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A New Approach to Dynamic Aperture Problems

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ABSTRACT

We develop the theory of a passive magnetic system intended to suppress nonlinear orbit distortion in high energy proton storage rings. The system is designed immediately to reduce "Collins distortion functions," which describe the size of nonlinear orbit distortion in first-order perturbation theory. Such a scheme could permit one significantly to decrease the physical aperture of a storage ring over most--but not necessarily all--of its length. This work was motivated by design needs of the proposed Superconducting Super Collider (SSC).

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I. GENERAL PHILOSOPHY

Concerns about the high cost of the proposed Superconducting Super Collider (SSC), and about discouraging results from single-beam numerical simulation[1] of the SSC Reference Design[2], have recently generated heightened interest in the general problem of predicting the dynamic aperture of a high energy proton (nonradiative) storage ring. Loosely speaking, dynamic aperture is the region of phase space within which beam can be injected without risk of being lost soon afterward. The small dynamic aperture predicted by orbit tracking of the SSC Reference Design is due largely to the rich multipole content expected from proposed designs for SSC dipoles.

Hopes for an increase in SSC dynamic aperture are fueling theoretical research in two areas: numerical simulation,[3] and analytic perturbation theory.[3,4] However, these are problematical. Simulation can in principle tell us when a design cannot work, but no one knows at present how to turn this into a concrete prescription for design modifications. High order perturbation theory is also not yet sufficiently intuitive to suggest specific remedies for orbital difficulties. Perturbation theory is attractive at this time mainly because of the hope that it may be able to provide the same sort of information as numerical simulation, in greater detail, in much less computing time.

Recently, we proposed[5] an example of a third, intermediate approach to dynamic aperture problems: a passive magnetic correction scheme, suggested by first order perturbation theory, to be applied when orbit tracking indicates trouble, and to be tested with further orbit tracking. This would be a designer's tool, whereas simulation and perturbation theory are only suited at present primarily for screening. In this paper we analyze our proposal in detail.

Even if this particular example is ultimately impractical, we feel strongly that the general philosophy of this intermediate approach is sound, and should be pursued further.

Here is a quick sketch of our ideas: Our scheme is based on a simple reformulation of first order perturbation theory due to Collins.[6] In this formulation, Collins writes the first order corrections to the betatron amplitudes I_x and I_y (constants in zeroth order) as sums of terms of the form

$$I_x^{|m|/2}\, I_y^{|n|/2}\, d(s)\exp[i(n\nu_x + m\nu_y)(2\pi s/C)]\exp i\phi \quad , \tag{1.1}$$

where m and n are integers, the ν's are unperturbed tunes, C is the storage ring circumference, ϕ is an arbitrary constant phase, and the complex periodic functions--a sort of basis for first order nonlinear orbit distortions--generically written as d are called "distortion functions." As a working principle, we adopt Collins' viewpoint that a well-behaved storage ring has small d's, although this is not yet a theorem in any sense.

Let N be the total number of dipole magnets (presumed to be the most important sources of unwanted higher multipoles) in the storage ring. If the error multipoles on these magnets are random and uncorrelated, with vanishing averages, then, as we shall show, the mean square magnitude of any particular distortion function is roughly (ignoring variation in beta functions) proportional to N. Our idea is to decrease distortion functions by partitioning the accelerator into M sectors, with correction magnets in each sector powered so that the distortion functions in any sector receive contributions only from the magnets in that sector. In this way, mean square distortion functions become roughly proportional to N/M instead of N. I. e., we expect reduction by a factor 1/M, times some position-dependent factor ξ that is to be determined numerically.

In the next section we shall develop in detail the general theory of uncorrected sextupole distortion functions. In Section III we develop in detail the mechanics of our correction scheme. In Section IV we prove a theorem that tells us how much improvement to expect in certain regions of the storage ring, and a theorem that places an important restriction on the positioning of correction magnets. In Section V we apply our scheme to sextupole distortion in a (somewhat modified) model lattice for SSC magnet design evaluation that Norman Gelfand[2] developed for use with the orbit-tracking program TEVLAT.[7] As we shall see, our scheme significantly reduces distortion functions away from the correction magnets, perhaps permitting a reduction in physical aperture throughout most of the storage ring. Reduction can be much more modest near the correction elements. For this reason we have not tracked orbits in a model lattice corrected in this way, since TEVLAT at present assumes a position-independent physical aperture.

Ideally, one would like to correct nonlinear orbit distortion actively, using model orbit tracking--or even post-construction measurements of real orbits in the real storage ring--directly to determine correction strengths. Active and passive schemes alike might be improved by combination with magnet shuffling.[8]

Originally, we had wanted to develop a completely optimized procedure for reducing distortion functions with a restricted number of correction magnets. To this end, we explored schemes that determined correction strengths by minimizing quadratic figures of merit of the form

$$\sum_{d} \oint ds \; |d(s)|^2 W_d(s) \quad , \tag{1.2}$$

(in an obvious notation) where the W's are positive weight functions. The W's might be chosen, for example, to emphasize regions with some beta function large. However, we eventually abandoned this idea, for two reasons:

(i) It required a great deal of computing time. One must invert an $L\times L$ matrix in order to arrange for L normal sextupole magnets to reduce the five normal sextupole distortion functions by minimizing (1.2). By contrast, the M-sector scheme detailed below, with 12M=L, requires inversion of L/12 matrices, each only 12×12; thus, computing time grows only linearly with L.

(ii) We know of no way, short of large-scale calculation, to make even a crude preliminary estimate of the extent of the distortion-function reduction in this least-squares scheme. In particular, we therefore could not a priori justify to ourselves the expected large expenditure of computing time. By contrast, in the scheme described below we can say with confidence that reduction will be substantial in certain regions of the storage ring.

Of course, there may be some variation on this minimization theme that does not suffer from these defects.

II. DEFINITION OF DISTORTION FUNCTIONS

We derive Collins'[6] formulation of first order betatron perturbation theory assuming, for simplicity, that non-linearities are due only to normal sextupole magnets, and ignoring skew quadrapole magnets.

The general Hamiltonian for this problem is

$$H = \frac{1}{2}\left(p_x^2 + k_x(s)x^2\right) + \frac{1}{2}\left(p_y^2 + k_y(s)y^2\right)$$

$$+ \frac{eB_y''(s)}{6E}\left(x^3 - 3xy^2\right) \quad , \tag{2.1}$$

where most notation here and below is standard[9] (B_y'' is $\partial^2 B_y/\partial x^2$ on the design orbit). The usual canonical action and angle variables for this system are

$$I_x = (2\beta_x)^{-1}\left[x^2 + \left(\beta_x p_x - \frac{1}{2}\beta_x' x\right)^2\right]$$

$$\theta_x = -\,\Phi_x(s) - \mathrm{Arctan}\left[\beta_x p_x - \frac{1}{2}\beta_x' x\right]/x \quad , \tag{2.2}$$

where Φ_x is defined by

$$\Phi_x(s) = \int_0^s \frac{ds'}{\beta_x(s')} \quad , \tag{2.3}$$

and similarly for y. (The reference point s=0 is arbitrary.) In terms of (2.2), (2.1) becomes

$$H = I_x(2\pi\nu_x/C) + I_y(2\pi\nu_y/C) + \frac{eB_y''}{3E\sqrt{2}} \Big\{ \frac{1}{2} (I_x\beta_x)^{3/2}\cos 3[\Phi_x + \theta_x]$$

$$+ \frac{3}{2} (I_x\beta_x)^{3/2}\cos [\Phi_x + \theta_x] - 3(I_x\beta_x)^{1/2}(I_y\beta_y) \cos [\Phi_x + \theta_x]$$

$$- 3(I_x\beta_x)^{1/2}(I_y\beta_y) \cos [\Phi_x + 2\Phi_y + (\theta_x + 2\theta_y)] \qquad (2.4)$$

$$- 3(I_x\beta_x)^{1/2}(I_y\beta_y) \cos [\Phi_x - 2\Phi_y + (\theta_x - 2\theta_y)]\Big\} .$$

To first order in the sextupole strengths, the actions I_x and I_y are given by $I_x^0 + \delta I_x$ and $I_y^0 + \delta I_y$, where I_x^0 and I_y^0 are independent of s, and where δI_x satisfies

$$\delta\dot{I}_x = - \frac{\partial H}{\partial \theta_x} \Bigg|_{\theta_x = \phi_x + 2\pi\nu_x s/C,\ \theta_y = \phi_y + 2\pi\nu_y s/C} ,$$

and similarly for δI_y. Since the general differential equation

$$\left(\frac{d}{ds} + i\omega\right)d(s) = R(s) \qquad (2.6)$$

has, for periodic R(s), the unique periodic solution

$$d(s) = \int_0^C G_\omega(s,s')R(s')ds' \quad , \qquad (2.7)$$

with

$$G_\omega(s,s') = e^{i\omega(s'-s)}[1-e^{i\omega C}]^{-1} \cdot \begin{cases} e^{i\omega C} & 0<s'<s<C \\ 1 & 0<s<s'<C, \end{cases} \tag{2.8}$$

we can integrate (2.5) to obtain

$$\begin{aligned} \delta I_x = \frac{e}{6E\sqrt{2}} \,\mathrm{Im} \Big\{ & \tfrac{1}{2} d_{30}(s)(I_x^0)^{3/2} \exp 3i(\phi_x + 2\pi\nu_x s/C) \\ & + \tfrac{3}{2} d_{10}(s)(I_x^0)^{3/2} \exp i(\phi_x + 2\pi\nu_x s/C) \\ & - 3\bar{d}_{10}(s)(I_x^0)^{1/2} I_y^0 \exp i(\phi_x + 2\pi\nu_x s/C) \\ & - 3d_{12}(s)(I_x^0)^{1/2} I_y^0 \exp i(\phi_x + 2\phi_y + 2\pi(\nu_x + 2\nu_y)s/C) \\ & - 3d_{1-2}(s)(I_x^0)^{1/2} I_y^0 \exp i(\phi_x - 2\phi_y + 2\pi(\nu_x - 2\nu_y)s/C) \Big\} \end{aligned}$$

where, for example, d_{12} is defined by (2.7) with $\omega = 2\pi(\nu_x + 2\nu_y)/C$, and $R(s)$ given by $B_y'' \, \beta_x^{1/2} \, \beta_y \cdot \exp i(\Phi_x + 2\Phi_y)$, etc. In a similar fashion, δI_y is formed from d_{12} and d_{1-2} alone.

In this way, first order perturbation theory has been decomposed into the effects of initial conditions (I^0's and ϕ's) and intrinsic properties (ν's and d's) of the storage ring. The periodic functions $\bar{d}_{10}$ and the d's are the normal sextupole distortion functions. Note that these functions are normalized differently here than in Reference [6]. However, the equations that follow are not sensitive to normalizations.

III. DEFINITION OF THE CORRECTION SCHEME

Imagine now partitioning the storage ring into M nonoverlapping sectors. For simplicity we imagine them to be of equal length, although this is not necessary. According to (2.6), the contribution of any magnet within a sector to a distortion function outside that sector is a constant times $\exp(-i\omega s)$, for some ω. If, in the sector in question, we place two real correcton magnets (of the appropriate multipolarity) for every one complex distortion function that we want to reduce, we can ensure that all the constants add to zero. In this way, we ensure that no magnet contributes to any designated distortion function outside that magnet's own sector. This procedure is formally equivalent to an "orbit bump" calculation.

In the case of d_{30}, for example, the cancellation of these constants takes the specific form

$$0 = \int ds \beta_x^{3/2}(s) B_y''(s) \exp 3i \int_0^s \frac{ds'}{\beta_x(s')} , \tag{3.1}$$

where the integration is restricted to the sector in question. The cancellation conditions for the other distortion functions are similar. If we imagine that the sextupole moments are lumped

$$B_y''(s) = \sum_i \delta(s-s_i) b_i + \sum_i \delta(s-s^c_j) b^c_j , \tag{3.2}$$

where the b^c's correspond to correction magnets, then (3.1) becomes

$$\sum_j b_j^c \beta_x^{3/2}(s_j^c) \exp 3i \int_0^{s_j^c} \frac{ds'}{\beta_x(s')} \quad ,$$

$$= - \sum_i b_i \beta_x^{3/2}(s_i) \exp 3i \int_0^{s_i} \frac{ds'}{\beta_x(s')} \quad , \tag{3.3}$$

The sum in (3.3) is of course restricted to locations within the sector in question.

We now explain why this scheme leads to the 1/M scaling law claimed in Section I. We suppose, for fixed $\{s_i\}$ (the centers of dipoles, for example), that the various b_i, whose distorting effects we want to supress, are independent random variables, with zero mean and common mean square σ^2. Then the mean square magnitude of any sextupole distortion function is a sum of contributions due to the various s_i, with no cross terms, with each contribution proportional to σ^2. If there were no correction magnets at all, then we would have, for example,

$$\langle |d_{12}(s)|^2 \rangle = \frac{\sigma^2}{4\sin^2\frac{2\pi}{2}(\nu_x+2\nu_y)} \sum_i \beta_x(s_i)\beta_y^2(s_i) \quad , \tag{3.4}$$

using (2.8). Ignoring variation in the β's, this is proportional to N, as claimed earlier.

When correction magnets are in place, the contribution of any s_i to any $\langle |d|^2 \rangle$ must take into account the correction strengths that the pre-existing random sextupole at s_i induces according to (3.3) etc. The additions that arise in this way are formed from the beta functions at the s_i in question, and from betatron phase advances from some reference point to s_i, and from the elements of the inverse of the matrix of coefficients, in (3.1), of the b_j^c in s_i's sector.

How do these ingredients vary from one s_i to another? If the uncorrected storage ring is composed of identical (up to random multipole errors)cells--as it is in the model discussed in Section V--then beta functions repeat from cell to cell. Further, if betatron phase advances per cell are close to simple fractions of 2π ($\pi/3$ in the model of Section V) then, when exponentiated, the betatron phase advances from a fixed reference point ($s = 0$ in (3.3)) repeat after several cells. Thus, in any sector, the full contributions of different random multipoles to some $\langle|d(s)|^2\rangle$ at any s very nearly repeat after a number of cells that is independent of M.

(According to (2.8), this last statement is complicated by a phase shift when the random-multipole s_i passes the point s at which $\langle|d|^2\rangle$ is evaluated. Strictly speaking, we should say that the contributions very nearly repeat as s_i moves away from s in either direction.)

Finally, if correction magnets are placed in sectors in a way that is independent of M in some sense (for example, four per cell in the first three cells, as discussed below) then it follows from the foregoing that any $\langle|d(s|)^2\rangle$ is roughly proportional to the number of cells per sector--itself proportional to 1/M--times an s-dependent factor that is independent of M as long as s is taken to refer to the same point <u>relative to the correction system</u> (for example, in the middle of the seventh cell from the start of the sector, in the configuration discussed below) for all M. We report on a check of this scaling law in Section V.

Notice that we have ignored sextupole magnets that are deliberately included in designs in order to cancel the storage ring's natural (quadrupole-induced) chromaticity. The strengths of these chromaticity correction magnets are not uncorrelated random numbers. In the original SSC model that we modified for the discussion in Section V, they repeat from cell to cell (with strengths comparable to those of the random sextupoles), so that their contributions to distortion functions add coherently. With betatron phase advances close to 60° per cell, it follows from (2.7) and (2.8) that their contributions to sextupole distortion functions nearly cancel in groups of six cells (for d_{10}, $\bar{d}_{10}$, d_{1-2}) or two cells (for d_{30} and d_{12}). Thus the total chromaticity correction contribution to any distortion function in any sector is comparable to the contribution of just a few cells of random sextupoles; and therefore any associated non-1/M piece in the scaling law should be negligible unless the total number of cells in a sector is small. For this reason, standard chromaticity correction magnets are ignored in the calculations described in Section V.

A real storage ring may also need correction magnets to cancel chromaticity induced by random, construction-related sextupole inaccuracies. Accordingly, in the calculations of Section V, we include two such magnets per sector, beyond the minimal ten needed to reduce the five complex normal sextupole distortion functions. Sector-by-sector cancellation of chromaticity induced by random fields requires

$$\sum_j \beta_x(s_j^c)b_j^c = -\sum_i \beta_x(s_i)b_i$$

$$\sum_j \beta_y(s_j^c)b_j^c = -\sum_i \beta_y(s_i)b_i \quad , \tag{3.5}$$

where each sum covers only one sector at a time, and where standard natural-chromaticity-compensating sextupoles are not included.

IV. TWO THEOREMS

The model storage ring discussed in the next section consists of 480 identical (up to random sextupole moments) cells, each with approximately 60^o horizontal and vertical betatron phase advances per cell. The number 480 was chosen for its many factors that could serve as the number M of structurally identical correction sectors. (This is a variant of an SSC model due to N. Gelfand, which has 481 cells.) For programming simplicity, we imagined each sector beginning with several cells in which identical numbers of correction magnets are placed in identical positions, followed by a long string of cells containing no corrections at all. Of course other configurations are possible.

The following theorem describes in quantitative terms the distortion function reduction to be expected from this sort of arrangement, at least in certain parts of the storage ring.

Theorem 1: Call the correction end of a sector its head, and the other end its tail. Assume that the random sextupole moments (we state the theorem in terms of sextupoles only for concreteness; an analogous theorem can be proved for any multipole moment) are concentrated in Dirac delta functions, as in (3.2), with assumptions on the b_i as stated in the previous section. Number the discrete random-sextupole sites in a sector from the tail. Then, with corrections, ignoring variation in the beta functions, the mean absolute square of any sextupole distortion function, between random sextupoles K and K + 1 in any sector, is at most equal to $4(K/N)\sin^2\omega c/2$ times the mean absolute square of that distortion function with all correction magnets absent (ω is the combination of frequencies appearing in the definition of the d's according to (2.7) and (2.8)), as long as random magnet K + 1 is not in a correction cell.

This is significant because even if our scheme does not markedly decrease distortion functions everywhere, we may still be reassured that distortion functions can reliably be reduced by at most a factor 4/M over a substantial fraction of the storage ring. For example, if M is 24 and $\omega C/2$ is nearly $\pi/2$ mod 2π (as it is in our model for d_{30} and d_{12}), and if the correction magnets occupy the first three cells in any sector, then on the average distortion functions in 85% of the ring can reliably be reduced in absolute square by anywhere from 0 to 4(17/480)~.14, roughly in steps of 4/480 ~ .008. Of course one would like to have tightly focused orbits everywhere. Failing that, perhaps our scheme can enable one to cut SSC cost by reducing physical aperture in, say, 85% of the machine.

Proof of Theorem 1: By the definition of our correction scheme, the corrected distortion function in question vanishes identically between the last magnet at the tail of a sector and the first magnet in the adjacent head of the next sector. Integrating the defining equation (2.6) into the tail of a sector, past the K tail-most random sextupoles, yields $e^{i\omega s}$ times

$$\sum_{i=1}^{K} (\text{phase})_i \beta_x^{m/2}(s_i)\beta_y^{n/2}(s_i)b_i \quad , \tag{4.1}$$

for some integers m and n, so that its mean absolute square is

$$\sigma^2 \sum_{i=1}^{K} \beta_x^m(s_i)\beta_y^n(s_i) \quad . \tag{4.2}$$

Comparison with (3.4), and its generalization to the other d's, yields the desired result.

Theorem 2: In this scheme, when the betatron phase advances per cell are exactly 60^{o}, a necessary condition for linear independence of the equations that determine the correction strengths (the b^{c}'s) is that every correction cell at the head of a sector contain at least four correction magnets.

Thus, with nearly 60^{o} phase advances per cell, as in the model of the next section, we cannot spread out sextupole corrections too much without paying the price of very large correction strengths, since they would arise from inversion of a nearly singular matrix.

Note, incidentally, that linear independence may impose different constraints on correction placement in the case of multipoles other than sextupole, and also when the phase advances per cell significantly differ from 60^{o}.

Proof of Theorem 2: The real and imaginary parts of the linear conditions $d_{30} = d_{12} = 0$ have coefficients of the b^{c} that change sign from one correction cell to the next, when phase advances are close to 60^{0}. If there were less than four correction magnets per cell, then the real coefficients in these four real equations, restricted to any one cell, would be linearly dependent, since any four n-tuples with $n \leq 3$ are linearly dependent. Linear dependence of $d_{30} = d_{12} = 0$ would follow immediately, since only the signs change in moving to other correction cells.

Had we applied that same viewpoint to d_{10}, $\bar{d}_{10}$, and the complex conjugate of d_{1-2}, we would have spoken about three complex conditions whose coefficients would each have been multiplied by the same exp $\pm i\pi/3$ in moving from cell to cell. Real linear dependence of these six (real and imaginary) real conditions sector-wise would imply only complex linear independence of these three complex conditions cell-wise, which would imply at least three correction magnets per cell. Similarly, linear independence of the two chromaticity constraints (in which coefficients of the b^c in adjacent cells are identical) would require at least two corrections per cell. Thus, the strongest constraints comes from the argument in the preceding paragraph.

V. NUMERICAL RESULTS

Theorem 1, above, justifies our "ξ/M" intuition outside cells that contain correction elements. In this section we report a check of our intuition globally, including the correction cells.

In particular, we have applied our scheme to the normal sextupole distortion functions in a model SSC containing 480 cells as shown in Figure 1. The random sextupole moments are concentrated in the centers of the dipoles, with

$$\sigma/B\rho \cong .011 \quad , \tag{5.1}$$

(Gaussian distribution) where $B\rho$ is the magnetic rigidity of a 20 TeV proton. Following Theorem 2, four correction magnets are placed in each of the first three cells in each sector: At the centers of each of the two dipoles, and at the centers of the D and F quadrupoles.

For a particular choice of the 960 random sextupole moments, we have computed the ratios of the maximum values of the absolute squares of the five normal sextupole distortion functions with corrections to their maximum absolute values without corrections, for M equal to every factor of 480; and we have least-squares fit each ratio to the functional form k/M. The results for k are, approximately, 24.1, 41.5, 16.9, 9.2, and 10.2 for d_{10}, d_{30}, d_{12}, d_{1-2}, and $\bar{d}_{10}$, respectively. The computed with-correction-to-without-correction ratio and the k/M fit for d_{10} are shown together in Figures 2 and 3. Agreement is encouraging, for such a rough theory.

Strictly speaking, this is not a fully statistical calculation, since we have used only one configuration of 960 random magnets. Presumably, fluctuation effects can average out for large values of N/M. For large M, on the other hand, only a small number of random magnets contribute in any one sector, so that fluctuations may be important.

Note that the data point for M = 1 in Figure 2 is far below the smooth fit. This is true for the other d's as well. The computer program that generated our random sextupoles constrained their sum to be zero, for reasons unrelated to the present project. Presumably, this had its most dramatic effect for M = 1, which ought to be the sector multiplicity most sensitive to biases connecting all the magnets. Accordingly, M = 1 was omitted from the least-squares fitting procedure.

Our results indicate that global reduction of all sextupole distortion functions is not achieved until M is significantly greater than about forty, at which point one worries about the cost of so many correction elements in so many sectors. Significant reduction of distortion over a large (but not exhaustive) fraction of the ring, as in Theorem 1, may be the best that one can afford to do with this scheme.

Ultimately, one should submit a lattice corrected in this way to a numerical orbit-tracking program with physical apertures that are not constant throughout the model machine. Moreover, since the small dynamic aperture of the SSC Reference Design appears to be due not only to sextupole errors, but also[1] at the same time to skew quadrupole, octupole, and even decapole, a fair test of this scheme should also include correction magnets for some of these multipoles as well.

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FIGURE CAPTIONS

1. Standard cell in our version of N. Gelfand's SSC model. Lengths are approximate. F and D quadrupoles have strength ~ .04 m^{-2}. A dipole has bend angle ~ 6.5 mrad.

2. Ratio of maximum square modulus of distortion function d_{10} with corrections to maximum square modulus without, vs. number M of correction sectors, by numerical computation (squares) and by least squares fit (ignoring M=1) to (const.)/M.

3. Continuation of Fig. 2.

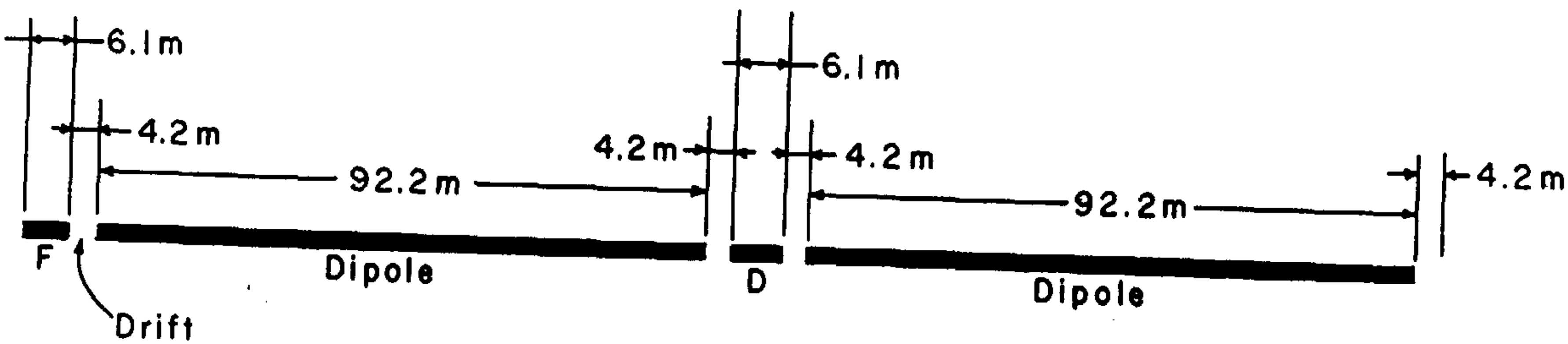

Figure 1

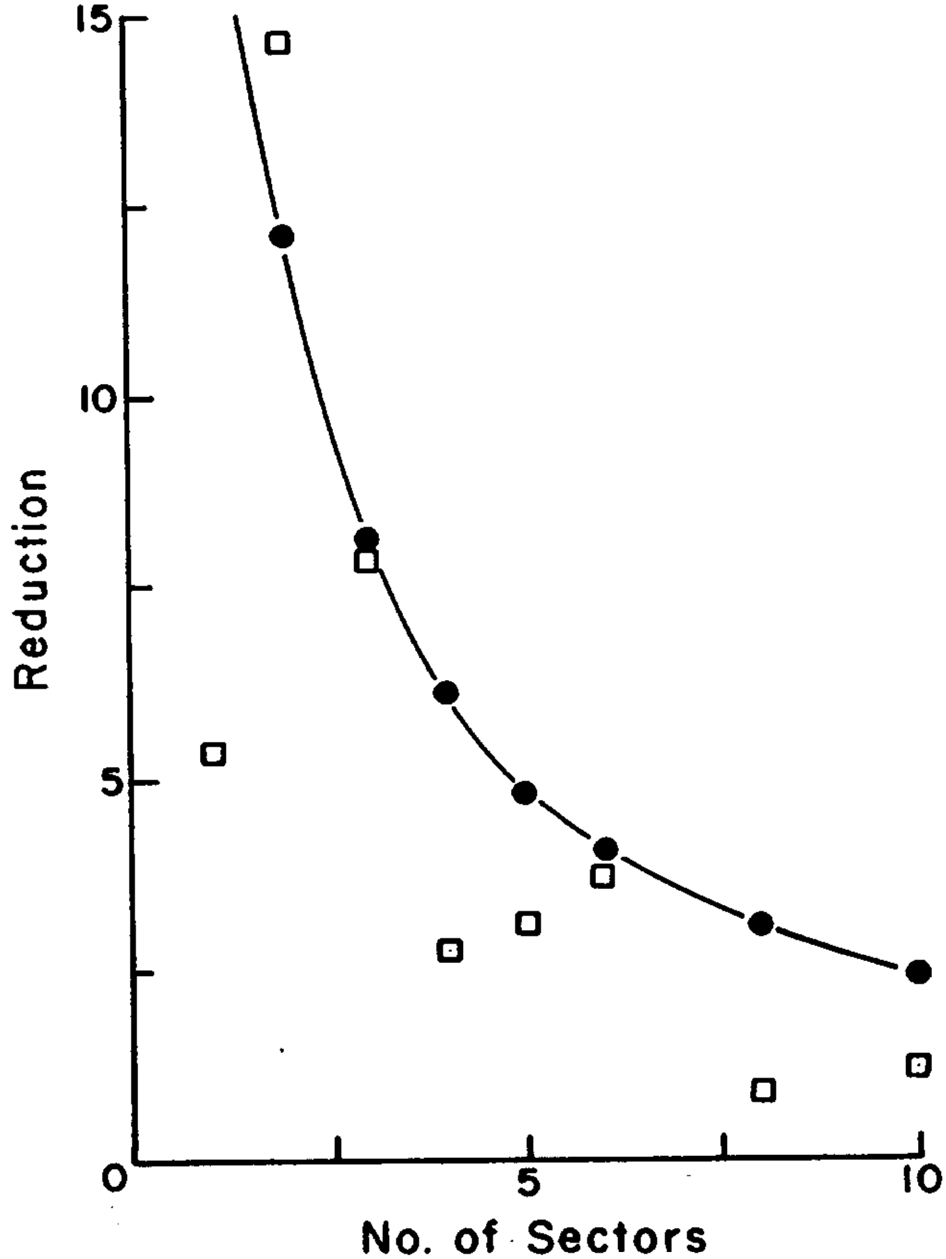

Figure 2

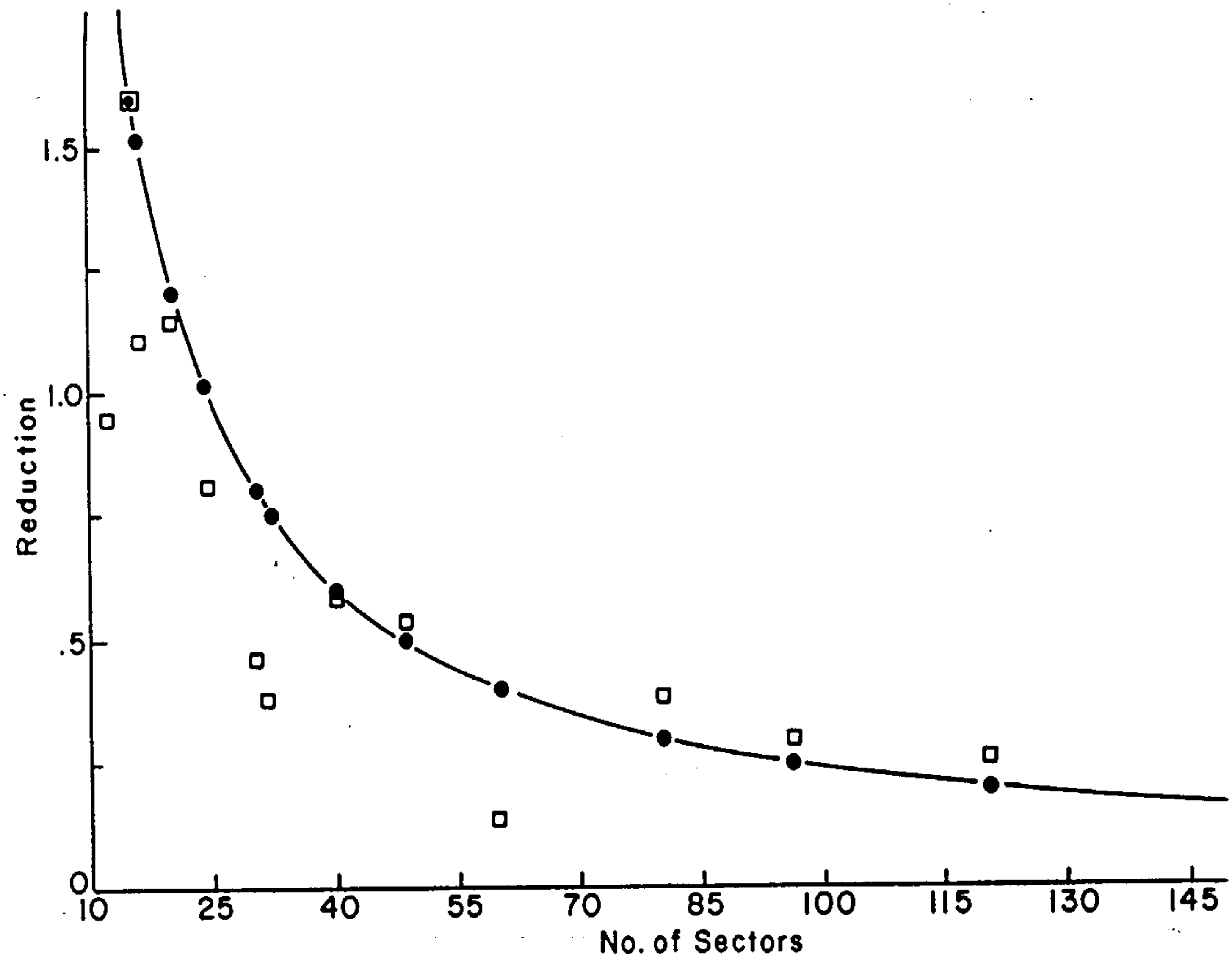
1.5
1.0
.5
0
10
25
40
55
70
85
100
115
130
145
Reduction
No. of Sectors

Figure 3