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Geometry induced by dynamical fields in String theories

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ABSTRACT

Propagation of fermionic and bosonic strings in background weak graviton field is studied, identifying the graviton field to the massless closed string excitation states. It is shown that for a bosonic string, the presence of a background graviton field is equivalent to a shift in the background metric at the first quantized level, thus producing a bosonic sigma model. For fermionic strings, the background graviton field gives rise to a supersymmetric sigma model at the first quantized level. Finally, it is shown that the presence of a background antisymmetric tensor field associated with massless excitations of type II closed strings gives rise to a Wess-Zumino term for the first quantized bosonic strings, and the supersymmetric extension of the Wess-Zumino term for the fermionic strings. Similar results for the heterotic string are also discussed.

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String theories[1] are of interest at present, since they may provide a realistic theory of nature[2-5]. Consistency of the theory demands, however, that these theories should be defined either in 10 (fermionic strings) or 26 (bosonic strings) dimensions. In order to establish connection with nature, the extra dimensions must curl up to form a compact space, with radius of the order of the Planck length[6-8]. So far, the only known way to consistently define interacting string theories with some of the extra dimensions compactified is to consider manifolds in which some of the extra dimensions form a multi-dimensional torus. Since a torus has zero curvature, the metric can be taken to be Minkowskian, the only new ingredient in this theory being that the string coordinates are periodic in nature. By compactifying the six extra dimensions in ten dimensional superstring theories in this manner, we may get various extended supersymmetric Yang-Mills and supergravity theories in four dimensions in the zero slope limit[1].

In order to make connection with nature, however, we need to compactify string theories on more complicated manifolds with non-vanishing curvature. Two different approaches[9-14] have been taken so far in compactifying string theories. One is the field theoretic approach, in which we study the zero slope limit of the string theory. Non-trivial compactification is then obtained by giving a non-zero background value to the metric[9-11]. In the second quantized string theory, this corresponds to giving a non-zero vacuum expectation value to the field, which, acting on the vacuum, creates a massless closed string state corresponding to a graviton. The second approach to string compactification is to study the first quantized string in an arbitrary background metric[12-14,10]. In this approach, the background metric is non-dynamical, and may be treated as a parameter of the theory.

The purpose of this paper is to study the connection between these two approaches; in particular, to find a relation between the background graviton field of the field theory limit, and the background metric in the first quantized string theory. We shall use the light-cone gauge formalism. In this gauge, the action for the bosonic string at the first quantized level is given by,

$$S = -\frac{1}{4\pi\alpha'} \int d\tau \int_0^\pi d\sigma \, (\partial_\tau X^i \partial_\tau X^i - \partial_\sigma X^i \partial_\sigma X^i) \qquad (1)$$

where α' is the string tension, τ, σ are the two parameters characterizing the string world sheet, and $X^i(\tau, \sigma)$ are the transverse coordinates of a particular point on the string ($i=1,..24$). In the rest of the paper, we shall denote τ, σ by z^α ($\alpha=0,1$), and raise and lower the α indices with a diagonal metric with eigenvalues 1, -1. We shall also set the string tension α' to be 1/2.

The spectrum of closed string states described by (1) has massless spin 2 excitations. The vertex for the emission of such a state from an arbitrary string state is given in the light cone gauge by the operator,

$$\frac{i\kappa}{\pi}\int_0^\pi d\sigma\ \zeta_{ij}\ (\partial_\tau-\partial_\sigma)X^i(\partial_\tau+\partial_\sigma)X^j\ e^{ik\cdot X(\tau,\sigma)} \tag{2}$$

where $\kappa\sim 1/M_{Planck}$. ζ_{ij} is the polarization tensor of the external graviton and k is its momentum. For simplicity, we assume that the external graviton has only transverse polarizations. We must also take the $k^+\to 0$ limit, since the light cone vertex given in (1) is valid only in this limit. These, however, are not strong restrictions, since ultimately we want to compactify only some of the transverse directions, and we want the components of the metric to be different from identity only in these directions.

Let us now assume that there exists a weak graviton field $h_{ij}(x)$ in the transverse directions, whose Fourier transform with respect to the transverse coordinates is given by $\tilde{h}_{ij}(k^i)$. The total transition matrix element from a string state $|\psi\rangle$ to a string state $|\phi\rangle$ in light cone 'time' T in this background is then obtained by calculating the matrix element of the operator[15],

$$\frac{i\kappa}{\pi}\int_0^T d\tau\int_0^\pi d\sigma\int d^{D-2}k\ \tilde{h}_{ij}(k)\,(\partial_\tau-\partial_\sigma)X^i(\partial_\tau+\partial_\sigma)X^j\ e^{ik\cdot X(\tau,\sigma)}$$

$$= \frac{i}{\pi} \kappa \int_0^T d\tau \int_0^\pi d\sigma \; (\partial_\tau X^i \partial_\tau X^j - \partial_\sigma X^i \partial_\sigma X^j) \, h_{ij} (X(\tau,\sigma)) \tag{3}$$

between these two states. Here we have ignored the quadratic and higher powers of the field h. Comparing this with Eq.(1) we see that we obtain the same transition matrix element to order h, if, instead of (1), we use the first quantized string Lagrangian,

$$S = -\frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \, \{ \delta_{ij} + 2\kappa \, h_{ij}(x) \} (\partial_\alpha X^i \partial^\alpha X^j)$$

$$\equiv -\frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \;\; g_{ij}(x) \, \partial_\alpha X^i \partial^\alpha X^j \tag{4}$$

Thus the presence of a background graviton field is equivalent to shifting the background metric in the first quantized string theory.

Let us now turn to the ten dimensional superstring theory. In our analysis, we shall use the old formalism of Ramond, Neveu and Schwarz[16], instead of using the new formalism of Green and Schwarz, since the two dimensional supersymmetry is more explicit in the old formalism. In the light-cone gauge, the superstring action is given by,

$$S = -\frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \, (\partial_\alpha X^i \partial^\alpha X^i + i \bar{\lambda}^i \rho^\alpha \partial_\alpha \lambda^i) \tag{5}$$

where each λ^i is a two dimensional Majorana spinor, and ρ^α ($\alpha=0,1$) are the two dimensional γ-matrices. We work in the Majorana representation,

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{6}$$

The graviton emission vertex is given by[15],

$$\frac{i\kappa}{\pi}\int_0^\pi d\sigma\, \zeta_{ij} \{(\partial_\tau - \partial_\sigma) X^i + \tfrac{1}{2} k_\ell \lambda^{\ell\dagger} (1+\gamma_p) \lambda^i\} \{(\partial_\tau + \partial_\sigma) X^j + \tfrac{1}{2} k_m \lambda^{m\dagger} (1-\gamma_p) \lambda^j\} e^{ik\cdot X(\tau,\sigma)} \tag{7}$$

where,

$$\gamma_p = \rho^0 \rho^1 \tag{8}$$

Thus, in a weak graviton field, the transition matrix element from one string state to another is obtained by calculating the matrix element of the operator,

$$\frac{i\kappa}{\pi}\int_0^T d\tau \int_0^\pi d\sigma\, [\partial_\alpha X^i \partial^\alpha X^j h_{ij}(X) - i\partial_\tau X^i \lambda^{\ell\dagger} \lambda^j h_{ij,\ell}(X) - i\partial_\sigma X^i \lambda^{\ell\dagger} \gamma_p \lambda^j h_{ij,\ell}(X) - \tfrac{1}{4} \lambda^{\ell\dagger} (1+\gamma_p) \lambda^i \lambda^{m\dagger} (1-\gamma_p) \lambda^j h_{ij,\ell m}(X) \tag{9}$$

where the comma denotes the derivatives of $h_{ij}(x)$ with respect to the x^i's. The second and the third term in (9) may be combined into the form,

$$-i\,\partial_\alpha X^i \,\bar{\lambda}^\ell \rho^\alpha \lambda^j \; h_{ij,\ell} \tag{10}$$

where,

$$\bar{\lambda}^\ell = \lambda^{\ell\dagger} \rho^0 \tag{11}$$

Since λ's are the Majorana spinors, they are real in the Majorana representation. Using this fact and the explicit representations for ρ^0 and ρ^1, we may show that $\bar{\lambda}^\ell \rho^\alpha \lambda^j$ is antisymmetric in ℓ and j. Thus $h_{ij,\ell}$ in (10) may be replaced by,

$$\frac{1}{2}\left(h_{ij,\ell} - h_{i\ell,j} - h_{j\ell,i}\right) = \frac{1}{4K}\left(g_{ij,\ell} - g_{i\ell,j} - g_{j\ell,i}\right)$$

$$\equiv -\frac{1}{2K}\,\Gamma_{\ell ij} \tag{12}$$

where g_{ij} has been defined in Eq.(4), and $\Gamma_{\ell ij}$ is the affine connection, with g_{ij} as the background metric. Using the same antisymmetry property, we may replace $h_{ij,m\ell}$ in (9) by,

$$\frac{1}{8K}\left(g_{ij,\ell m} - g_{\ell j,im} - g_{im,\ell j} - g_{\ell m,ij}\right)$$

$$= -\frac{1}{4K}\, R_{i\ell jm} + O(h^2) \tag{13}$$

Using explicit representation for ρ^0 and ρ^1 we may show that,

$$\lambda^{\ell\dagger}(1+\gamma_p)\lambda^{i}\ \lambda^{m\dagger}(1-\gamma_p)\lambda^{j}$$

$$= -\lambda^{j\dagger}\rho^0(1+\gamma_p)\lambda^{\ell}\ \lambda^{m\dagger}\rho^0(1+\gamma_p)\lambda^{i} \tag{14}$$

Finally, note that we may add to (9) a term proportional to,

$$i\, h_{ij}(X)\, \bar{\lambda}^{i}\rho^{\alpha}\partial_{\alpha}\lambda^{j} \tag{15}$$

whose effect vanishes to order h, since $\rho^{\alpha}\partial_{\alpha}\lambda^{j}$ vanishes by equations of motion. Thus the full effective action at the first quantized level is given by,

$$S = -\frac{1}{2\pi}\int d\tau \int_0^{\pi} d\sigma \,[\partial_{\alpha}X^{i}\partial^{\alpha}X^{j} g_{ij}(X)$$
$$+ i\bar{\lambda}^{i}\rho^{\alpha}\partial_{\alpha}\lambda^{j} g_{ij}(X) + i\partial_{\alpha}X^{i}\bar{\lambda}^{\ell}\rho^{\alpha}\lambda^{j}\Gamma_{\ell ij}(X)$$
$$+ \frac{1}{8}R_{i\ell mj}(X)\bar{\lambda}^{j}(1+\gamma_p)\lambda^{\ell}\ \bar{\lambda}^{m}(1+\gamma_p)\lambda^{i}\,] \tag{16}$$

which is precisely the action for a supersymmetric sigma model in two dimensions[17-19] with a background metric g_{ij}.

Next we consider the effect of introducing a background antisymmetric tensor field which corresponds to massless excitations of type II closed strings. The emission vertex for such a field is given by the same expressions as Eq.(2) and Eq.(7), except for the fact that the external polarization tensor is antisymmetric instead of symmetric in the indices i and j. Thus if $b_{ij}(x)$ denotes a background antisymmetric tensor gauge field, the action for the first quantized bosonic string has an extra piece given by,

$$-\frac{k}{\pi}\int d\tau \int_0^\pi d\sigma \quad \epsilon^{\alpha\beta}\, \partial_\alpha X^i\, \partial_\beta X^j\; b_{ij}(X) \tag{17}$$

which is precisely the Wess-Zumino term[20] for the bosonic sigma model in two dimensions, with torsion potential[18] b_{ij}.

In the case of superstring, straightforward algebraic manipulation shows that the effective action for the first quantized string acquires an extra term besides (17),

$$\begin{aligned}
-\frac{k}{\pi}\int d\tau \int_0^\pi d\sigma\, \Big[&-i\, \bar\lambda^i \rho^\alpha \lambda^j\, \epsilon_{\alpha\beta}\, \partial^\beta X^k\, S_{ijk}(X) \\
&-\frac{i}{2}\, \partial_\alpha \left(\bar\lambda^i \gamma_p\, \rho^\alpha \lambda^j\, b_{ij}(X) \right) \\
&+\frac{1}{8}\, \tilde R_{ijkl}\, \bar\lambda^i (1+\gamma_p)\lambda^k\; \bar\lambda^j (1+\gamma_p)\lambda^l \Big]
\end{aligned} \tag{18}$$

where,

$$S_{ijk} = \frac{1}{2}\left(b_{jk,i} + b_{ki,j} + b_{ij,k} \right) \tag{19}$$

$$\tilde{R}_{ijkl} = -\frac{1}{2}\left(b_{jk,il} - b_{ik,jl} - b_{jl,ik} + b_{il,jk} \right) \tag{20}$$

This is precisely the supersymmetric extension of the Wess-Zumino term.

Thus we have shown that in the presence of background weak graviton field $h_{ij}(x)$, and antisymmetric tensor field $b_{ij}(x)$, the bosonic string theory reduces to a bosonic sigma model with a background metric $\delta_{ij}+2\kappa h_{ij}(x)$ and a Wess-Zumino term proportional to $b_{ij}(x)$, at the first quantized level. The superstring theory reduces to a two dimensional supersymmetric sigma model with metric $\delta_{ij}+2\kappa h_{ij}(x)$, and a supersymmetric extension of the Wess-Zumino term proportional to b. The appearance of the Wess-Zumino term in the presence of the anti-symmetric tensor field is not surprising, since, as was shown in Ref.21, a background anti-symmetric tensor field may be interpreted as the presence of torsion in the manifold. On the other hand, the presence of the Wess-Zumino term may also be interpreted as the presence of torsion in the manifold[18]. This probably shows that the string theory has a much richer geometrical structure than even Einstein gravity, although we do not see it at present.[22]

One may try to repeat our analysis with background gauge field. In the old way of introducing gauge group in a string theory, the gauge bosons are identified with zero mass excitations of the open strings, and couples only to the ends of the open strings. If we repeat our analysis with a background gauge field, we find that the effective two dimensional field theory at the first quantized level has new terms only at the end points of the string, and hence the presence of the background gauge field cannot be interpreted as a change of the geometry of the internal manifold. For the heterotic string, however, the gauge charge is distributed on the string, and the presence of a background gauge field may be interpreted as the change in geometry of the internal manifold. It may be shown that in the presence of background graviton and antisymmetric tensor field, the string action is given by Eqs.(16), (17) and (18), the only new feature being that the λ^i's satisfy the constraint $(1-\gamma_p)\lambda^i=0$, whereas in the presence of background gauge fields $A_{iI}(x)$ associated with the diagonal generators of the gauge group, the effective action acquires an extra term,

$$
\begin{aligned}
&-\frac{k}{\pi}\int d\tau\int_0^\pi d\sigma\,\Big[A_{iI}(X^j)(\partial_\tau-\partial_\sigma)X^i(\partial_\tau+\partial_\sigma)X^I \\
&-\frac{i}{2}A_{iI,\ell}(X^j)\,\bar{\lambda}^\ell(\rho^0+\rho^1)\lambda^i(\partial_\tau+\partial_\sigma)X^I \qquad (21)
\end{aligned}
$$

where X^I's are the internal coordinates $(10\leq I\leq 25)$ satisfying the constraint $(\partial_\tau-\partial_\sigma)X^I=0$. The effective action in the presence of the off-diagonal gauge fields may be obtained from (1) using the generators of global gauge transformation derived in Ref.23.

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