the shape of the ultra high energy spectrum and its associated neutrinos may be related to the epoch of galaxy formation and energetic early sources like quasars.

We will now look in detail at two specific areas where cosmology and particle physics are very close, at inflation and the dark matter problems in connection with galaxy formation.

2. INFLATION

Instead of beginning this section with a review of the cosmological problems solved by Guth's inflationary scenario[11], (and the new inflationary scenario[12]) we would like to first look at some of the pre-Guth work that went into the making of the inflationary Universe. The earliest reference we know to a scenario that resembles what occurs in inflation, is a paper by Gliner[13] in 1966. In that paper, Gliner looks at the various possible forms for the eigenvalues of the energy-momentum tensor and their description as different types of matter. He concludes that the case when all four eigenvalues are equal (as in the case of with a cosmological constant and no ordinary matter) corresponds to "matter" with the properties of a vacuum. Hence a vacuum dominated Universe with positive energy density must correspond to a De Sitter model.

In a later paper, Gliner and Dymnikova[14] came very close to what is the present theory of inflation. In this paper they assume a transition from a vacuum dominated state to a radiation dominated one. Their idea was actually to remove the initial singularity with De Sitter space. Their model is then restricted by ensuring that the total entropy of the Universe agrees with the observed entropy. They also choose two possible values for the energy density of the vacuum: 1) the scale set by weak interactions $\rho \sim (10^2 GeV)^4$; and 2) the planck scale $\rho \sim (10^{19} GeV)^4$. Although grand unification was introduced[14a] a year earlier, they can hardly be faulted for not discussing a GUT transition. Their results show that the transition produces an enormous growth of the scale factor and indeed for the planck-scale vacuum, there is a change in the scale factor by about 30 orders of magnitude, remarkably similar to the present goals of inflationary models.

More recently, but still prior to the observations of Guth, a number of papers came very close to the inflationary scenario without really hitting it. What they lacked in terms of explaining the present abundance of entropy, they made up for in being much more explicit in terms of a detailed phase transition. Kolb and Wolfram[15] studied the cosmological consequences of the SU(2) x U(1) phase transition and showed in detail that for a first order transition, the Universe could have been dominated by the vacuum energy density of the symmetric phase and that acting like a cosmological constant, the expansion rate of the Universe was exponential rather than a simple power law. In addition, they noted that if strong enough, the phase transition could produce a great deal of entropy and perhaps even density inhomogeneities.

Sato[16] also studied the effects of a first order phase transition in the early Universe. Looking at a GUT phase transition, he showed that the horizon could be stretched exponentially large but was mainly concerned with domain walls, due to spontaneously broken CP and preserving a baryon symmetric Universe. Such a scenario, however, has little hope in deriving a baryon to photon ratio of the right order of magnitude. In a second paper, Sato[17] looked carefully at the mechanism in which the phase transition proceeds, i.e., through the nucleation of bubbles. He realized that unless the nucleation rate was fairly large, such a phase transition might never be completed. A preview to the fate of the original model of inflation.

Independently, Kazanas[18] also showed that the effects of a first order transition could have greatly changed the expansion laws of the early Universe. More importantly, Kazanas had asked whether or not the exponential expansion could have lasted long enough to account for the observed isotropy of the Universe today, i.e. one of the key problems which inflation sets out to solve.

This brings us then to the inflationary Universe model of Guth[11]. The motivation for such a model was to solve two classical cosmological problems: the horizon problem and the flatness problem. Guth's initial motivation, however, was to find a solution to the monopole problem[19]. As we will see, there are several other problems which are potentially solved by inflation as well. The horizon problem basically refers to the question of the high degree of overall isotropy observed today. The horizon volume or the volume of a casually connected region today is simply related to the present age of the Universe $V_o \propto t_0^3$. The microwave background radiation with temperature $T_\gamma \sim 3\ {}^\circ K$ has been decoupled from itself since the epoch of recombination at $T_d \sim 10^4\ {}^\circ K$. The horizon volume at that time was $V_d \propto t_d^3$. Now the present horizon scaled back to the time of decoupling will be $V_o' = V_o(T_o/T_d)^3$ and the ratio of this volume to the horizon volume at decoupling is

$$V_o'/V_d \sim (V_o/V_d)(T_o/T_d)^3 \sim (\frac{t_o}{t_d})^3 \sim 10^5 \tag{2.1}$$

Where we have taken $t_d \sim 3\times10^{12}$ s., $t_o \sim 5\times10^{17}$ s. This ratio corresponds to the number of causally disconnected regions at recombination which grew into our present visible Universe. The problem is that it is difficult to understand why all of these regions had the same temperature. The limits on the anisotropy of the microwave background radiation indicates that on small scales[20] (but still causually connected at recombination)

$$\Delta T/T \leq (2\text{–}5) \times 10^{-5} \tag{2.2}$$

(Even on the largest scales[21] $\Delta T/T \leq 10^{-3}$)

Why 10^5 causually disconnected regions all had the same temperature at T_d is the horizon or isotropy problem.

The flatness problem refers to the lack of certainty as to the overall geometry of the Universe, i.e., we do not know if the Universe is open or closed. To see this, let us go back to the Friedman equation governing the expansion rate of the Universe,

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi\rho}{3M_p^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{2.3}$$

Where H is the Hubble parameter, a is the Robertson-Walker scale factor, ρ is the total mass energy density of the Universe, $M_p = 1.22\times10^{19}$ GeV is the Planck mass; $k = \pm1,0$ is the curvature constant determining whether the Universe is open (-1), closed (+1) or flat (0), and finally, Λ is the cosmological constant. Neglecting the cosmological constant, we can rewrite (2.3) as

$$k/a^2 = (\Omega-1)\ H^2 \tag{2.4}$$

where

$$\Omega = \rho/\rho_c \tag{2.5}$$

is the cosmological density parameter and

$$\rho_c = \frac{3H^2}{8\pi} M_P^2 = 1.88 \times 10^{-29} h_o^2 \ g cm^{-3} \tag{2.6}$$

is the critical energy density today and h_c is the present value of the Hubble parameter

$$h_c = H_o/(100 km \ Mpc^{-1} \ s^{-1}) \tag{2.7}$$

Observational limits indicate only that $\Omega<4$ and $h_o<1$. It is possible to form a dimensionless constant from (2.4) if we assume that the expansion of the Universe has always been adiabatic $a \propto T^{-1}$

$$\hat{k} = \frac{k}{a^2 T^2} = (\Omega - 1) \ H_o^2/T^2 < 3H_o^2/T_o^2 \lesssim 2 \times 10^{-58} \tag{2.8}$$

If $\hat{k}$ were any larger the Universe would either have expanded away or recollapsed long ago. Equation (2.8) then becomes an initial condition to one of the parameters in the standard cosmological model. The smallness of k or the proximity of Ω to 1, initially represents the flatness or curvature problem. Other cosmological problems which can be solved by inflation include the monopole problem[19], the rotation problem[22], the gravitino problem[23] and perhaps, the origin of the initial fluctuations that eventually form galaxies and clusters[24].

The key to Guth's inflationary model is then to break the assumption that the Universe has always been expanding adiabatically. If, for example, there was a first order phase transition in the very early stages of the Universe in which the scale factor, a, increased many orders of magnitude relative to T, both the horizon and flatness problems would be solved, in particular, Ω would be driven naturally to one. In Guth's original model, the phase transition most natural to consider was the breaking of SU(5) to SU(3) x SU(2) x U(1). If the scalar potential for the Higgs field Σ (a 24 to break SU(5)) had a shape such as that given in Fig. 1, the Universe would have supercooled to the state with $<\Sigma> = 0$. The barrier between $<\Sigma> = 0$ and the global minimum at $<\Sigma> = \upsilon$ prevents the transition from occurring rapidly. Because the energy density of radiation falls as $\rho \sim T^4$, eventually, the constant vacuum energy density $V_o \equiv V(0)$ begins to dominate the expansion rate of the Universe. V_o acts then as a cosmological constant

$$\Lambda = 8\pi V_o/M_p^2 \tag{2.9}$$

the expansion then shifts from a simple power law $a \sim t^{1/2}$ to an exponential

$$a \sim \exp(Ht) \tag{2.10}$$

If the timescale for completing the transition τ, is long enough so that $H\tau > 65$, (a would be increased by a factor $e^{65} \approx 10^{28}$) the cosmological problems would be solved.

In the original inflationary scenario, the phase transition given by a potential with a large barrier as in Fig. 1, proceeds via the formation of bubbles[25]. The Universe would reheat, i.e., the release of entropy must occur through bubble collisions and the transition is completed when the bubbles fill up all of space. It is now known[26], however, that the requirement for a long timescale τ is not compatible with the completion of the phase transition. The Universe as a whole remains trapped in the exponentially expanding phase containing only a few isolated bubbles of the broken SU(3) x SU(2) x U(1) phase.

The well-known solution to this dilemma is called the new inflationary scenario[12]. If the shape of the potential V (Σ) resembles that of Fig. 2 rather than Fig. 1, the phase transition would proceed not through the formation of bubbles, but rather, by the long rollover in which Σ picks up a vacuum expectation value. During the rollover, the vacuum energy density would remain essentially constant for a long period of time triggering the exponential expansion. Completion of the transition is thus guaranteed and reheating occurs through the dissipation of energy due to field oscillations about the global minimum. Early models for new inflation utilized Coleman-Weinberg[27] type symmetry breaking for SU(5). These too, turned out to be problematic as we will soon see.

A scalar potential with a shape as in Fig. 2, is subject to several requirements[28] in order to produce a satisfactory model for inflation. We will here, only outline the two key requirements. The first is obviously that the rollover time scale τ be long. The timescale is generally determined by the classical equations of motion for a field ϕ moving under the influence of a potential $(V\phi)$

$$\ddot{\phi} + (3H+\Gamma)\,\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \tag{2.11}$$

where Γ is the rate of interactions of the scalar field ϕ. Initially we can neglect $\ddot{\phi}$ and the $\Gamma\dot{\phi}$ term is only relevant for $\Gamma \geq H$. Inflation (a slow rollover) can only occur if $\Gamma < H$ so that $3H\dot{\phi} + \partial V/\partial\phi = 0$

$$\tau^{-1} \approx \dot{\phi}/\phi \sim (\partial^2 V/\partial\phi^2)/3H \tag{2.12}$$

Hence near the origin $\phi < H << v$, we must require

$$\partial^2 V/\partial\phi^2|_{\phi\approx o} < 3H^2/65 \tag{2.13}$$

i.e., we must have a flat potential near the origin.

A second key constraint concerns the production of density fluctuations during the phase transition. In general there will be a time spread over which in certain regions of space, ϕ rolls down faster or slower than in others. Density perturbations have been calculated[24] in terms of this time spread

$$\frac{\delta\rho}{\rho} \propto H\delta\tau \tag{2.14}$$

where $\frac{\delta\rho}{\rho}$ is the magnitude of the perturbation as it enters the horizon and $\delta\tau$ is calculated in terms of H and ϕ. Limits coming from the istotropy of the microwave background radiation imply that

$$\frac{\delta\rho}{\rho} \leq 10^{-4} \tag{2.15}$$

These two constraints are alone sufficient to rule out SU(5) - Coleman- Weinberg type inflation. Mass scales in GUTs tend to be much larger than required by (2.13), and $\frac{\delta\rho}{\rho}$ is calculated to be about five orders of magnitude larger than (2.15), although the spectral shape is of the Harrison-Zeldovich type which has been argued as good for galaxy formation. Other maladies are present as well.[29,30]

In the past few years, two variants on the new inflationary scenario have emerged: 1) primoridal supersymmetric inflation[31] and 2) chaotic inflation.[32] Primoridal inflationary models are simply those in which the scalar field responsible for inflation is no longer associated with the 24, of SU(5), but rather with some other field ϕ, dubbed the inflaton, which picks up a vacuum expectation value $<\phi> \; >> \; <\Sigma>$, thus allowing for a longer rollover timescale. Supersymmetry offers the possibility for keeping small those radiative corrections responsible for large scalar masses, i.e. supersymmetric theories naturally have flatter potentials.[30,31]

Chaotic inflation[32] which may or may not be supersymmetric, is an interesting variant of the new inflationary model. The potential for the inflaton may be something as simple as $V(\phi)=\lambda\phi^4$. The idea is that, at early enough times, the initial value for ϕ in some regions of space may be quite large ($\phi > M_p$) for small enough λ. In this case the rollover occurs as ϕ tends toward the minimum at $\phi = 0$. The magnitude of λ is determined again by conditions like (2.13) and (2.15). The major problem with this type of scenario is that unless the gradiant of ϕ, $\partial_\mu\phi$ is very small, $(\partial_\mu\phi)^2 \lesssim \lambda\phi^4$, inflation cannot occur. This requires the length scale for the uniformity of ϕ to be at least 10^5 H^{-1}, i.e. much larger than the scale of the horizon.

In all models of new inflation, the Universe is reheated due to the dissipation of the coherent field oscillations about the global minimum.[33] Thus the inflaton must not be too decoupled from the rest of the particle world. If Γ_ϕ denotes the rate of interactions between ϕ and other fields, it is not hard to show that for $\Gamma_\phi < H_I$, where H_I represents the value of H at the onset of inflation, then the temperature to which the Universe reheats is given by

$$T_R \sim (\Gamma_\phi M_p)^{1/2} \tag{2.16}$$

If M_H is the mass of the lightest Higgs boson carrying baryon number violating interactions, then the baryon to photon ratio produced after the reheating is

$$\frac{n_B}{s} \sim (T_R/M_H)\epsilon \tag{2.17}$$

where ϵ is the net baryon number produced in the decay of a H,H^* pair. For $n_B/s \gtrsim 10^{-10}$ we have the limit $T_R \gtrsim 10^{-10} \; M_H/\epsilon \; \geq \; 10^4 GeV$ for $M_H > 10^{10}$ GeV and $\epsilon < 10^{-4}$.

Because of its capability of producing flat potentials, supersymmetric inflation has been of great interest lately. For example, attention has been focused on finding a theory in $N = 1$ supergravity which has inflationary capabilities.[34] In these theories one specifies a superpotential f from which one derives[35] the scalar potential V(ϕ)

$$V(\phi)=e^G\left[\frac{\partial G}{\partial\phi}\frac{\partial G}{\partial\phi^*}\left(\frac{\partial^2 G}{\partial\phi\partial\phi^*}\right)^{-1}-3\right] \tag{2.18}$$

where

$$G \;=\; \phi\phi^* \;+\; \ln|f|^2 \tag{2.19}$$

is the Kähler potential in minimal N = 1 supergravity, f must then be chosen so that V(ϕ) complies with the conditions for inflation discussed earlier. The simplest example for f, satisfying the inflationary requirements, is[36]

$$f \;=\; m^2\left(1-\frac{\phi}{M_p}\right)^2 M_p \tag{2.20}$$

The mass scale m is determined to be O (10^{-4}) from the calculation of $\delta\rho/\rho$.

Because the inflaton is generally taken to be a gauge non-singlet, there is an additional concern regarding initial conditions. Normally, one expects that symmetries are restored at high tempertures so that a natural initial condition for $<\phi>$ is $<\phi>=0$. Inflation then takes place as $<\phi>$ moves from 0 to $v\approx M_p$ In these models "symmetry restoration" must be added as an additional constraint[37]. Namely, we must also require that at very high temperatures there exists a minimum at $\phi=0$. It has been shown,[38] however, that no choice of a superpotential can simultaneously satisfy the inflationary constraints and the thermal constraint.

Another constraint on the initial conditions comes from the fact that at high temperatures ϕ is not localized near $\phi\sim 0$ but rather, $\phi\sim T$ and hence, as the Universe cools ϕ may fall directly to the global minimum at $\phi=v$[39]. Recently, it has been shown that these difficulties are most easily avoided in models of primordial inflation.[40]

A route to tackle the problem regarding the thermal constraint and finding a model with a suitable superpotential is to look at non-minimal supergravity models, i.e., those in which a

$$\partial^2 G/\partial\phi\partial\phi^* \;\neq\; 1 \tag{2.21}$$

If, instead, one considered[41] a general form for G

$$G = g_\phi\,(\phi + \phi^*) \;+\; g_\phi^\phi\;(\phi\phi^*) \;+\; g_{\phi\phi}\;(\phi^2 + \phi^{*2}) + g_{\phi\phi}{}^\phi(\phi\,\phi^*)\,(\phi + \phi^*) + \cdots \tag{2.22}$$

where g_ϕ, g_ϕ^ϕ etc. are couplings. It is then possible to derive relations between the g_ϕ's to satisfy the inflationary constraints as well as the thermal constraint. The extra degrees of freedom in G allowed the constraints to be satisfied without the inclusion of several scales.

There are also specific non-minimal models based on SU(n,1) supergravity[42] which satisfy all constraints.[43] In these models the Kähler potential has the form

$$G \;=\; -3\ln(z+z^*-\phi\phi^*/3)+|\ln|f|^2 \tag{2.23}$$

where z is the field which breaks supergravity (see references[44] for a discussion on the z field in minimal supergravity theories) ϕ is the inflaton and f is the superpotential. In these theories it is possible to write down a superpotential as in eq. 2.20 which in addition satisfies the thermal constraint[43]

$$f \;=\; m^2(\phi-\phi^4/4M_p^3). \tag{2.24}$$

SU(n, 1) models also prove to supply very simple superpotentials for the chaotic inflationary scenario as well[45]

$$f \;=\; \lambda^{1/2}\phi^3 \tag{2.25}$$

Hence, it appears that supergravity indeed, can supply some simple models for the inflationary scenario.

To conclude this section, we note that other types of inflationary models are available. Here we can only list the different types. These include the non-singular cosmologies which begin with a De Sitter inflationary period[46], models employing SU(5) singlets which can also help solving the strong CP problem[47], inflation driven by exotic objects such as doman walls[48], and inflation produced in Kaluza-Klein models utilizing extra dimensions[49]. In addition, Gott [50] has shown that something like the inflationary scenario is needed for purely geometric reasons.

3. DARK MATTER

There are several different dark matter problems in astrophysics.[51] The first is the local disk problem.[52] where local dynamics implies that there are about equal amounts of unseen matter and observed matter. However, this problem is probably connected to baryons in low luminosity forms in the disk (white dwarfs, low mass stars, "Jupiters") and may not have any connection to cosmology nor particle physics. There are only limits which rule out massive objects and frown on black holes[53]. The other three classes of dark matter problems are on larger scales and will be the focus of the rest of this section. They include:

(1) Dynamical Halos (including evidence from rotation curves, dwarf spheroidals and binaries and small groups and large clusters)

(2) Galaxy Formation and Clustering

(3) The $\Omega = 1$ of inflation.

Each of these will be discussed.

It should be emphasized that at least some of the dark matter must be baryonic, and in fact all of the halo material could, in principal, be baryonic. However, if problems (2) and (3) are real then we are forced to require additional non-baryonic material. Simple one particle dark matter hypotheses, whether "hot" or "cold" or "warm", are shown to not simultaneously solve all 3 problems and even hybrids of two kinds of stable non-baryonic matter fail. ["Hot" particles are those which are relativistic, like 10 eV neutrinos, until shortly before recombination; "cold" particles are ones which are slow moving well before decoupling like 10 GeV gravitinos or axions; and warm ones are in between.] More complex "ugly" solutions can be made to work but, at present, each needs more fine tuning than one would like. Solutions with strings, seeds from the quark-hadron-chiral symmetry transition, decaying particles and light not being an unbiased tracer of mass will be discussed.

(a) Halos

The classical dark matter problem is the now well established observational fact that galaxies have dark halos. This problem is nicely described and documented in numerous reviews[54] so it won't be gone into detail here. A simple summary of the results will suffice. Basically, galactic masses, M, are measured using simple dynamics,.

$$M \sim v^2 r/G \tag{3.1}$$

where v is the orbital velocity, r the separation distance and G, Newton's constant. When this is applied to the visible regions of spiral galaxies, the typical mass obtained is $\sim 10^{11} M_\odot$ with a mass-to-luminosity ratio M/L $\sim 10 h_o$ in solar units. Even this M/L is in excess of what is directly seen.

When equation (3.1) is applied to binaries and small groups, it is found that the implied masses increase by a factor of $\sim$ 10 while the amount of light is not increased at all, thus M/L approaches $\sim 100 h_0$. This is known as the dark halo problem. The mass must be there so it is not the mass which is missing but the "light", thus Steigman and DNS[55] referred to it as the "missing light problem". The need for dark halos has also been discussed on theoretical grounds[56] as necessary for disk stability. As well described in the reviews, it is also supported by measurements of rotation velocities versus radius for distant material such as stars and gas as well as other galaxies. While mentioning dark halos it is important to note that dark halos may even surround small dwarf spheroidal galaxies[57] as well as spirals and ellipticals. If true, this has important implications on what material could form these halos since phase- space arguments[58] would not allow neutrinos to work on these small scales.

As we go to the still larger scales of large clusters and superclusters the apparent mass per galaxy and thus the best estimate for M/L continues to rise, however, the uncertainties and scatter in the data also increase. The range for M/L's implied from these large scale measurements using the virial theorem (where averages for $<v^2>$ and $<r>$ are used in eq. (3.1) and from looking at the deviations in the Hubble flow caused by infall into the Virgo cluster[59] is from $\sim 100 h_o$ to $\sim 500 h_o$. Nothing gives a significantly larger M/L. It should also be noted that whether M/L keeps rising beyond $\sim 100 h_o$ or not at large scales is still not unambiguous.

The M/L's can be made cosmologically relevant by multiplying by

$$\mathcal{L}=2\times 10^{8}h_{o}L_{\odot}/Mpc^{3} \tag{3.2}$$

the average luminosity density[60] (care needs to be taken to use the same filter bands for $\mathcal{L}$ and the L's in the M/L's, different M/L's than listed are frequently quoted but they correspond to a different $\mathcal{L}$ thus maintaining the resultant product.) The product $\rho=M/L\cdot\mathcal{L}$ is the implied matter density *if* that M/L applies to the average light in the universe. The density parameter Ω thus obtained is independent of h_o since $\rho_{crit}\propto h_o^2$ and $(M/L)\mathcal{L}\propto h_o^2$. The results are summarized in Table I. Note that since most galaxies are not in the largest clusters, their M/L may not be associated with L, but perhaps, is only related to some special process involved in forming these things. Thus, while we can say with some confidence that $\Omega\gtrsim 0.07$, we are not forced to make it significantly larger on the grounds of unambiguous observational evidence. Note also that while the M/L and the implied Ω do tend to rise with scale, no observation yields an implied Ω of unity or larger. The only way to achieve an $\Omega\gtrsim 0.4$ would be to have significant amounts of material that does not cluster within the bounds of the largest clusters.

(b) Galaxy Formation

To form a galaxy requires a density fluctution, δn_b in the baryon density, n_b. Such a fluctuation can come from a primordial fluctuation or it can be created by shocks coming from explosions of pre-existing seeds[61], with the origin of the seeds still requiring some primordial occurrence. Classically[62], two kinds of primordial fluctuations were discussed;

1) isothermal, where $n_\gamma = constant$

2) adiabatic, where $n_b/n_\gamma = constant$

where n_γ is the photon density.

We now know that baryons can be produced by GUT interactions in the early universe[8] and we have no other convincing way to produce the observed excess of baryons over antibaryons. Turner, DNS and Press[63] noted that such production is only easy to make compatible with primordial adiabatic fluctuations since in such schemes n_b is a unique function of temperature T, thus δn_b must be accompanied by a δT yielding a δn_γ. Once primordial adiabatic fluctuations are accepted then there is a direct connection between $\delta n_b/n_b$ and the hoped-to-be observed variations in the 3 °K background. As mentiond earlier, recent limits on the 3 °K anisotropy[20] tell us that

$$\delta T/T \leq 2\times 10^{-5} \tag{3.3}$$

at the decoupling of the radiation from the matter which occurs at T $\sim$ 3000 °K. We know that density perturbations grow linearly with 1/T in an expanding universe once the universe is matter dominated. (Growth will cease in an open universe at redshift z $\sim 1/\Omega$). Since baryons are coupled to the radiation, their perturbations must be small at T $\sim$ 3000 °K. (Naively $\delta\rho/\rho \approx 3\delta T/T$ but detailed calculations[64] through the decoupling epoch taking into account the averaging techniques in the measurements show that the proportionality is a little different from 3.)

We know that at the present epoch of T $\sim$ 3 °K, that density variations $\frac{\delta\rho}{\rho} \gtrsim 1$ exist on scales up to at least the large clusters of galaxies. Linear growth tells us that this requires $\frac{\delta\rho}{\rho} \geq 10^{-3}$ at $T \sim 3000$ °K. But, from limits on $\delta T/T$ it is known that $\delta n_b/n_b$ syntax error file CosmoPaper, between lines 688 and 688 must be $<<10^{-3}$ at $T = 3000$ °K, thus $\delta\rho/\rho >> \delta n_b/n_b$ *at* $T \sim 3000$ °K. We are forced to non-baryonic matter if we assume adiabatic perturbations and linear growth. Detailed calculations[64] even with non-baryonic matter, have noted that growth is cut by a factor $\sim 1/\Omega$ and find that $\Omega \sim 1$ (at least $\Omega > 0.4$) is required to get $\delta\rho/\rho \approx 1$ today. [Remember once $\delta\rho/\rho \geq 1$, non-linear growth can occur so the existence of some objects with $\delta\rho/\rho >> 1$ is not a problem unless the scale is so large that $\delta\rho/\rho$ could not have reached unity.]

(c) Galaxy Clustering

An important constraint on the dark matter involved with galaxy formation is how galaxies are clustered and how clusters are clustered. There are two important considerations here. The first is the galaxy and cluster correlation functions. The second is the existence of large scale filaments and voids. Let us begin with the latter.

Although there is still no unambiguous, unbiased statistical study of the problem, there is definitely a growing trend among observers to note large scale holes in space and to note the lining up of the largest clusters along filamentary lines.[65] The scales of such ordering corresponds to mass scales $\delta M > 10^{16} M_\odot$. Such structure requires density fluctuations $\delta\rho/\rho$ exceeding unity on extremely large scales. It has been estimated[66] that with random fluctuations the probability of such large scales having $\delta\rho/\rho \geq 1$ so that non-linear growth can set-in is about equivalent to a 4σ event.

The existence of these very large scales has been used by some to argue for neutrinos[68,55,67], (or other "hot" matter) as the dark matter candidates or to favor non-random phases.[69,70] However, it may also be possible through statistical fluctutions to obtain a few rare such cases in "cold matter" scenarios.[71] The test will be whether larger surveys reveal these very large structures to be rare or common.

The use of 2, 3, and even 4 point correlation functions has been developed by Peebles[62] and his co-workers to a fine art that has now become a cornerstone of modern cosmology. In particular the 2-point galaxy-galaxy correlation function $\xi(r)$ which is defined as the excess probability over random for a galaxy to be at a distance r from another galaxy is found to be proportional to $r^{-1.8}$ which is equivalent to a fractal of dimension 1.2. That is, galaxies do not fill all space and they are correlated. The correlation may deviate from this power law at large scales and may even go negative[66] for r $\gtrsim 20 Mpc$. It is also interesting that the 3-point function is what one would expect for a hierarchical clustering scenario where large scale builds up from small. This used to be a strong argument in favor of primordial isothermal fluctuations before grand unified theories, since a pure baryonic isothermal model produced hierarchical clustering whereas a pure baryonic adiabatic model produced large scales first and required fragmentation. However, we now know that hierarchical clustering can be achieved with cold (or warm) particles in adiabatic scenarios. In addition, Fry[72] has shown that scenarios which produce large scale filaments will also yield a 3-point function which fits the data.

An exciting new result by Bahcall and Soniera and by Klypin and Khlopov[73] following earlier explorations by Hauser and Peebles[74] is the recognition that the correlation function between clusters also has the $r^{-1.8}$ power law dependence but is ~ 20 times stronger than the galaxy-galaxy function on the same scale and is definitely non-zero on scales up to at least 100Mpc. This seemed somewhat perplexing, and was not a simple quantitative consequence of pure baryonic nor "cold" nor "hot" models.[75] One possible explanation was that clusters are 3σ effects[76] and the correlation of such effects would be significantly enhanced over the rest of the fluctuations. Such an interpretation would mean a proportional amplication, thus if the galaxy-galaxy function goes negative at ~ 20 Mpc, so should the cluster-cluster function. This seems to contradict the observations, however, both the negativity at 20 Mpc in the galaxy-galaxy function and the strength of the cluster-cluster function at large scales are not yet beyond question. More observational work is clearly required. However, it should be remembered that any reasonable galaxy formation fluctuation spectrum yields negative galaxy-galaxy correlations at a few tens of Megaparsecs and the cluster-cluster function is definitely positive on these scales.

An alternative way to look at the cluster-cluster versus galaxy-galaxy functions is use a dimensionless approach[61,77]. In particular, instead of using the same units for r for both galaxies and clusters one "renormalizes" and uses a unit of the average separation distance of the object being studied. In these dimensionless units the cluster-cluster function is actually weaker than the galaxy-galaxy function by a factor of about 3. But as one goes to higher richness, clusters with longer dimensionless length scales, the renormalized amplitude stays roughly constant (in Mpc units, higher richness classes yield stronger correlations). Such a renormalized approach also means that negative correlations in the galaxy-galaxy function at 20Mpc might not manifest themselves onto the cluster-cluster function until $\sim$ 200Mpc since the renormalized length units for clusters are $\sim$ 10 times those for the galaxy sample. If the dimensionless approach has any physical merit it must mean that there is some other physical process at play on large scales which is scale free. Since the $r^{-1.8}$ or 1.2 fractal character holds in both limits this would imply that the process giving the scale free character is the 1.2 fractal producing process. Possible physical processes which yield large linear scale structure include explosion percolation[61,78,78a] or strings.[79]

(d) Inflation

In the previous section, we discussed inflation and how it required $\Omega=1$. This yields a different dark matter problem since it requires matter that is outside of galaxies and clusters, since galaxies and clusters dynamics never yield $\Omega>0.4$. Note that even given the many different scales for Ω in table 1, they are all different than the universal value $\Omega=1$.

(e) Baryons

In addition to the arguments presented above, there are other constraints on what normal baryonic matter can do for these problems. In particular, a detailed comparison of the state-of-the-art Big Bang Nucleosynthesis calculations and the current observed abundances yields[80] an extreme upper bound on the baryonic density, Ω_b, of 0.19 with a reasonable bound put at 0.14. These limits can be lowered to 0.15 and 0.10 respectively if the new limits[80a] on the background temperature are used. Yang, et al.[80] also point out the existence of an extreme lower bound on Ω_b of $\sim$0.01. This lower bound can be tightened[67] $\Omega_b>0.03$ using limits on the age of the universe from nucleochronology and globular clusters. This range on Ω_b is intriguing. On the one hand it tells us that the halos even in the large clusters can be completely baryonic. On the other, it tells us that at least some of the baryons are not shining. We know that some of these non-optically shining baryons are shining in x-rays as evidenced by the x-ray gas associated with large clusters[81]. If all the galaxies have as many non-optical baryons associated with them as do the ones in large clusters, then we know the answer to the dark halo problem[82] - baryons. However, it has been argued[83] that it would take a very peculiar baryonic object to work. Jupiters or low mass stars work, but only if produced in large excess of any extrapolation of observed stellar initial mass functions. Similarly, stellar mass black holes work only if they're not produced with accompanying heavy element producing supernovae. (Stellar mass black holes count as baryons since they would have been in the form of baryons during Big Bang nucleosynthesis.) The safest possibility lies in very massive ($M > 100 M_\odot$) black holes which were formed by gravitational instabilities leaving no ejected material. But as we have seen, although baryons might solve galaxy formation with adiabatic fluctuations, they cannot account for $\Omega = 1$ from inflation and they do not naturally give the large scale structure.

(f) Candidates

Let us now examine possible solutions to the cosmological dark matter problems. Single particle non-baryonic candidates have been divided into "hot", "cold", and "warm" following Bond and Szalay[68]. The logic to this division comes from the effective Jeans mass[84]

$$(M_J)_i = \frac{3 \times 10^{18} M_\odot}{m_i^2 (eV)} \tag{3.4}$$

This is the smallest scale which can initially collapse when particle i first dominates the mass density of the universe. At times earlier, when the temperature T $>$ m_i species i would be relativistic and damp out all adiabatic fluctuations out to the horizon. m_J is related to the horizon mass at $T \approx m_i$. For light-hot paticles, m_J is large and large cluster scales form first and eventually fragment to make galaxies, for heavy-cold particles,[85] M_J is small, so small scales can form. [Axions[86] have a small mass but were never in thermal equilibrium so they have a low velocity and thus a small M_J.] Table II lists various proposed particles and their classification.

Massive neutrinos are the least exotic of the proposals since they are known particles and although their massiveness is not required it is also not forbidden. Since neutrino interactions and spins are well known, it is easy to calculate the exact density of them produced in the big bang (cf. ref. 6 and references therein). In particular it can be shown that they decouple at $\sim$ 1 MeV so their present temperature will be $\sim$2 °K compared to a photon temperature of 3 °K, due to subsequent e^+e^- annihilation heating the photons relative to the neutrinos. The net result, including spin factors, is that the number density of a neutrino species $\nu_i + \bar{\nu}_i$ is $\sim 150/cm^3$ as compared to $\sim$ 450 for photons. Other more weakly interacting species like gravitinos decouple sooner allowing more annihilations to heat up ν's and γ's. Therefore, the temperature and number densities of these ultra weak species will be still lower[87], thereby allowing larger single particle masses without exceeding cosmological density limits.

Planetary mass black holes behave just like any elementary cold particles,[88] but their production requires a first order phase transition to occur when the cosmological horizon exceeds $\sim 10^{15} g$, so the black holes don't disintegrate via the Hawking process, and yet they must form before nucleosynthesis if the light element abundances are not to constrain their total density. The two transitions that fall into this range are the electro-weak (T $\sim$100 GeV) and the quark-hadron-chiral symmetry transitions at T $\sim$1 GeV. The possible production of planetary black holes has been discussed in each.[89,90]

At present, it appears that the quark-hadron-chiral transition might be first order and thus, may have some chance. The electro-weak does not appear capable of significant planetary mass black hole production[90] Quark nuggets as proposed by Witten[90a] would behave similarly to these black holes and may instead form at the quark-hadron transition.

It has been noted[61,67,71,91,91a,92] that neither a single cold nor a single hot, nor even a single warm[93] particle can simultaneously solve all three cosmological dark matter problems in the simple model with non-interacting free particles undergoing gravitational clustering.

Hot particles have $M_J > 10^{16} M_\odot$ so they give the large scale structure and their large clustering scale can put the bulk of them outside of the largest clusters thus enabling Ω=1 without conflicting with the observation that Ω cluster $\sim$0.2 to within a factor of 2. However, such models need to have galaxies form late[94] ($z \leq 1$) which conflicts with observations of quasars at $z \geq 3.5$. Thus, they don't make galaxies well. In addition, phase space arguments prevent them from being the dark halo matter of dwarf spheroidals,[67] but that is not critical since we know some dark baryons must exist somewhere. While cold matter has received much praise recently[71,91] due to its being able to solve the galaxy formation problem and fit galaxy-galaxy correlations as well as serve as halos even on the small scales of dwarf galaxies, it does have the serious flaw[67,95] of putting all of its matter on scales that should be measured by cluster dynamics if light traces mass in some unbiased way. Thus, if $\Omega_{cold} \sim 1$, then $\Omega_{cluster} \sim 1$ in conflict with observations. No warm particle mass[93] has been found which doesn't fall into either the cold or hot difficulty. Thus, there is no simple solution. Hybrid two particle models have also been tried using a hot and a cold particle.[91a] These also fail because the hot particles will damp out the growth of the cold density fluctuations until the hot particles become non-relativistic.[96,67] Such damping occurs unless $\Omega_{cold} >> \Omega_{hot}$ but from observations $\Omega_{cluster} < 0.4$, thus $\Omega_{cold} < \Omega_{hot}$ if $\Omega \sim 1$.

This dilemma is now forcing various groups to look at more complex models, all of which seem somewhat contrived at this time.

g. Solutions

Table III lists "ugly" solutions which have been proposed and can, with enough tweaking of the parameters, be made to simultaneously solve the three dark matter problems. The "ugliness" differs from case to case as is listed in Table III. While none are compelling at the present time, they at least have the advantage of making different specific predictions which might eventually be checked. In particular the "light-not-a-tracer" and the "decay" scenarios make statements about large scale structure and cluster-cluster correlations which future large sky surveys should be able to resolve. They will also tell us whether the large superclusters and voids are rare or common. If common, this would argue for non-random phases and perhaps for the GUT phase transition going via strings. It also seems that the biasing to get $\Omega_{cluster} \lesssim 0.4$ is inconsistent with the biasing necessary to get the large cluster-cluster correlation function.[96a]

A very nice way to begin to resolve the dark matter problem would be to find some of the stuff in the lab. If neutrinos are found to have a mass or if a 10GeV photino is found this would immediately collapse the degrees of freedom in the proposals. Note also that different proposals end up with the dark baryons in different locations,[51] halos versus voids. Thus, finding the dark baryons may resolve the problem.

4. CONCLUSION

The previous two sections give a sample of what the cosmology/particle physics interface involves. Clearly, every period of the early Universe represents events concerning particle interactions; at the earliest of times, we expect quantum gravity and/or supergravity to be important; then, through GUTs, through the desert to the period of electro-weak symmetry-breaking, quark-confinement and chiral symmetry-breaking, and on to big bang nucleosynthesis. At each stage, our understanding of the big bang relies most heavily on our understanding of fundamental interactions.

Now, more than ever before, there is a feedback effect where our understanding of particle physics relies on cosmology. Clearly, model builders avoid particle mases and lifetimes which lead to an $\Omega > 2$ (or if timescales are included $\Omega h_o^2 > 0.3$) Universe. In more subtle ways too, however, the nature of phase transitions and symmetry breaking also requires an acceptable cosmological framework. In the case of inflation, the early models discussed a strong first order phase transition for SU(5) to SU(3) x SU(2) x U(1). When that became problematic, people looked towards second order transitions. The point is that the structure (representations, couplings, etc.) dictating the type of symmetry breaking is dependent on an acceptable cosmological scenario.

In various ways described in section 3, what knowledge we have of galaxy formation also makes certain demands on particle physics. The strongest case made, is the need for dark matter. It will still take time for a fuller interplay to take place. On the one hand, there are too many dark matter candidates, while on the other, none of them by themselves seem to work well and it is as yet unclear what cosmologists themselves would like for galaxy formation. We expect, however, that eventually our understanding of galaxies will be linked to our understanding of the particle world as is the case with most of the rest of cosmology.

TABLE I

Scale	r	M/L*	Implied Ω
Stars	<1 pc	~ 1 to 2	$\lesssim 10^{-3}$
Visible regions of galaxies	$\sim 10^4$ pc	$\sim 10h_o$	.01
Binaries and small groups	$\sim 10^6$ pc	$\sim 100h_o$	~ 0.07
Large clusters	$\sim 3 \times 10^7$ pc	$\sim 100h_o$ to $\sim 500h_o$	~ 0.07 to 0.4

*Uses bandwidth consistent with $= 2 \times 10^8 h_o\ L_{\odot}/\mathrm{Mpc}^3$

TABLE II

Popular Candidates

Name	Mass	Classification
Neutrinos	$5 \leq m_{nu} \lesssim$ 50eV	hot
Neutral heavy leptons	≥ 3 GeV	cold
gravitinos sneutrinos	arbitrary	hot, warm or cold
photinos	$\gtrsim 2$ GeV	cold
higgsinos	$m > 5$ GeV or $\lesssim 100$ eV	cold hot
axinos	$m \lesssim 10^2$ eV	hot
Topological beasts	$m \gtrsim 10^{17}$ GeV	cold
Axions	$<<1$ eV	cold
Planetary black holes	$10^{15}g \lesssim M \lesssim 10^{33}g$	cold

TABLE III
Ugly Solutions

Solution	Ugliness	References
LIGHT IS NOT AN UNBIASED TRACER OF LIGHT	In the extreme, this means observational astronomy is a waste of time.	71
Version 1 - cold matter	Requires semi-ad hoc assumption that only 3σ density fluctuations lead to light emitting galaxies.	97
Version 2 - hot matter	Requires "special" hydrodynamics or magnetohydrodynamics to prevent large dark pancakes from becoming observable x-ray sources. Also requires assumptions about fragmentation of some pancakes into galaxies. (May be aided by shock induced galaxy formation.)	68
A COLD OR WARM PARTICLE DECAYS TO A HOT ONE AFTER GALAXIES FORM ($\nu_{heavy} \rightarrow \nu_{light}$ + X or gravitino → axino + axion or ?	Requires a finely tuned particle model with no other current reason for the tuning than the solution to these problems	96, 98 99, 100 101
NON-RANDOM PHASES (strings?)	Opens up a tremendous range in multiparameter space one the assumption that the fluctuations are random is thrown out. Different physical models, like strings, do not provide some constraints but their model parameters have no strong motivation other than this class of problems	69, 70
SHOCK ENHANCED GALAXY FORMATION	Early fluctuations not carried by the matter (strings or isothermals) enable hot matter to work, since small scales not damped, but again require ad hoc model	61, 88
	Requires initial seeds which either come from cold or hot models with their problems or from baryons falling onto clusters of planetary mass black hole whose production is dependent on the physics of poorly understood phase transitions	102
NON-ZERO COSMOLOGICAL CONSTANT	Traditionally invoked to solve cosmological problems, requires that we live at a special epoch.	
GRAVITY DEVIATING FROM $1/r^2$ AT LARGE r	No known reason for gravity to have any scale other than the Planck scale	103

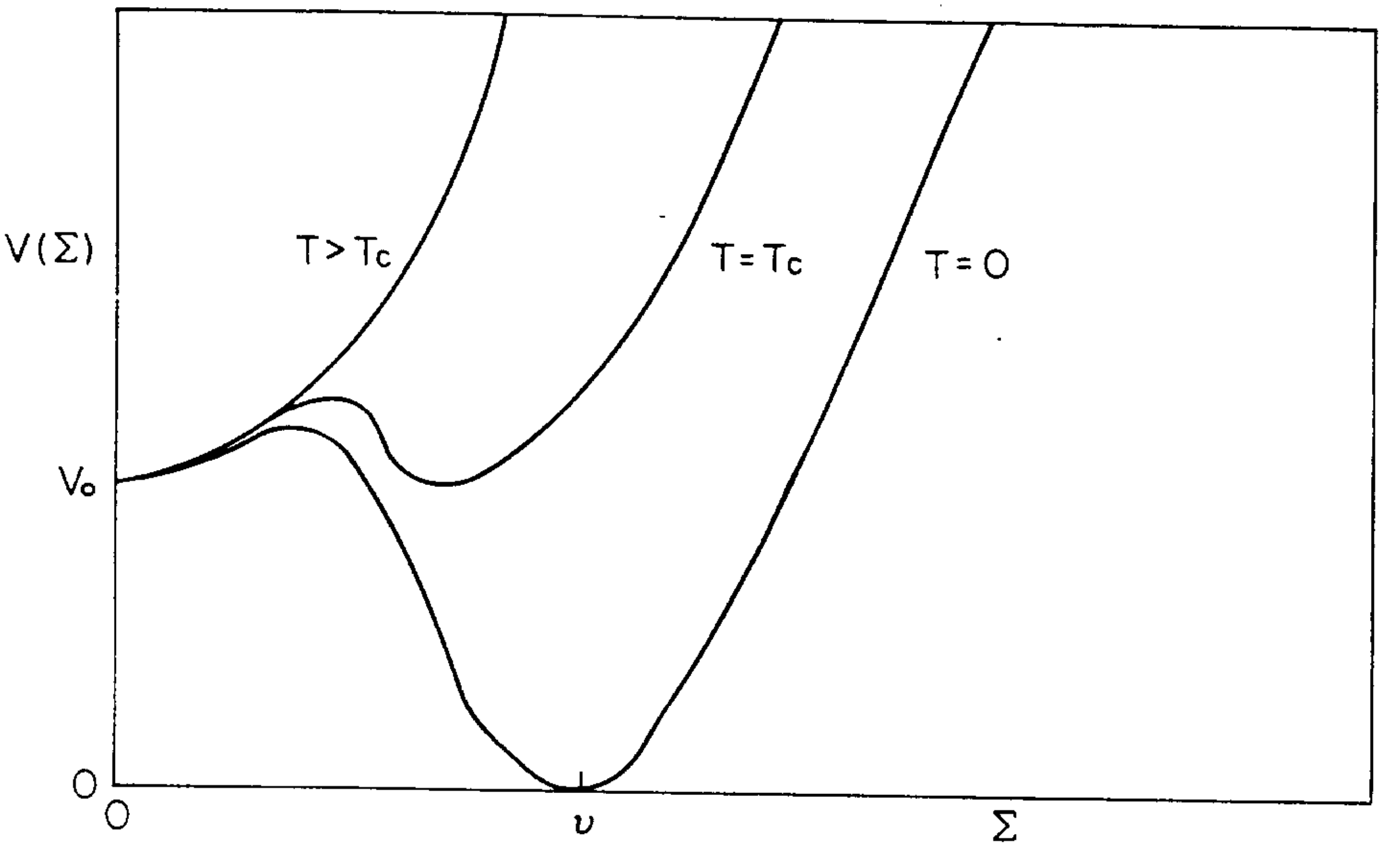

FIGURE 1 A schematic plot of the scalar potential for the 24 of SU(5) at various temperatures leading to a 1st order phase transition.

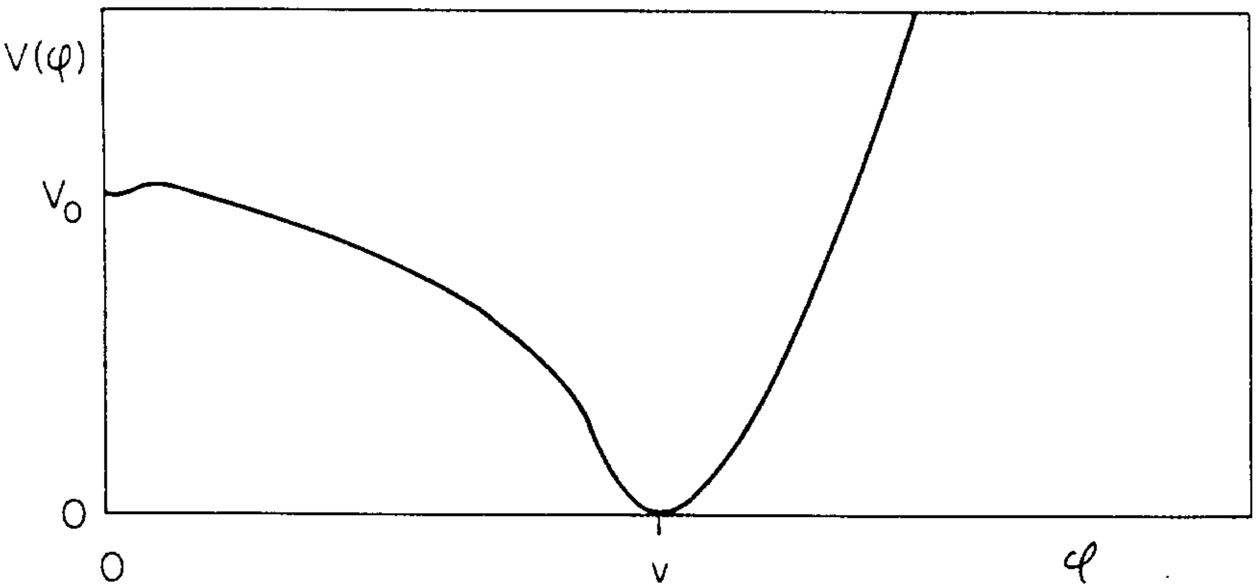

FIGURE 2 A schematic plot of the type of scaler potential needed for the new inflationary scenario.

REFERENCES

1. Unity of Forces in the Universe, ed. A. Zee, World Scientific, Singapore (1983) and Inner Space-Outer Space (ISOS), ed. E. Kolb, M.S. Turner, D. Lindly, K.Olive and D. Seckel, University of Chicago Press (1985)
2. P. Richards in ISOS (1985)
3. K. Olive, D. Schramm, G. Steigman, M. Turner, and J. Yang Ap. J. **246**, 547 (1981)
4. G. Steigman, D. Schramm and J. Gunn, Phys. Lett **B66**, 202 (1977)
5. D. Schramm and G. Steigman, Phys. Lett **B141**, 337 (1984)
6. G. Steigmann, Ann. Rev. Nuc. & Part. Sci. **29**, 313 (1979)
7. Proceedings of Bielefield Conference on Phase Transitions, a list on The Early Universe, Nuc. Phys. **B252** (1985)
8. E. Kolb and M. Turner. Ann. Rev. Nucl. & Part. Sci. **33**, 645 (1983) and references therein
9. Proc. Int. Cosmic Ray Cont., Banglelone (1983)
10. K. Griesen Phys. Rev. Lett **16**, 748 (1967); G. Zatsepin and V.A. Kuz'min, JETP Lett. **4**, 78 (1966)
11. A. Guth, Phys. Rev. D **23**, 347 (1981)
12. A.D. Linde, Phys Lett. **108B**, 389 (1982); A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982)
13. E. Gliner, JETP **22**, 378 (1966)
14. E. Gliner and T. G. Dymikova, Sov. Astro. Lett. **1**, 93 (1975)

14a. H. Georgi and S. Glashow, Phys.Rev.Lett. **32**, 438 (1974)

15. E. Kolb and S. Wolfram. Ap J. **239**, 428 (1980)
16. K. Sato, Phys Lett. **99B**, 66 (1981)
17. K. Sato, Mon. Not. Roy. Astron. Soc. **195**, 467, (1981)
18. D. Kazanas, Ap.J. **241**, L59 (1980)
19. Y. Zeldovich and M. Khlopov, Phy. Lett 79B, 239 (1978); J. Presskil, Phy. Rev. Lett. 43, 1365 (1979)
20. J. Uson and D. Wilkinson, in: ISOS
21. G. Smoot, M.V. Gorenstein and R.A. Muller, Phys.Rev.Lett. **39**, 898 (1977)
22. J. Ellis and K. Olive, Nature, **303**, 679 (1983)
23. J. Ellis, A. Linde and D. Nanopolous, Phys. Lett. **118B**, 39 (1982)
24. W. H. Press, Phys. Scr. **21**, 702; S. W. Hawking, Phys. Lett. **115B**, 295 (1982); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); A.A. Starobinski, Phys. Lett. **117B**, 175 (1982); J. M. Bardeen, P.J. Steinhardt, and M. S. Turner, Phys. Rev. **D28**, 679 (1983)
25. S. Coleman, Phys. Rev. **D15**, 2929 (1977); C. Callan and S. Coleman, Phys. Rev. **D16**, 1762. (1977)
26. A. H. Guth and E. Weinberg, Phys. Rev. **D23**, 826 (1981); and Nucl. Phys. **B212**, 321 (1983)
27. S. Coleman and E. Weinberg, Phys. Rev. **D7**, 1888 (1973)
28. P. Steinhardt and M.S. Turner, Phys. Rev. **D29**, 2105 (1984)
29. A. D. Linde, Phys. Lett. **116B**, 335 (1982); J. Breit, S. Gupta and A. Zaks, Phys. Rev. Lett. **51**, 1007 (1983)
30. J. Ellis, D. Nanopoulos, K. Olive, and K. Tamvakis, Phys. Lett. **118B**, 335 (1983)
31. J. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis, Nucl. Phys. **B221**, 224 (1983)

32. A. D. Linde, Phys. Lett. **129B**, 177 (1983)

33. L. F. Abbot, E. Fahri and M.B. Wise, Phys. Lett. **117B**, 29 (1982)

34. D. V. Nanopoulos, K. A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. **123B**, 41 (1983)

35. E. Cremmer, B. Julia, J. Scherck, S. Ferrara, L. Girardello; and P. Van Nieuwenhuizen, Phys. Lett. **79B**, 231 (1978); and Nucl. Phys. **B147**, 105 (1979); E. Cremmer, S. Ferraro, L. Girardello and A. Van Proeyen, Phys. Lett. **116B**, 231 (1982); and Nucl. Phys. **B212**, 431 (1983)

36. R. Holman, P. Ramond and G. G. Ross, Phys. Lett. **137B**, 343 (1984)

37. G. Gelmini, D. V. Nanopoulos and K. A. Olive, Phys. Lett. **131B**, 53 (1983)

38. B. Ovrut and P. Steinhardt, Phys. Lett. **133B**, 161 (1983)

39. G. Mazenko, W. Unruh and R. Wald, Phys.Rev. **D31**, 273 (1985)

40. A. Albrecht and R. Bradenberg, preprint (1984); Coughlan and G. G. Ross, preprint (1984); L. Jensen and K.A. Olive, preprint (1985)

41. L. Jensen and K. A. Olive, preprint (1985)

42. E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. **133B**, 287 (1983); J. Ellis, C. Kounnas, D. V. Nanopoulos, Nucl. Phys. **B241**, 406 (1984) and Phys. Lett. **143B**, 410 (1984) and CERN preprint Th. 3824

43. J. Ellis, K. Enquist, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Phys. Lett. **152B**, 175 (1985)

44. G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, Phys. Lett. **131B**, 59 (1983); M. Dine, W. Fischler and D. Nemeschansky, Phys. Lett. **136B**, 169 (1984); G. D. Coughlan, R. Holman, P. Ramond and G. G. Ross, Phys. Lett. **140B**, 44 (1984)

45. A. S. Goncharov and A. D. Linde, Lebedev Physical Institute, preprint (1984)

46. R. Brout, F. Englert and E. Gunzig, Ann. Phys. **115**, 78 (1978) A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980)

47. 2A. Shafi and A. Vilenkin, Phys. Rev. Lett. **52**, 691 (1984); S. Y. Pi, Phys. Rev. Lett. **52**, 1725 (1984)

48. D. Seckel, Fermilab preprint 84-92 (1984)

49. E. Alvarez and M. Belen - Gavela, Phys. Rev. Lett. **51**, 931 (1983); Q. Shafi and C. Wetterich, Phys. Lett. **129B**, 387 (1983); D. Sahdev, Phys. Lett. **137B**, 155 (1984); E. W. Kolb, D. Lindley and D. Seckel, Phys. Rev. **D30**, 1205 (1984); R. B. Abbot, S. M. Barr, and S. D. Ellis, Phys. Rev. **D30**, 720 (1984)

50. J. R. Gott, Nature **295**, 304 (1982)

51. D. N. Schramm, Nuc.Phys. **252**, 53; D.N. Schramm, Nuovo Cimento (1985) in press

52. J. Bahcall, Ap. J. 276, 169 (1984)

53. J. Bahcall, P. Hut and S. Tremaine, preprint (1984); D. Hegyi, E. W. Kolb and K. A. Olive preprint (1985); J. McDowell, preprint (1985)

54. S. Faber and J. Gallagher, Ann. Rev. Astron. and Astrophys. **17**, 135 (1979) P.J.E. Peebles, in: *Physical Cosmology*, eds. J. Audouze, R. Balian and D. Schramm (Reidel, Amsterdam, 1980)

55. D. Schramm and G. Steigman, Ap. J. **243**, 1 (1981)

56. J. Ostriker and P.J.E. Peebles, Ap. J. **186**, 467 (1973)

57. M. Aaronson Ap. J. **266**, L11 (1983); S. Faber and D. Lin **266**, L17 (1983)

58. S. Triemaine and J. Gunn, Phys. Rev. Lett. **42**, 407 (1979)

59. M. Davis and P.J.E. Peebles, Ann. Rev. Astron. and Astrophys. **21**, 109 (1983) and references therein

60. R. Kirshner, G. Oemler and D. Schecter, Ap.J. **84**, 951 (1979)

61. J. Ostriker and L. Cowie, Ap.J. **243**, L127 (1980)

62. Y. Zeldovich and I. Novikov, in: *Relativistic Astrophysics, Vol. II* (U. of Chicago Press, Chicago, 1983); P.J.E. Peebles, *Large Scale Structure of the Universe* (Princeton University Press, 1981)

63. M. Turner and D. N. Schramm, Nature **279** 303 (1978); also W. Press, Physica Scripta **21**, 702 (1980)

64. N. Vittorio and J. Silk, Ap. J. (in press) (1985); J. R. Bond and G. Efstathiou, in: ISOS (1985)

65. M. Davis, in: ISOS, and references therein (1985)

66. M. Davis and N. Kaiser, UC Berkeley preprint (1984)

67. K. Freese and D. N. Schramm, Nuclear Physics **B233**, 167 (1984)

68. R. Bond and A. Szalay, Ap.J. **274**, 443 (1983), and references therein

69. P.J.E. Peebles, Ap.J. **277**, 470 (1984)

70. N. Turok and D. N. Schramm, Nature **312**, 598 (1984)

71. M. Davis, G. Efstathiou, C. Frenk and S. White, Ap. J. (Suppl.) (in press, 1984)

72. J. Fry, Ap. J. **270**, 131 (1983)

73. N. Bahcall and R. Soniera, Ap. J. **270**, 20 (1983); A. Klypin and N. Khlopov in: Proc. COSPAR (Bulgaria 1983, communicated by R. Sunyaev)

74. M. G. Hauser and P.J.E. Peebles, Ap. J. **185**, 757 (1973)

75. G. Efstathiou, in: Proc. COSPAR Mtg. (Bulgaria, 1983)

76. N. Kaiser, in: ISOS (1984)

77. A. S. Szalay and D. N. Schramm, Nature (in press) (1985)

78. B. J. Carr and I. Ikeuichi, Cambridge Univ. Preprint (1984)

78a. J. Charlton and D.N. Schramm, (in preparation 1985)

79. A. Vilenkin, Physics Reports (in press, 1985)

80. J. Yang, M. Turner, G. Steigman, D. N. Schramm, and K. Olive, Ap.J. **281**, 493 (1984)

80a. B. Partridge, *Proceedings of Rome Symposium on Background Radiation* Nuovo Cimento (in press 1985)

81. S. Lea, J. Silk, E. Kellogg and S. Murray, Ap.J. **184**, L105 (1973)

82. J. R. Gott, J. Gunn, D. N. Schramm and B. Tinsley, Ap.J. **194**, 543 (1974)

83. D. Hegyi and K. Olive, Phys. Lett. 126B 28 (1983); and Ap.J. (submitted) (1985)

84. J. R. Bond, G. Efstathiou and J. Silk, Phys. Rev. Lett. **45** 1980 (1980); Ya. B. Zeldovich and R. A. Sunyaev, Pisma Astr. Zh. **6**, 457 (1980)

85. P.J.E. Peebles, Ap.J. 263, L1 (1982) and Ap.J. **277**, 440 (1984)

86, M. S. Turner, F. Wilczek and A. Zee, Phys.Rev.Lett. B125, 35; J. Ipser and P. Sikivie, Phys. Rev. Lett. **50**, 925 (1983)

87. K. Olive, D. Schramm and G. Steigman, Nucl.Phys. **B180**, 497 (1981)

88. K. Freese, R. Price and D. Schramm, Ap.J. **275**, 405 (1983)

89. M. Crawford and D. N. Schramm, Nature **298**, 538 (1982) D. N. Schramm and K. Olive, Nucl.Phys. **A418**, 289 (1984)

90. M. Crawford, Ph.D. Thesis (U. of Chicago, 1984)

90a. E. Witten, Phys.Rev. D30, 272 (1984)

91. G. R. Blumenthal, S. Faber, J. Primack and M. Rees, Nature **311**, 517 (1984)

91a. F. Occhinaro, *Proceedings of Rome Symposium on Background Radiation* loc.cit (1985)

92. A. Dekel and M. West, Ap. J. **279**, 1 (1984)

93. A. Melott and D. N. Schramm, Ap.J. (in press) (1985)

94. C. Frenk, S. White and M. Davis, Ap.J. **271**, 417 (1983)

95. J. Gunn, B. Lee, I. Lerche, D. Schramm and G. Steigman, Ap.J. **223**, 1015 (1978)

96. M. Davis, M. LeCar, C. Pryor and E. Witten, Ap.J. **250**, 423 (1981)

96a. J. Bardeen, N. Kaiser and A. Szalay (in preparation 1985)

97. A. Szalay and N. Kaiser, UC Berkeley Preprint (1985); S. White, M. Davis and C. Frenk, M.N.R.A.S. **209**, 27 (1984)

98. P. Hut and S.D.M. White Nature **310**, 637 (1984)

99. K. Olive, D. Seckel and E. Vishniac, Ap.J. (in press, 1985)

100. M. Turner, G. Steigman, and L. Krauss, Phys. Rev. Lett. **52**, 2090 (1984); G. Gelmini, D. N. Schramm, J.W.F. Vallee, Phys.Lett. **146**, 311 (1984)

101. K. Olive, D. Schramm and M. Srednicki, Nucl.Phys. **B255**, 495 (1985)

102. C. Hogan and J. Applegate, Caltech preprint (1985)

103. J. Beckenstein, Proc. Jerusalem School on Cosmology (1983)