Fermi National Accelerator Laboratory

FERMILAB-Pub-85/52-A
April 1985

Comment on Vittorio and Silk's "The Microwave Background Anisotropy and Decaying Particle Models of a Flat Universe"

Michael S. Turner
NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, IL 60510

Abstract

In a recent PRL Vittorio and Silk calculate the microwave anisotropy in a Universe dominated by the relativistic decay products of neutrinos which decayed in the recent past (redshift z_d). They use the upper limit of Uson and Wilkinson on the anisotropy of the microwave background at 4.5' to obtain the constraint: $z_d \leq 8$. The constraint on decaying warm particles presumably lies somewhere between 4 and 8.

Submitted to Physical Review Letters 4/85.

COMMENT ON SILK AND VITTORIO's "THE MICROWAVE BACKGROUND ANISOTROPY AND DECAYING PARTICLE MODELS OF A FLAT UNIVERSE"

Theoretical prejudice argues strongly for the $\Omega = 1$, Einstein-deSitter cosmological model (Ω is the ratio of the present energy density to the critical density). However, the observational data suggest that $\Omega_{OBS} \simeq 0.2 \pm 0.1$. Since the observational data is only sensitive to the component that clumps, theory and observation can be reconciled if there exists a smooth component that accounts for $\Omega_{SMOOTH} = 1 - \Omega_{OBS} \simeq 0.8 \pm 0.1$. Several authors[1,2] have suggested that the smooth component could be relativistic particles produced by the recent decay of a massive, relic particle species. As Turner etal.[2] point out, such "decaying particle cosmologies" (DPC)[3] are severely constrained by the observed isotropy of the 3K background. In particular, they argued that $1+z_d$ must be less than O(10) (z_d is the redshift of decay). In a recent PRL Vittorio and Silk[4] have used the measurements of Uson and Wilkinson[5] to put stringent constraints on z_d in the case that the decaying particle is a neutrino: $z_d \lesssim 4$. [The Uson-Wilkinson measurement places an upper limit of $\delta T/T < 3 \times 10^{-5}$ on the angular scale of 4.5'.] In this comment we point out that their result can be easily generalized to the case where the decaying particle is a cold relic (i.e., heavy, mass >> 1 keV), and obtain an upper limit to the redshift of decay: $z_d \lesssim 8$.

As discussed by Vittorio and Silk[4] the anisotropy at 4.5' is related to the power spectrum P(l) of density perturbations on the scale corresponding to 4.5' at the epoch of decoupling[6] ($z_{dec} \simeq 1500$). The angular scale ϕ and linear scale l are related by: $\phi \simeq 1.1' h(\ell/\mathrm{Mpc})/H_o d_o$, where the Hubble parameter today is $H = 100h$ km sec^{-1} Mpc^{-1}, the μ-wave temperature is 2.7θ K, and $H_o d_o$ is the distance to the last scattering surface in Hubble units. Here $P(l) = k^{3/2}|\delta_k|$, δ_k is the Fourier component of $\delta\rho/\rho$, $k = 2\pi/l$ is the wavenumber and l the wavelength of the Fourier component δ_k. Physically, P(l) measures the RMS fluctuation of $\delta M/M$ on the scale l. For cold relics P is a universal function[7] $P(\ell) = g\, f(\ell/\ell_{eq})$, where g is an overall normalization, and l_{eq} is the scale just entering the horizon when the Universe became matter-dominated: $\ell_{eq} \simeq 12\,\mathrm{Mpc}\, h^{-2}(1+z_d)^{-1}\theta^2$. In general $f(x) \propto x^{-m}$, where m depends on the initial spectrum of perturbations and x. For the Zel'dovich spectrum m = 0 for x<<1, m ≃ 1 for x ≃ 1, and m = 2 for x>>1.

For non-decaying cold relics $H_o d_o = 2$. Using the fact $P(l_c) = 1$ on the scale $l_c \simeq 7h^{-1}$ Mpc today and that $P(l > l_c)$ has grown linearly with the scale factor since decoupling we can solve for $P_{dec}(l_{4.5})$:

$$P_{dec}(\ell_{4.5}) \simeq (1500)^{-1}\, \frac{f(0.7h/\theta^2)}{f(0.6h/\theta^2)} \simeq 1.2^{-m}/1500$$

In the DPC things are a little difference. At a redshift of z_d the growth of perturbations begins to slow down because the Universe is becoming radiation-dominated and the growth since decoupling can be written as 1500/df, where df is the deficit factor.[3] A reasonable fit to the numerical results of ref. 3 (good to about 10%) is: $df = .5(1 + z_d)^{2/3}$, valid for $4 \lesssim z_d \lesssim 100$. In the DPC $H_o d_o = 2\,\Omega_{NR}^{-1}\left[1 - \Omega_R(1+z_d)^{1/2}/(1+\Omega_R z_d)^{1/2}\right] \simeq$

1.2-1.4 for the parameters of interest. Again we can use $P(l_c) = 1$ to solve for $P^{DPC}_{dec}(l_{4.5})$:

$$P^{DPC}_{dec}(l_{4.5}) = (1500)^{-1} df \frac{f(0.4h(1+z_d)(H_0d_0/1.3)/\theta^2)}{f(0.6h(1+z_d)/\theta^2)},$$

$$\simeq 0.8^{-m} df (H_0d_0/1.3)^{-m}/1500$$

Comparing to the previous result we see that $P^{DPC}_{dec}(l_{4.5})$ is larger by a factor of

$$P^{DPC}_{dec}/P_{dec} \simeq df\, 1.6^m (H_0d_0/1.3)^{-m}.$$

There are two effects--first the deficit factor and second the fact that $l_{4.5}$ is smaller in the DPC. For $h = .5$ (in the DPC h, must be .4-.5 in order to have a sufficiently old Universe) the predicted anisotropy for non-decaying cold relics is within a factor of 3 of the measured upper limit. Taking $m = 1$ (for the Zel'dovich spectrum) and requiring that $P^{DPC}_{dec}(l_{4.5})$ be no larger than 3.5 times $P_{dec}(l_{4.5})$ results in the upper limit: $z_d \lesssim 8$. We note that since the small-scale anisotropy restricts z_d to be less than 4 for hot relics and less than 8 for cold relics, we would expect the constraint for warm relics to lie between 4 and 8.

We thank Nicola Vittorio for stimulating our interest in this problem and for many useful conversations. This work was supported by the DOE (at Fermilab and The University of Chicago), NASA (at Fermilab), and the A.P. Sloan Foundation.

Michael S. Turner
Theoretical Astrophysics
Fermi National Accelerator Laboratory
PO Box 500
Batavia, IL 60510

References:

1. D.Dicus, E.Kolb, and V.Teplitz, Astrophys.J.221,327(1978); G.Gelmini, D.Schramm, and J.Valle, Phys.Lett.146B,311(1984); K.Olive, D.Seckel, and E.Vishniac, Astrophys.J.,in press(1985).

2. M. Turner, G. Steigman, and L. Krauss, Phys. Rev. Lett.52,2090(1984).

3. The DPC is analyzed in detail by M. Turner, Phys. Rev. D31,1212(1985). The redshift of decay is defined by $1 + z_d = (\Omega_{NR}/\Omega_R)/\beta$, where $\Omega_R(\Omega_{NR})$ is the fraction of critical density in relativistic(nonrelativistic) particles and β^{-1} is the ratio of density in decaying particles (before decay) to stable particles.

4. J. Silk and N. Vittorio, Phys.Rev. Lett.   and references therein.

5. J. Uson and D. Wilkinson, Astrophys. J.277,L1(1984).

6. To be more precise, the RMS temperature fluctuation at 4.5' depends upon an integral over a range of wavelengths around $l_{4.5}$; see ref. 4. A more careful treatment which takes this fact into account may give results which differ slightly from our's.

7. P.J.E. Peebles, Astrophys. J.263,L1(1982).