Fermi National Accelerator Laboratory

FERMILAB-Pub-85/93-A
June 1985

ON THE COSMIC AND LOCAL MASS DENSITY OF 'INVISIBLE' AXIONS

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ABSTRACT

If the Universe is axion-dominated it may be possible to detect the axions which comprise the bulk of the mass density of the Universe. The feasibility of the proposed experiments depends crucially upon knowing the axion mass (or equivalently, the PQ symmetry breaking scale) and the local mass density of axions. In an axion-dominated Universe our galactic halo should be composed primarily of axions. We calculate the local halo density to be at least 5×10^{-25} gcm^{-3}, and at most a factor of 2 larger. Unfortunately, it is not possible to pin down the axion mass, even to within an order of magnitude. In an axion-dominated Universe we place an upper limit to the axion mass of 4.5×10^{-5} eV. We give precise formulae for the axion mass in an axion-dominated Universe, and clearly point out all the uncertainties involved in pinning down the precise value of the mass.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

INTRODUCTION

One of the most pressing issues in cosmology is the nature of the ubiquitous dark matter which prevades the Universe. In the past few years one of the most popular and attractive explanations has been that the dark matter is comprised of a cosmic sea of very weakly-interacting relic particles left over from an early, very hot epoch of the Universe. Candidate relics include massive neutrinos, photinos, superheavy monopoles, and axions just to mention a few. Of these many candidates, axions[1-3] are in many ways the most intriguing possibility. The energy density in axions corresponds to large-scale, coherent scalar field oscillations, set into motion by the initial misalignment of the field with the minimum of its potential.[4,5] These axions are very weakly-interacting (and in fact were originally dubbed 'invisible') and very cold (i.e., $v/c \ll 1$). Axions behave as cold dark matter[6] and as a result should be found in, and should be the dominant component of, the halos of spiral galaxies (including our own Galaxy).

Although cosmic axions were originally thought to be so weakly-interacting as to be invisible and undetectable, Sikivie[7] has recently pointed out that they might be detected by using a very strong, inhomogeneous magnetic field to convert cosmic axions to photons (taking advantage of the axion- photon-photon coupling through the axial anomaly). The feasibility of this experiment depends upon a number of factors including the mass density of halo material (presumed to be axions) in the solar neighborhood and the mass (or equivalently, the PQ symmetry breaking scale) of these axions. In this brief paper we comment on both of these issues.

COSMIC DENSITY OF AXIONS

Cosmic axions come into existence as coherent scalar field when the temperature of the Universe is about a GeV. These oscillations are set into motion by the initial misalignment of θ, the axion degree of freedom. Initially, when the Peccei-Quinn (PQ) symmetry is broken (temperature of order f_a) θ is left undetermined because the axion is massless. However, at low temperatures ($\lesssim$ order a GeV) the axion develops a mass due to instanton effects. When the temperature of the Universe ($T \simeq T_1$) is such that the mass of the axion is about 3 times the expansion rate of the Universe the coherent field oscillations begin.[4] Estimating the energy density in these oscillations depends upon many things, including, f_a ; θ_1, the initial misalignment angle; and the finite-temperature behavior of the axion mass. A careful estimate of the energy density gives[5,11]

$$\left(\Omega_a h^2/T_{2.7}^3\right) \simeq 1.5\,\theta_1^2 N \left(f_a/10^{13}\,\mathrm{GeV}\right)^{1.22} \gamma^{-1} \Lambda_{200}^{-.67}, \quad (1a)$$

$$\left(f_a \lesssim 5\Lambda_{200}^{-2.4} \times 10^{17}\,\mathrm{GeV}\right)$$

$$\simeq 2.2\times 10^{6}\,\theta_1^2 N^{1/2} \left(f_a/10^{18}\,\mathrm{GeV}\right)^{1.5} \gamma^{-1}; \quad (1b)$$

$$\left(f_a \gtrsim 5\Lambda_{200}^{-2.4} \times 10^{17}\,\mathrm{GeV}\right)$$

where $\Omega_a = \rho_a / \rho_{crit}$, $\rho_{crit} = 1.88 h^2 \times 10^{-29}\,g\,cm^{-3}$ is the critical density, H = 100h km Mpc^{-1} sec^{-1} is the present value of the Hubble parameter, 2.7 $T_{2.7}$ K is the present value of the temperature of the microwave background radiation, Λ_{200} 200MeV is the QCD scale factor, N depends upon the PQ charges of the quarks (N = 6 in the simplest models[8]), and γ is the ratio of the entropy per comoving volume now to that when $T \simeq T_1$. [Any entropy production since $T \simeq T_1$ dilutes the energy density in axions, which can only be calculated relative to that in photons. Entropy production could result due to the very out-of-equilibrium decay of a massive particle species, such as the gravitino.[9]] The initial misalignment angle θ_1 must be in the interval $[-\pi/N, \pi/N]$ as the axion potential is periodic with period $2\pi/N$. Although θ_1 is most likely to be of order unity, in inflationary models all values of θ_1 occur in some bubble or fluctuation region with finite probability.[10,11] The zero-temperature mass of the axion and the PQ symmetry breaking scale are related by

$$m_a \simeq \frac{\sqrt{z}}{1+z} \frac{f_\pi m_\pi}{f_a} N \simeq 0.48\, N f_\pi m_\pi / f_a \quad , \qquad (2a)$$

$$\simeq 0.62 N \times 10^{-6}\,eV\, (f_a / 10^{13}\,GeV)^{-1} \quad , \qquad (2b)$$

where $z = m_u / m_d \simeq 0.55$.

Bringing together Eqns(1,2) we can solve for the predicted axion mass or symmetry breaking scale

$$(f_a / 10^{13}\,GeV) \simeq 0.72 (\Omega_a h^2 / T_{2.7}^3)^{.82}\, \theta_1^{-1.64} N^{-.82} \gamma^{.82} \Lambda_{200}^{.55} \; , \qquad (3a)$$

$$(m_a / 10^{-6}\,eV) \simeq 0.86 (\Omega_a h^2 / T_{2.7}^3)^{-.82}\, \theta_1^{1.64} N^{1.82} \gamma^{-.82} \Lambda_{200}^{-.55} \; , \qquad (3b)$$

$$(f_a \lesssim 5 \Lambda_{200}^{-2.4} \times 10^{17}\,GeV)$$

$$(f_a/10^{18}\,\mathrm{GeV}) \simeq 5.9\times10^{-5}(\Omega_a h^2/T_{2.7}^3)^{.67}\theta_1^{-1.33}N^{-.33}\gamma^{.67}, \quad (3$$

$$(m_a/10^{-11}\,\mathrm{eV}) \simeq 1.1\times10^{4}(\Omega_a h^2/T_{2.7}^3)^{-.67}\theta_1^{1.33}N^{1.33}\gamma^{-.67}. \quad (3c$$

$$(f_a \gtrsim 5\Lambda_{200}^{-2.4}\times10^{17}\,\mathrm{GeV})$$

Even taking $\Omega_a = 1$, and ignoring the dependence of m_a on the initial misalignment angle there is a great deal of leeway; allowing the following uncertainties: $\frac{1}{2} \le h \le 1$, $1 \le T_{2.7} \lesssim 1.1$, $\frac{1}{2} \le \Lambda_{200} \le 2$ results in a factor of 9 uncertainty in the predicted axion mass. Eqn(3) does not take into account the theoretical uncertainities in calculating the finite-temperature axion mass,[12] which is crucial for determining T_1, or axion damping mechanisms which have been recently discussed (although the damping expected seems likely to be very small).[13] The uncertainty in T_1 is apt to be considerable, as the finite-temperature mass is calculated in the limit $T \gg \Lambda_{QCD}$, and T_1 is typically of the order of a GeV,

$$T_1 \simeq 670\,\mathrm{MeV}\,\Lambda_{200}^{.67}(f_a/10^{13}\,\mathrm{GeV})^{-.22} \quad (f_a \lesssim 5\Lambda_{200}^{-2.4}\times10^{17}\,\mathrm{Ge}$$

$$\simeq 45\,\mathrm{MeV}(f_a/10^{18}\,\mathrm{GeV})^{-.5} \quad (f_a \gtrsim 5\Lambda_{200}^{-2.4}\times10^{17}\,\mathrm{GeV}$$

[Note that the predicted value of $\Omega_a \propto T_1^{-1}$.] For very large values of f_a this is not a problem, because the axion oscillations do not begin until T is less than Λ_{QCD} when the zero-temperature mass is the appropriate one to use. In sum, it is probably fair to say that one cannot predict the axion mass to better than an order of magnitude. To this I might add that one can however place an upper limit to the axion mass in an $\Omega_a = 1$, axion-dominated Universe, by taking: $N=6$, $\theta_1 = \pi/N$, $\gamma = 1$, $h = 1/2$, $T_{2.7} = 1.1$, $\Omega_a = 1$

$$m_a \lesssim 4.5 \times 10^{-5}\ \mathrm{eV} \tag{4}$$

LOCAL MASS DENSITY OF AXIONS

If axions are the dark matter, then they should provide the halo material in our Galaxy and other spiral galaxies.[14] Predicated upon this assumption Sikivie[7] has proposed an axion detection scheme which might be capable of detecting the local reservoir of axions. The feasibility of his (and other) detection scheme(s) depends upon the local mass density of axions which should just be the local halo mass density. As an estimate Sikivie takes

$$\rho_{halo} \simeq 10^{-24}\ g\,cm^{-3} \tag{5}$$

In this section I will derive an estimate for ρ_{halo} based upon Bahcall's models[15] of the Galaxy which is about a factor of 2 smaller, and I believe more realistic.

The Galaxy is thought to consist of three components; the disk, the spherical bulge, and the extended halo (see Fig. 1). The mass density then is given by

$$\rho_{TOT} = \rho_{disk} + \rho_{bulge} + \rho_{halo}. \tag{6}$$

The halo is believed to be well represented by an isothermal sphere model

$$\rho_{halo}(r) = \rho_0 / (r^2 + a^2) \,, \qquad (7$$

where a is the core radius of the isothermal sphere. [Such models are believed to describe self-gravitating systems of non-interacting particles; in particular, they predict the flat rotation curves, i.e., $v_{rot} \simeq$ constant, that are observed in virtually all spiral galaxies.]

Kepler's third law implies that the orbital velocity of a star in a circular orbit is given by

$$r v_{rot}^2 = G M_{halo}(r) + G M_{bulge}(r) + G M_{eq}(r) \,, \qquad (8)$$

where $M_{halo}(r)$ is the halo mass interior to the orbital radius r, $M_{bulge}(r)$ is the bulge mass interior to r, and $M_{eq}(r)$ is the equivalent central mass which is needed to account for the gravitational effect of the disk.

For r >> R the contribution of the bulge and of the disk to the rhs of Eqn(8) is negligible, while for r < R, $r v_{rot}^2$ is almost totally accounted for by the disk and bulge components. Here R $\simeq$ 9 kpc is the distance from our position to the center of the Galaxy. For reference

$$1\,\mathrm{pc} \simeq 3.09\times10^{18}\,\mathrm{cm}$$
$$1\,M_\odot \simeq 1.99\times10^{33}\,\mathrm{g}$$
$$1\,M_\odot\,\mathrm{pc}^{-3} \simeq 6.7\times10^{-23}\,\mathrm{g\,cm^{-3}}$$

In terms of ρ_o and a, $M_{halo}(r)$ is given by

$$M_{halo}(r) = 4\pi\int_0^r \rho_{halo}(r)\,r^2 dr, \qquad (9)$$
$$= 4\pi\rho_o r\,(a/r)\int_0^{r/a} x^2 dx/(1+x^2),$$
$$\equiv 4\pi\rho_o r\; J(r/a).$$

The integral J is tabulated in Table 1 for r/a = .1, .3, 1., 3., 10., 30., and 100. For r/a << 1, $J \simeq \frac{1}{3}(r/a)^2$, and for r/a >> 1, J $\simeq$ 1. Using the fact that $r v_{rot}^2 \simeq GM_{halo}(r)$ for r >> R and $v_{rot}(r \gg R) \simeq 220\,\mathrm{km\,s^{-1}}$, we can solve for ρ_o (for a discussion of the rotation curve of the Galaxy see ref. 17)

$$\rho_o \simeq 5.8\times10^{20}\,\mathrm{g\,cm^{-1}}, \qquad (10a)$$
$$\rho_{halo}(R) \simeq 7.5\times10^{-25}\,\mathrm{g\,cm^{-3}}/[1+(a/R)^2], \qquad (10b)$$
$$M_{halo}(R) \simeq 1.0\times10^{11} M_\odot\; J(R/a), \qquad (10c)$$
$$[GM_{halo}(r)/r]^{1/2} \simeq 220\,\mathrm{km\,sec^{-1}}\; J^{1/2}(r/a), \qquad (10d)$$
$$\sigma_{halo}(R) \equiv \int_{-\infty}^{\infty} \rho_{halo}[(R^2+z^2)^{1/2}]\,dz = \pi R\rho_{halo}(R)\,[1+(a/R)^2]^{1/2},$$
$$\simeq 313\,M_\odot\,\mathrm{pc}^{-2}/[1+(a/R)^2]^{1/2}, \qquad (10e)$$
$$\sigma_{halo}(R,\mathrm{few\,kpc}) \simeq 55\,M_\odot\,\mathrm{pc}^{-2}/[1+(a/R)^2]. \qquad (10f)$$

Here $\sigma_{halo}(R)$ is the total column density of halo material at our position and $\sigma_{halo}(R, \text{few kpc})$ is the column density of halo material within a few kpc of the plane of the Galaxy at our poition. Note that based upon $v_{rot}(r \gg R)$ alone, the local halo density can be at most 7.5×10^{-25} g cm^{-3}.

The orbital velocity at our radius is about 240 km sec^{-1}. The Galaxy models of Bahcall and his collaborators[15] indicate that about half the orbital velocity squared at our position is accounted for by the gravitational effects of the bulge and disk components. From Eqn(10d) and Table 1 this indicates that R/a must be about 2, implying that

$$\rho_{halo}(R) \simeq 5 \times 10^{-25} \text{ g cm}^{-3}, \tag{11a}$$

$$\sigma_{halo}(R, \text{few kpc}) \simeq 40 \, M_\odot \text{ pc}^{-2}. \tag{11b}$$

Bahcall and his collaborators have constructed detailed, 3-component models of our Galaxy. Their models[15] indicate that

$$\rho_{halo}(R) \simeq 6 \times 10^{-25} \text{ g cm}^{-3}, \tag{11c}$$

a local density which is very consistent with my estimate. In addition, Bahcall[16] has constructed detailed models of the distribution of matter in the vicinity of the sun. He uses the observed motions of stars perpendicular to the galactic disk to determine the total amout of local matter, and obtains:

$$\rho_{TOT}(R) \simeq 0.2\, M_\odot\, pc^{-3} \simeq 1.2 \times 10^{-23} g\, cm^{-3}, \qquad (12a)$$

$$\sigma_{TOT}(R, \text{few kpc}) \simeq 70\, M_\odot\, pc^{-2}, \qquad (12b)$$

Some of the quantities that he calculates in his models can be determined by a direct inventory of material in the solar neighborhood. In particular

$$\rho_{seen}(R) \simeq 0.095\, M_\odot\, pc^{-3} \simeq 6 \times 10^{-24} g\, cm^{-3}, \qquad (13a)$$

$$\sigma_{seen}(R) \simeq 30\, M_\odot\, pc^{-2}, \qquad (13b)$$

where the 'seen' component includes all the material that has been detected one way or another--stars, gas, dust, etc. Based upon his model of the solar neighborhood and the local inventory, Bahcall (as well as Oort[18] earlier) conclude that there are about equal amounts of seen and unseen material in the solar neighborhood.

Could this unseen material be halo material? Bahcall (and I believe any reasonable person would) concludes NO. To see how implausible this hypothesis is, assume that the local halo mass density were this large and that the halo density interior to R is constant (i.e., a >> R). We would then find that due to the halo material alone

$$[GM_{halo}(R)/R]^{1/2} \simeq 360 \text{ km sec}^{-1}, \qquad (14a)$$

$$\sigma_{halo}(R, \text{few kpc}) \simeq 450 \, M_\odot \, pc^{-2}, \qquad (14b)$$

$$M_{halo}(R) \simeq 2.7 \times 10^{11} \, M_\odot, \qquad (14c)$$

which is clearly in conflict with the observation data. [Bahcall has not used his models of the local neighborhood to place an upper limit on $\sigma(R, \text{few kpc})$; however, it seems likely that such a large column density of material would have a big effect on the motions of nearby stars perpendicular to the Galactic plane.] In addition, such a local density of halo material would result in:

$$v_{rot}(r \gg R, a) \simeq 620(a/R) \text{ km sec}^{-1}, \quad (15)$$

(based upon the isothermal model)--which too is clearly absurd. Bahcall[16] concludes that the unseen material must be in the form of a dark, disk component. Clearly this cannot be axions as they have no way to dissipate their gravitational energy, which they would have to do in order to settle into the disk.[19]

All of these estimates for the halo density are predicated on the assumption that the halo is well described by an isothermal sphere. Our knowledge of the ellipticity of the halo is poor; however the few observations[20] which bear on this question seem to indicate that the ratio of minor/major axes is $\gtrsim 0.8$. One might have expected that the presence of the disk would tend to flatten the halo. Numerical simulations done by Barnes[21] indicate that this is likely to be a small effect, perhaps causing a spherical halo to be compressed to an elliptical halo with minor/major axis ratio of 0.8-0.9. If this were the case for the Galaxy, then the halo density in the Galactic plane might be 10%-20% larger than my estimates.

Based upon my simple analysis of the Galactic rotation curve and Bahcall's detailed models of the Galaxy, one would conclude that the local mass density of halo material must be at least

$$\rho_{halo}(R) \simeq 5\times 10^{-25}\ \mathrm{g\ cm^{-3}}, \qquad (16)$$

in order to support the observed rotational velocities at $r \gg R$. If a/R is $\ll 1$, if v_{rot} is significantly larger than 220 km sec^{-1}, or if the halo is highly non-spherical, then the local halo density could be a factor of 2 or so higher. My estimate is about a factor of 2 smaller than Sikivie's estimate.[7]

I thank J. Bahcall, W. Bardeen, R. Pisarski, D. Seckel, P. Sikivie, and G. Steigman for valuable conversations. This work was supported in part by the DOE (at Chicago and Fermilab), the

NASA (at Fermilab), and an Alfred P. Sloan Fellowship.

REFERENCES

1. R.Peccei and H.Quinn, Phys.Rev.Lett.38, 1440(1977).

2. S.Weinberg, Phys.Rev.Lett.40, 223(1978); F.Wilczek, Phys.Rev.Lett.40, 279(1978).

3. J.Kim, Phys.Rev.Lett.43, 103(1979); M.Dine, W.Fischler, and M. Srednicki, Phys.Lett.104B, 199(1981).

4. J.Preskill, M.Wise, and F.Wilczek, Phys.Lett.120B,127(1983); L.Abbott and P.Sikivie, Phys.Lett.120B, 133(1983); M.Dine and W.Fischler, Phys.Lett.120B,137(1983).

5. P.Steinhardt and M.S. Turner, Phys.Lett.129B, 51(1983).

6. J.Ipser and P.Sikivie, Phys.Rev.Lett.50, 925(1983); M.S.Turner, F.Wilczek, and A.Zee, Phys.Lett.125B, 35, 519(E)(1983); G.Blumenthal, S.Faber, J.Primack, and M.J.Rees, Nature 311, 517(1984).

7. P.Sikivie, Phys.Rev.Lett.51, 1415(1983). L.L.Krauss, J.Moody, D. Morris, and F.Wilczek(UCSB/ITP preprint,1985)have recently suggested a new type of axion detector which exploits the spin coupling of axions to matter. Several groups are designing experiments to detect axions in the halo of the Galaxy; they include Sikivie etal (U. of Florida), Lubin etal (LBL), and Melissinos etal (Rochester).

8. P.Sikivie, Phys.Rev.Lett.48, 1156(1982).

9. S.Weinberg, Phys.Rev.Lett.48, 1303(1982); R.Scherrer and M.S.Turner, Phys.Rev.D31, 681(1985).

10. S.-Y.Pi, Phys.Rev.Lett.52, 1725(1984); in a recent preprint, A. Linde has also emphasized the point that all values of should occur with finite probability in an axion-dominated Universe which is inflationary; A.Linde, Lebedev Physical Institute preprint number 65 (1985).

11. D.Seckel and M.S.Turner, Phys.Rev.D, submitted(1985).

12. D.Gross,R.Pisarski,and L.Yaffe, Rev.Mod.Phys.53, 43(1981).

13. W.Unruh and R.Wald, Phys.Rev.D, in press(1985); M.S.Turner, Phys.Rev.D, in press(1985).

14. The most convincing evidence for the existence of galactic halos are the flat rotation curves observed in spiral galaxies. See, e.g., V.Rubin etal, Astrophys.J.225, L107(1978); 261, 439(1980); Science 220, 1339(1983); A.Bosma, Astron.J.86, 1721, 1825(1981).

15. J.Bahcall and R.M.Soneira, Astrophys.J.Suppl.44, 73(1980); J.Bahcall, M.Schmidt, and R.M.Soneira, Astrophys.J.265, 730(1983).

16. J.Bahcall, Astrophys.J.287,926(1984) and refs. therein.

17. D.Mihalas and J.Binney, Galactic Astronomy: Structure and Kinematics (Freeman, San Francisco, 1981). It is only fair to mention that for our Galaxy v_{rot} has only been measured out to about 15 kpc, which is not really >> R. This implies an additional uncertainty in determining ρ_0 (and in turn ρ_{halo}), as $\rho_0 \propto v_{rot}^2$.

18. J. Oort, Bull.Astr.Inst.Netherlands 6,249(1932); 15,45(1965).

19. For a discussion of the constraints on the composition of the dark disk component, see D.Hegyi, E.W.Kolb, and K.Olive, Astropys.J., in press(1985); B.J.Carr, Fermilab preprint (1985).

20. See, e.g., Proceedings of IAU Symposium 117: Dark Matter in the Universe, eds. J.Kormendy and E.Turner (1985).

21. J.Barnes, in preparation(1985).

Table 1 - Numerical Evolution of
$J \equiv (a/r)\int_0^{r/a} x^2 dx/(1+x^2)$

r/a	J	$J^{1/2}$
0.1	0.00331	0.0575
0.3	0.0285	0.169
1.0	0.215	0.464
3.0	0.584	0.764
10.0	0.853	0.924
30.0	0.949	0.974
100.	0.984	0.992

Figure 1