Fermi National Accelerator Laboratory

FERMILAB-PUB-85/50-T
April, 1985

Radiative corrections in grand unified theories based on N=1 supergravity II. Gauge theories

Ashoke Sen

Fermi National Accelerator Laboratory
P.O.Box 500, Batavia, IL 60510

ABSTRACT

The effect of radiative corrections in supersymmetric grand unified theories with soft supersymmetry breaking terms induced by N=1 supergravity is analyzed. It is shown that any mass hierarchy present in the limit of unbroken supersymmetry is stable under radiative corrections induced by soft supersymmetry breaking terms, provided the theory does not contain a light singlet field that transforms as a singlet under the unbroken subgroup of the theory below the grand unification scale.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

I. INTRODUCTION

In a previous paper[1] (hereafter refered to as I) we analyzed the effect of radiative corrections in supersymmetric grand unified theories (GUT's) based on N=1 supergravity. In I we ignored all the gauge interactions in the theory. In this paper we extend our analysis to theories with gauge interactions. As in I, we shall consider theories in which supersymmetry is spontaneously broken at a scale m_S of order 10^{10}-10^{11}GeV in the hidden sector[2]. If we freeze the hidden sector fields at the minimum of the potential, then the effective theory involving the observable sector superfields ϕ_i is given by a supersymmetric theory with a superpotential $W(\phi)$, together with some explicit soft supersymmetry breaking terms in the Lagrangian of the form,

$$-\{m_g z_i \frac{\partial W}{\partial z_i} + m_g (A-3) W(z) + H.c.\} - m_g{}^2 z_i^* z_i \qquad (1.1)$$

where z_i's denote the scalar components of the observable sector superfields ϕ_i, m_g denotes the gravitino mass which is of order $m_S{}^2/M_P \sim 10^2$-10^3GeV, and A is a constant of order unity, whose exact value depends on the details of the hidden sector superpotential. The presence of the explicit scalar mass term proportional to $m_g{}^2$ in (1.1) gives a common mass m_g to the scalar partners of all the light fermion fields, and as a result, such models may be made to be consistent with phenomenology.

Since the original motivation[3] for considering supersymmetric models was to solve the gauge hierarchy problem[4], we must ensure that the presence of the explicit soft supersymmetry breaking terms do not destroy the mass hierarchy. In particular, if in the supersymmetric limit (m_g=0) the model contains some masless fields z_α, these fields must acquire a mass$\lesssim m_g$ after the inclusion of the soft supersymmetry breaking terms and the effect of radiative corrections in the Lagrangian. Although the effect of radiative corrections in specific models has been discussed by various authors[5] to one loop order, no systematic analysis has been made for the general class of models of this kind. In this paper we show that the fields z_α which are massless in the supersymmetric limit, acquire a mass at most of order m_g after the inclusion of the soft supersymmetry breaking terms and radiative corrections, provided there is no light field in the theory which transforms as a singlet under the unbroken subgroup. The instability of mass hierarchy in the presence of a light singlet field has been noted before by several authors[6].

The rest of the paper will be organized as follows. In Sec.II we shall analyze the tree level potential including the soft supersymmetry breaking terms. We calculate the effective potential involving the light fields by eliminating the heavy fields using their equations of motion, and show that the result agrees with the result of Hall et. al.[7], who calculated the effective potential involving the light fields by starting from the full potential involving the observable and the hidden sector fields and eliminating all the heavy fields using their equations of motion. The reason for this agreement has been discussed in I. In Sec.III we write down all possible radiatively generated terms in the theory using superfield techniques[8,9], and estimate the order of magnitude of various terms appearing in the effective potential. In Sec.IV and V we eliminate the heavy fields using their equations of motion, and obtain an effective potential involving the light fields only. Sec.IV is devoted to theories without any light singlet fields, and without any broken generator of the gauge group transforming as a singlet of the unbroken subgroup. In Sec.V we discuss the complications arising in the presence of a broken generator of the gauge group which transforms as a singlet under the unbroken subgroup, and show that despite this complication we get a stable mass hierarchy. We summarize our results in Sec.VI.

II. LOW ENERGY EFFECTIVE POTENTIAL AT THE TREE LEVEL

We shall consider a supersymmetric gauge theory with superpotential $W(\phi)$ which is assumed to be invariant under some gauge group G. We shall denote by z_i the scalar components of the chiral superfields ϕ_i. In the globally supersymmetric theory, the potential involving the scalar fields is given by,

$$V_0 = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 + \frac{1}{2} \sum_a \left(\sum_{i,j} z_i^\dagger (T_a)_{ij} z_j \right)^2 \tag{2.1}$$

where the sums over i and j run over all the scalar fields, and the sum over a includes all the generators of the gauge group. We assume that the potential given in (2.1) has a supersymmetric minimum at $z_i = z_i^{(0)}$, where,

$$\frac{\partial W}{\partial z_i} = 0 \quad \forall\, i, \qquad \sum_{i,j} z_i^\dagger (T_a)_{ij} z_j = 0 \quad \forall\, a \tag{2.2}$$

Some of the scalar fields are assumed to have a non-zero vacuum expectation value (vev) of order M at this minimum, which breaks the gauge group G to one of its subgroups H. We assume that M is the only mass scale in the superpotential $W(\phi)$, hence the fields z_i and their fermionic partners either have mass of order M, or are massless. We shall denote by z_A, z_B, z_C,.. the fields which acquire a mass of order M at the minimum from the F term of the potential (the first term on the right hand side of Eq.(2.1)), and by z_α, z_β, z_γ,.. the massless fields which are not the goldstone bosons corresponding to the spontaneous breaking of the gauge group G to H. There is a third set of fields which we shall denote by z_K, z_L, z_M,.. whose imaginary parts are the Goldstone bosons corresponding to the gauge symmetry breaking, and are absorbed by the gauge bosons through Higgs mechanism. The real parts of these fields acquire mass of order M through the D terms of the potential (the second term on the right hand side of Eq.(2.1)), and become degenerate in mass with the gauge bosons, forming a complete vector supermultiplet. Thus, for each of the broken generators of the gauge group, there is one such field z_K.

Let us denote the broken generators of the gauge group by T_K, T_L, T_M,.. and the generators of the unbroken subgroup H by T_ρ, T_σ, T_τ,... Let us define,

$$(\mu^2)_{KL} = \sum_{i,j} z_i^{(0)\dagger} (T_K \; T_L)_{ij} \; z_j^{(0)} \tag{2.3}$$

μ^2 is a real, symmetric, positive definite matrix. Let μ_{KL} be the positive definite square root of this matrix. Then,

$$z_K = (\mu^{-1})_{KL} \{(T_L)_{ij} \; z_j^{(0)}\}^* z_i \tag{2.4}$$

The gauge invariance of W, together with Eq.(2.2) then implies that,

$$\frac{\partial^2 W}{\partial z_\alpha \, \partial z_K} = \frac{\partial^2 W}{\partial z_A \, \partial z_K} = \frac{\partial^2 W}{\partial z_K \, \partial z_L} = 0 \quad \text{at} \quad z = z^{(0)} \tag{2.5}$$

The fields z_A and z_α are orthogonal to the fields z_K. Hence,

$$\sum_i (T_K)_{Ai} \, z_i^{(0)} = \sum_i (T_K)_{\alpha i} \, z_i^{(0)} = 0$$

$$\sum_i (T_L)_{Ki} \, z_i^{(0)} = \mu_{LK} \tag{2.6}$$

Since the generators T_ρ of the unbroken subgroup H annihilates the vacuum, we have,

$$\sum_j (T_\rho)_{ij} \, z_j^{(0)} = \sum_j z_j^{(0)\dagger} (T_\rho)_{ji} = 0 \quad \forall i = \alpha, A, K \tag{2.7}$$

Also, since the fields z_α are massless, and the fields z_A acquire mass of order M, we have,

$$\frac{\partial^2 W}{\partial z_\alpha \, \partial z_\beta} = \frac{\partial^2 W}{\partial z_\alpha \, \partial z_A} = 0 \quad \text{at} \quad z_i = z_i^{(0)} \tag{2.8}$$

$$\left(\frac{\partial^2 W}{\partial z_A \partial z_B}\right)_{z^{(0)}} = M_{AB} \tag{2.9}$$

where M_{AB} is a non-singular matrix with eigenvalues of order M. If we define,

$$z_{KR} = \frac{1}{\sqrt{2}} (z_K + z_K^*) \tag{2.10}$$

as the real part of the field z_K, then the quadratic part of V_0 is given by,

$$z_A^\dagger (M^\dagger M)_{AB}\ z_B + (\mu^2)_{KL}\ z_{KR}\ z_{LR} \tag{2.11}$$

This gives the tree level mass matrix.

Let us now introduce explicit supersymmetry breaking terms in the potential of the form,

$$\Delta V = \{ m_g \sum_i z_i \frac{\partial W}{\partial z_i} + m_g (A-3) W(z) + H.c. \} + m_g^2 \sum_i |z_i|^2 \tag{2.12}$$

and write the full potential as,

$$V = V_0 + \Delta V$$

$$= \sum_i \left| \frac{\partial W}{\partial z_i} + m_g z_i^* \right|^2 + \frac{1}{2} \sum_a \Big(\sum_{i,j} z_i^\dagger (T_a)_{ij} z_j \Big)^2 + \big(m_g (A-3)\ W(z) + H.c.\big) \tag{2.13}$$

We have to minimize this potential with respect to the fields z_A and z_K for arbitrary values of the light fields z_α of order m_g, and eliminate them from the potential to get an effective potential as a function of the fields z_α. The minimization gives,

$$\sum_A \frac{\partial^2 W}{\partial z_A \partial z_B} \left(\frac{\partial W}{\partial z_A} + m_g z_A^* \right)^*$$
$$= - \sum_\alpha \frac{\partial^2 W}{\partial z_B \partial z_\alpha} \left(\frac{\partial W}{\partial z_\alpha} + m_g z_\alpha^* \right)^* - \sum_K \frac{\partial^2 W}{\partial z_B \partial z_K} \left(\frac{\partial W}{\partial z_K} + m_g z_K^* \right)^*$$
$$- m_g \left(\frac{\partial W}{\partial z_B} + m_g z_B^* \right) - m_g (A-3) \frac{\partial W}{\partial z_B}$$
$$- \sum_\alpha (z^\dagger T_\alpha z)(z^\dagger T_\alpha)_B \tag{2.14}$$

$$\sum_K (z^\dagger T_K z)(z^\dagger T_K)_L$$
$$= - \sum_\rho (z^\dagger T_\rho z)(z^\dagger T_\rho)_L - \sum_i \frac{\partial^2 W}{\partial z_i \partial z_L} \left(\frac{\partial W}{\partial z_i} + m_g z_i^* \right)^*$$
$$- m_g \left(\frac{\partial W}{\partial z_L} + m_g z_L^* \right) - m_g (A-3) \frac{\partial W}{\partial z_L}$$

$$(2.15)$$

Let us assume that the value of z_A and z_K at the minimum of the potential has the form,

$$z_A = z_A^{(0)} + z_A^{(1)} + z_A^{(2)} + \cdots \tag{2.16}$$

$$z_K = z_K^{(1)} + z_K^{(2)} + \cdots \tag{2.17}$$

where $z_i^{(n)}$ is of order $m_g{}^n/M^{n-1}$ (i=A or K). $z_K^{(0)}$ may be shown to vanish using Eqs.(2.2) and (2.4). Eqs.(2.14) and (2.15) may be solved iteratively, starting from the initial point which solves the equations,

$$\sum_{i,j} z_i^{\dagger} (T_K)_{ij}\, z_j = 0 \tag{2.18}$$

$$\frac{\partial W}{\partial z_A} + m_g z_A^{*} = 0 \tag{2.19}$$

We now assume that none of the light fields z_α transform as singlets of the unbroken gauge group H, so that $z_\alpha^{(0)}$ vanishes for all α. Using Eqs.(2.2), (2.5)-(2.8) and (2.18), (2.19), we may show that the right hand side of Eq.(2.14) is at most of order $m_g{}^2$ M, whereas the right hand side of (2.15) is at most of order m_g^3. The term of order m_g^2M on the right hand side of (2.14) is given by $m_g^2(A-3)z_B^{(0)*}$. Hence Eqs. (2.14) and (2.15) may be written as,

$$\frac{\partial W}{\partial z_A} + m_g z_A^* = m_g^2(A-3)(M^{-1})_{AB}\; z_B^{(0)*} + O(m_g^3/M) \tag{2.20}$$

$$\mu_{LK}\,(z^\dagger T_K z) = O(m_g^3) \tag{2.21}$$

If we now substitute these new solutions on the right hand side of Eqs. (2.14) and (2.15), we get back the same results, except for a change in the order m_g^3 terms, which, as we shall see, are irrelevant for calculating the effective potential to order m_g^4.

Eq. (2.20) may be solved exactly in the same way as in I to find $z_A^{(1)}$ and $z_A^{(2)}$. From Eq. (2.21) we get,

$$\begin{aligned} &\mu_{LK}\,(z^{(0)\dagger} T_K)_M\,(z_M^{(1)} + z_M^{(2)}) + \text{H.c.} \\ &+ \mu_{LK}\,(z^{(1)\dagger} T_K)_i\, z_i^{(1)} = O(m_g^3) \end{aligned} \tag{2.22}$$

Comparing terms of order m_gM^2 and m_g^2M from both sides, we get,

$$z_M^{(1)} + z_M^{(1)*} = 0 \tag{2.23}$$

$$z_M^{(2)} + z_M^{(2)*} = -(\mu^{-1})_{MK} \{ z_A^{(1)\dagger}(T_K)_{AB}\, z_B^{(1)} + z_\alpha^{\dagger}(T_K)_{\alpha\beta}\, z_\beta$$

$$+ z_A^{(1)\dagger}(T_K)_{A\alpha}\, z_\alpha + z_\alpha^{\dagger}(T_K)_{\alpha A}\, z_A^{(1)} \} \qquad (2.24)$$

Following I, we define,

$$W_{eff}(z_\alpha)$$

$$= W(z_A = z_A^{(0)} + z_A^{(1)},\ z_\alpha,\ z_K = 0) - W(z_A = z_A^{(0)} + z_A^{(1)},\ z_\alpha = 0,\ z_K = 0) \quad (2.25)$$

The potential may be written as,

$$V = \sum_\alpha \left| \frac{\partial W}{\partial z_\alpha} + m_g z_\alpha^* \right|^2 + \sum_A \left| \frac{\partial W}{\partial z_A} + m_g z_A^* \right|^2 + \{ m_g (A-3) W + H.c. \}$$

$$+ \sum_K \left| \frac{\partial W}{\partial z_K} + m_g z_K^* \right|^2 + \frac{1}{2} \sum_K |z^\dagger T_K z|^2 + \frac{1}{2} \sum_\rho |z^\dagger T_\rho z|^2$$

$$(2.26)$$

As was shown in I, the first three terms in (2.26) may be expressed as,

$$\left| \frac{\partial W_{eff}}{\partial z_\alpha} + m_g z_\alpha^* \right|^2 + m_g (A-3) \{ W_{eff}(z_\alpha) - W_{eff}^{(2)}(z_\alpha) - 2 W_{eff}^{(1)}(z_\alpha) \}$$

$$(2.27)$$

where $W_{eff}^{(2)}$ and $W_{eff}^{(1)}$ are the terms in W_{eff} quadratic and linear in z_α respectively.

Using Eqs.(2.2), (2.5) and (2.23) we may write,

$$\frac{\partial W}{\partial z_K} + m_g z_K^* = \frac{1}{2} \frac{\partial^3 W}{\partial z_K \partial z_\alpha \partial z_\beta} z_\alpha z_\beta + \frac{1}{2} \frac{\partial^3 W}{\partial z_K \partial z_A \partial z_B} z_A^{(1)} z_B^{(1)}$$

$$+ \frac{\partial^3 W}{\partial z_K \partial z_\alpha \partial z_A} z_\alpha z_A^{(1)} + O(m_g^3/M) \qquad (2.28)$$

As was shown in Ref.7, both, $(\partial^3 W/\partial z_K \partial z_\alpha \partial z_\beta)$ and $(\partial^3 W/\partial z_K \partial z_\alpha \partial z_A) z_A{}^{(1)}$ vanish at $z^{(0)}$ due to the gauge invariance of the potential. $z_A^{(1)}$, on the other hand, is given by $-m_g (M^{-1})_{AB} z_B^{(0)*}$ and is independent of z_α (see I). Hence the right hand side of (2.28) is independent of z_α to order $m_g{}^2$. As a result, the fourth term on the right hand side of (2.26) is independent of z_α to order $m_g{}^4$. As can be seen from Eq.(2.21), the fifth term in (2.26) vanishes to order $m_g{}^4$. To order $m_g{}^4$, the last term of (2.26) may be written as

$$\sum_\rho \left| z_A^{(1)\dagger} (T_\rho)_{AB} z_B^{(1)} + z_A^{(1)\dagger} (T_\rho)_{A\alpha} z_\alpha + z_\alpha^\dagger (T_\rho)_{\alpha A} z_A^{(1)} + z_\alpha^\dagger (T_\rho)_{\alpha\beta} z_\beta \right|^2$$

$$(2.29)$$

using Eq.(2.7).

Since M^{-1} must be invariant under transformations in the little group H, and z_A must be a singlet of H in order that $z_A^{(0)}$ is of order M, $z_A^{(1)} = -m_g (M^{-1})_{AB} z_B^{(0)*}$ must vanish unless the field z_A is a singlet of H. This gives,

$$(T_\rho)_{\alpha B} z_B^{(1)} = (T_\rho)_{AB} z_B^{(1)} = 0 \qquad \forall \alpha, A \qquad (2.30)$$

Hence (2.29) is given by,

$$\sum_\rho \Big| \sum_{\alpha,\beta} z_\alpha^{+} (T_\rho)_{\alpha\beta} z_\beta \Big|^2 \qquad (2.31)$$

and the full potential is given by,

$$V = \sum_\alpha \Big| \frac{\partial W_{eff}}{\partial z_\alpha} + m_g z_\alpha^* \Big|^2 + \sum_\rho \Big| \sum_{\alpha,\beta} z_\alpha^{+} (T_\rho)_{\alpha\beta} z_\beta \Big|^2$$
$$+ \Big[m_g (A-3) \{ W_{eff}(z_\alpha) - W_{eff}^{(2)}(z_\alpha) - 2 W_{eff}^{(1)}(z_\alpha) \} + H.c. \Big] \qquad (2.32)$$

which is identical to the result of Ref.7.

III. RADIATIVE CORRECTIONS

In this section we shall study the effect of radiative corrections that arise due to the presence of the soft supersymmetry breaking terms. We shall use the background gauge formalism for our analysis. Before proceeding further, however, we shall summarize the effect of radiative corrections in a general gauge theory with unbroken supersymmetry[10-12].

If F_i and D_a denote the auxiliary components of the scalar superfield ϕ_i and vector superfield V_a respectively, the only radiatively generated terms linear in the auxiliary fields have the form,

$$-D_a \, P_a(z, z^\dagger) \tag{3.1}$$

where P_a is some function of the scalar superfields. Terms containing two or more power of the auxiliary fields, on the other hand, do not have any important effect on the theory[12]. The total potential, expressed as a function of the auxiliary fields F_i, D_a and the physical scalar fields z_i, $z_i^\dagger$ may then be expressed as,

$$V = -F_i^\dagger F_i - \left(F_i \frac{\partial W}{\partial z_i} + \text{H.c.} \right) - \frac{1}{2} D_a D_a$$
$$- D_a \sum_{i,j} z_i^\dagger (T_a)_{ij} z_j - D_a \, P_a(z, z^\dagger) \tag{3.2}$$

Eliminating the auxiliary fields through their equations of motion we get,

$$V = \sum_{i} \left| \frac{\partial W}{\partial z_i} \right|^2 + \frac{1}{2} \sum_{\alpha} \left| \sum_{i,j} z_i^{\dagger} (T_{\alpha})_{ij} z_j + P_{\alpha} \right|^2 \tag{3.3}$$

Let $\{T_S\}$ denote the set of broken generators of G which transform as singlets under the unbroken subgroup H, and $\{T_N\}$ denote the set of broken generators of G which transform non-trivially under the group H. It was shown in Ref.11-12 that at any point, invariant under H,

$$P_{\rho} = P_N = 0 \qquad \forall \rho, N \tag{3.4}$$

while P_S is non-zero in general. The potential (3.3) has a zero at a new point $\tilde{z}^{(0)}$, given by,

$$\tilde{z}^{(0)}_{i} = \left[\exp\left(\sum_{s} \lambda_s T_s \right) \right]_{ij} z_j^{(0)} \tag{3.5}$$

where λ_S's are the solutions of the equations,

$$z^{(0)\dagger} e^{\lambda_{s'} T_{s'}} T_s e^{\lambda_{s''} T_{s''}} z^{(0)}$$
$$+ P_s \left(e^{\lambda_{s'} T_{s'}} z^{(0)}, z^{(0)\dagger} e^{\lambda_{s''} T_{s''}} \right) = 0 \tag{3.6}$$

The fields z_{α}, z_A and z_K are no longer eigenstates of the mass matrix at the new minimum of the new potential. However, it turns out that we may define new fields z'_α, z'_A and z'_K, which are eigenstates of the mass matrix.

$$z'_\alpha = U_{\alpha i} z_i = V_{\alpha i} \left(e^{-\lambda_s T_s}\right)_{ij} z_j \tag{3.7a}$$

$$z'_A = U_{Ai} z_i = V_{Ai} \left(e^{-\lambda_s T_s}\right)_{ij} z_j \tag{3.7b}$$

$$z'_K = U_{Ki} z_i = V_{Ki} \left(e^{-\lambda_s T_s}\right)_{ij} z_j \tag{3.7c}$$

where U is a non-singular, non-unitary (in general) matrix that may be calculated in terms of the functions P_S (see Ref.12). V has the property that the only non-zero off-diagonal elements of V^{-1} are $(V^{-1})_{KA}$ and $(V^{-1})_{K\alpha}$. The fields z'_α are massless, the fields z'_A receive mass only from the F term of the potential, the imaginary parts of z'_K are absorbed by the gauge bosons through Higgs mechanism, and the real parts of z_K receive mass only from the D term of the potential.

We shall now proceed to discuss the effect of the supersymmetry breaking terms. As in I, it is convenient to express the explicit supersymmetry breaking terms in terms of the spurion superfield[9] η,

$$\eta = m_g \theta^2 \tag{3.8}$$

The Lagrangian density, including the soft supersymmetry breaking terms, but ignoring the gauge fixing and the ghost terms, may be written as,

$$\left\{ \int d^2\theta\, W(\phi) + \text{H.c.} \right\} + \frac{1}{4} \left\{ \int d^2\theta\, W_\alpha W^\alpha + \text{H.c.} \right\}$$

$$+ \int d^2\theta\, d^2\bar{\theta}\; \bar{\phi}\, e^V \phi - \left[\int d^2\theta \left\{ \eta\, \phi_i \frac{\partial W}{\partial \phi_i} + \eta (A-3) W(\phi) \right\} + \text{H.c.} \right]$$

$$- \int d^2\theta\, d^2\bar{\theta}\;\; \bar{\eta}\, \eta\, \bar{\phi}_i\, \phi_i \qquad (3.9)$$

Here ϕ_i's are the chiral superfields, $V=V_aT_a$ is the vector superfield, and,

$$W_\alpha = \bar{D}\bar{D}\, e^{-V} D_\alpha\, e^V \qquad (3.10)$$

where D, $\bar{D}$ are the covariant derivatives in the superspace.

Let us now notice that we may replace ϕ_i by $\phi_i + \eta \tilde{z}_i^{(0)}$ everywhere in (3.9) without changing the theory. This is due to the fact that the above replacement amounts to replacing F_i by $F_i + m_g \tilde{z}_i^{(0)}$ everywhere in the Lagrangian, and, hence, after elimination of the F_i fields, yields the same potential. If we define,

$$\tilde{W}(\phi) = W(\phi) + m_g \tilde{z}_i^{(0)*} \phi_i \qquad (3.11)$$

then the new Lagrangian density may be written as,

$$\{\int d^2\theta\, \widetilde{W}(\phi) + H.c.\} + \frac{1}{4}\{\int d^2\theta\, W_\alpha W^\alpha + H.c.\}$$

$$+ \int d^2\theta\, d^2\bar\theta\ \bar\phi_i\, (e^V)_{ij}\ \phi_j$$

$$+ \int d^2\theta\, d^2\bar\theta\, [\{\bar\eta\, \widetilde{z}_i^{(0)*} (e^V - 1)_{ij}\, \phi_j + H.c.\}$$

$$+ \bar\eta\, \eta\, \widetilde{z}_i^{(0)*} (e^V - 1)_{ij}\ z_j^{(0)}]$$

$$- \int d^2\theta\, (\eta\, \hat\phi_i \frac{\partial W}{\partial \phi_i} + \eta\, (A-3)\, W(\phi) + H.c.)$$

$$- \int d^2\theta\, d^2\bar\theta\ \bar\eta\, \eta\, (\bar\phi_i\, \phi_i - \widetilde{z}_i^{(0)*} \widetilde{z}_i^{(0)}) \qquad (3.12)$$

where,

$$\hat\phi_i = \phi_i - \widetilde{z}_i^{(0)} \qquad (3.13)$$

Let us now divide each of the fields ϕ_i, $\bar\phi_i$ and V_a into background and quantum superfields as follows[11],

$$e^V = e^{V_a^{(b)} T_a/2}\ e^{V_a^{(q)} T_a}\ e^{V_a^{(b)} T_a/2} \qquad (3.14)$$

$$\phi = \phi^{(b)} + \phi^{(q)} \qquad (3.15)$$

$$\bar\phi = \bar\phi^{(b)} + \bar\phi^{(q)} \qquad (3.16)$$

and define,

$$\tilde{\phi}^{(b,q)} = e^{V_a^{(b)}T_a/2}\,\phi^{(b,q)} \qquad (3.17)$$

$$\bar{\tilde{\phi}}^{(b,q)} = \bar{\phi}^{(b,q)}\,e^{V_a^{(b)}T_a/2} \qquad (3.18)$$

The action (3.12) is invariant under a background gauge transformation, the details of which have been discussed in Ref.11. We may choose the gauge fixing term to be also invariant under background gauge transformation (see Ref.12 for a particular choice). As a result, the full effective action, including the quantum corrections, must also be invariant under the background gauge transformation. Using the background gauge transformation we may bring the background gauge fields in the Wess-Zumino gauge. The full effective potential may then be considered as a function of the auxiliary components D_a of $V_a^{(b)}$ and F_i, $F_i^\dagger$ of ϕ_i, $\bar{\phi}_i$, as well as the physical scalar components z_i, $z_i^\dagger$.

The possible radiatively generated terms in the effective action in this theory is constrained due to a theorem due to Grisaru, Rocek and Siegel[8], which states that it must be of the form,

$$\int\Big(\prod_i d^4x_i\Big)\int d^4\theta\; f\big(\phi^{(b)}(x_i,\theta), \bar{\phi}^{(b)}(x_j,\theta), V^{(b)}(x_k,\theta), \eta, \bar{\eta}\big) \qquad (3.19)$$

where f is a function of various superfields and their covariant derivatives at different space-time points, but at the same point in the θ, $\bar{\theta}$ space. For the time being, we shall analyze the effect of only those terms which contain at most one power of the auxiliary field. In the absence of supersymmetry breaking, the only possible terms of this kind are the ones given in Eq. (3.1), since we need two powers of θ and two powers of $\bar{\theta}$ in order to saturate the $d^4\theta$ integral in (3.19). In the presence of explicit supersymmetry breaking terms in the Lagrangian, we may use some powers of θ, $\bar{\theta}$ from the spurion fields η, $\bar{\eta}$ to saturate some of the θ integrals in (3.19). The most general radiatively generated term in the effective action, linear in the auxiliary fields, is then given by,

$$D_a P_a(z, z^\dagger) + m_g D_a \hat{P}_a(z, z^\dagger) + (m_g F_i g_i(z, z^\dagger) + H.c.)$$
$$- m_g^2 f(z, z^\dagger) \qquad (3.20)$$

In the above equation we have dropped the superscript (b) from all the fields. In the rest of the paper, any field without a superscript will always refer to background field. Adding (3.20) to (3.12), and using the equations of motion for the F_i and the D_a fields, we get,

$$F_i^* = -\left(\frac{\partial \tilde{W}}{\partial z_i} + m_g g_i\right) \equiv -\hat{F}_i^*(z_j, z_j^\dagger) \qquad (3.21)$$

$$D_a = -(z^\dagger T_a z + P_a + m_g \hat{P}_a) \equiv -\hat{D}_a(z_j, z_j^\dagger) \qquad (3.22)$$

and the full effective potential is given by,

$$V_{eff} = \hat{F}_i^* \hat{F}_i + \frac{1}{2} \hat{D}_a \hat{D}_a + [m_g (z_i - \tilde{z}_i^{(0)}) \frac{\partial W}{\partial z_i} + m_g (A-3) W + H.c.] + m_g^2 f(z, z^\dagger) + m_g^2 z_i^* z_i \qquad (3.23)$$

Sum over repeated indices is implied in the above equation.

We shall now estimate the order of magnitude of the various functions f, g_i and $\hat{P}_a$, and their derivatives with respect to the various fields. Since all the ultraviolet divergences in this theory are logarithmic[11], naive dimensional arguments tell us that the functions f, g_i, $\hat{P}_a$, $\partial f/\partial z_i$, $\partial \hat{P}_a/\partial z_i$ and $\partial P_a/\partial z_i$ are at most of order M^2, M, M, M, 1, and M respectively in general, up to logarithmic factors. This is based on the assumption that these functions do not contain any extra power of M/m_g, i.e. they do not suffer from any power law infrared divergence in the $m_g \to 0$ limit. Since all the operators under consideration have mass dimension less than or equal to 4, this is a reasonable assumption[13]. Gauge invariance puts further constraints on these functions. First, note that since all the light fields z_α transform non-trivially under H, g_α and $\partial f/\partial z_\alpha$ must vanish at $z=\tilde{z}^{(0)}$, and must be of order m_g for arbitrary values of z_α of order m_g. Similar arguments show that P_N vanishes at $\tilde{z}^{(0)}$, whereas P_S is in general of order M^2. On the other hand, $\partial P_N/\partial z_\alpha$ may, in general, be of order M, whereas $\partial P_S/\partial z_\alpha$ must be of order m_g for z_α of order m_g, since it transforms non-trivially under H. The z_α dependent part of P_S is thus of order $m_g{}^2$ for arbitrary vev of z_α of order m_g. Similar analysis for $\hat{P}_a$ shows that $\hat{P}_N$ is of order m_g for $z_\alpha \sim m_g$, whereas $\hat{P}_S$ may be of order M at $\tilde{z}^{(0)}$. The z_α dependent part of $\hat{P}_S$ is, of course, again of order m_g.

The analysis of P_ρ and $\hat{P}_\rho$ is more tricky. As was shown in Ref.12, in a theory with unbroken supersymmetry, the dependence of the radiatively generated terms linear in D_ρ on $V^{(b)}$ must come through the fields $\tilde{\phi}^{(b)}$, $\bar{\tilde{\phi}}^{(b)}$ defined in Eqs.(3.17) and (3.18). As a result, P_ρ must be of the form,

$$P_\rho = \sum_{i,j} \left(K_i(z, z^\dagger)^\dagger (T_\rho)_{ij}\, z_j + H.c. \right) \tag{3.24}$$

where K_i is some function of z, $z^\dagger$. Since (3.24) is quadratic in the fields K_i, z_j, both of which transform non-trivially under H, P_ρ is at most of order $m_g{}^2$ for arbitrary values of the z fields of order m_g. $\partial P_\rho/\partial z_i$, on the other hand, is at most of order m_g.

In order to carry out a similar analysis for $\hat{P}_\rho$, we must find out how the radiatively generated terms due to the presence of explicit soft supersymmetry breaking terms in (3.12) may depend on $V_\rho^{(b)}$. The analysis may be done by setting all the background gauge fields except the ones lying in the unbroken subgroup H to zero, since we are only interested in terms linear in the auxiliary fields D_ρ. The analysis is best carried out by using the improved supergraph rules developed by Grisaru and Zanon[14]. According to their analysis, the contribution from all the supergraphs, except the one loop graphs with only background gauge fields as external lines[16], may be cast in a form so that that the Feynman rules for evaluating the graph depend on $V_\rho^{(b)}$ only through the combination $W_\alpha^{(b)}$, $\bar{W}_{\dot\alpha}^{(b)}$, $\Gamma_{\alpha\dot\alpha}^{(b)}$, $\tilde\phi^{(b)}$ or $\bar{\tilde\phi}^{(b)}$, but not explicitly through the connections $\Gamma_\alpha^{(b)}$ or $\bar\Gamma_{\dot\alpha}^{(b)}$, or the gauge potential $V_\rho^{(b)}$. Here,

$$\Gamma_{\alpha\dot\alpha}^{(b)} = e^{-V_a^{(b)}T_a/2}\{D_\alpha, \bar{D}_{\dot\alpha}\}\, e^{V_a^{(b)}T_a/2} \qquad (3.25)$$

Since the terms linear in D_ρ appear with coefficient $\theta\bar\theta$ in $\Gamma_{\alpha\dot\alpha}$, θ in W_α and $\bar\theta$ in $\bar{W}_{\dot\alpha}$, none of these terms, by themselves, may saturate the $d^4\theta$ integral in (3.19) to give a non-zero result. We may try to saturate this integral by picking up some power of η from the explicit supersymmetry breaking terms, but we need at least one power of η and one power of $\bar\eta$. This gives a contribution of order $m_g{}^2$ to $m_g\hat{P}_\rho$. The dependence of the effective action on $V_\rho^{(b)}$ through the fields $\tilde\phi^{(b)}$ and $\bar{\tilde\phi}^{(b)}$, on the other hand, has the form of

(3.24), and must be of order m_g^2 for arbitrary values of z_α of order m_g. One may also wonder whether the dependence of the Lagrangian (3.12) on $V_\rho^{(b)}$ through the explicit soft supersymmetry breaking terms linear in η may produce a contribution to $\hat{P}_\rho$ of order M. The only potentially dangerous term is the fourth term in (3.12). However, the vertex generated by this term has the same structure as the vertices of a supersymmetric theory, except that the external background field $\phi_i^{(b)}$ is replaced by $\eta z_i^{(0)}$. Hence the dependence of the effective action on $V_\rho^{(b)}$ through this vertex may be analyzed in the same way as in the theory with unbroken supersymmetry, and the contribution to $\hat{P}_\rho$ may be shown to be at most of order m_g for $z_\alpha \sim m_g$. Thus the net result of our analysis is that $P_\rho + m_g \hat{P}_\rho$ is at most of order m_g^2 for arbitrary values of z_α of order m_g, whereas its first derivative with respect to any field is of order m_g.

IV. ELIMINATION OF THE HEAVY FIELDS

In this section we shall eliminate the heavy fields z'_A and z'_K from the potential by minimizing the potential with respect to these fields, and find an effective potential involving the light fields only. Let us define,

$$g_i' = g_j \, (U^{-1})_{ji} \tag{4.1}$$

$$\hat{F}_i'^{*} = \hat{F}_j^{*} (U^{-1})_{ji} = \left(\frac{\partial \widetilde{W}}{\partial z_j'} + m_g g_j' \right) \tag{4.2}$$

where the matrices U_{ij} are as defined in Eq.(3.7). The potential (3.23) may then be written as,

$$V = (U U^{\dagger})_{ij} \, \hat{F}_i'^{*} \hat{F}_j' + \tfrac{1}{2} \hat{D}_a \hat{D}_a$$
$$+ [m_g \{ \hat{z}_i' \frac{\partial W}{\partial z_i'} + (A-3) W \} + H.c.] + m_g^2 [f + (U U^{\dagger})^{-1}_{ij} z_i'^{*} z_j'] \tag{4.3}$$

where,

$$\hat{z}_i' = U_{ij} (z_j - \tilde{z}_j^{(0)}) = z_j' - U_{ij} \tilde{z}_j^{(0)} \equiv z_j' - z_j'^{(0)} \tag{4.4}$$

We now have to minimize the potential with respect to the fields z'_A and z'_K for arbitrary values ($\sim m_g$) of the fields z_α. Minimization of V with respect to the fields z'_C gives,

$$(UU^\dagger)_{BA} \frac{\partial \hat{F}'^*_B}{\partial z'_C} \hat{F}'_A$$

$$= -(UU^\dagger)_{\alpha i} \frac{\partial \hat{F}'^*_\alpha}{\partial z'_C} \hat{F}'_i - (UU^\dagger)_{Ki} \hat{F}'_i \frac{\partial \hat{F}'^*_K}{\partial z'_C} - (UU^\dagger)_{B\alpha} \frac{\partial \hat{F}'^*_B}{\partial z'_C} \hat{F}'_\alpha$$

$$- (UU^\dagger)_{BK} \frac{\partial \hat{F}'^*_B}{\partial z'_C} \hat{F}'_K - m_g (UU^\dagger)_{ij} \frac{\partial g'^*_j}{\partial z'_C} \hat{F}'^*_i - \hat{D}_a \frac{\partial \hat{D}_a}{\partial z'_C}$$

$$- m_g{}^2 \frac{\partial f}{\partial z'_C} - m_g{}^2 (UU^\dagger)^{-1}_{iC} z'^*_i - m_g(A-2) \frac{\partial W}{\partial z'_C} - m_g \hat{z}'_i \frac{\partial^2 W}{\partial z'_i \partial z'_C}$$

$$(4.5)$$

Minimization of the potential with respect to the fields z'_K yields,

$$\frac{\partial \hat{D}_L}{\partial z'_K} \hat{D}_L$$

$$= - \hat{D}_\rho \left(\frac{\partial \hat{D}_\rho}{\partial z'_K} \right) - (UU^\dagger)_{ij} \hat{F}'_j \frac{\partial \hat{F}'^*_i}{\partial z'_K} - m_g (UU^\dagger)_{ij} \hat{F}'^*_i \frac{\partial g^*_j}{\partial z'_K}$$

$$- m_g{}^2 \frac{\partial f}{\partial z'_K} - m_g{}^2 (UU^\dagger)^{-1}_{iK} z'^*_i - m_g(A-2) \frac{\partial W}{\partial z'_K}$$

$$- m_g \hat{z}'_i \frac{\partial^2 W}{\partial z'_i \partial z'_K} \qquad (4.6)$$

As in Sec.II, we seek solutions of the above equations of the form,

$$z'_A = z'^{(0)}_A + z'^{(1)}_A + z'^{(2)}_A + \cdots \tag{4.7}$$

$$z'_K = z'^{(0)}_K + z'^{(1)}_K + z'^{(2)}_K + \cdots \tag{4.8}$$

We start from the ansatz that at the minimum of the potential, for arbitrary values of z'_α of order m_g,

$$\hat{F}'_A \lesssim m_g^{\,2}, \qquad \hat{D}_K \lesssim m_g^{\,2} \tag{4.9}$$

We shall now try to estimate the various terms on the right hand side of Eq.(4.5) and (4.6) for arbitrary vev of the shifted fields $\hat{z}'_i$ of order m_g. For this we use the equations,

$$\frac{\partial W}{\partial z'_\lambda} = 0 \quad \text{at} \quad z'_\lambda = z'^{(0)}_\lambda \tag{4.10}$$

$$\frac{\partial^2 W}{\partial z'_\alpha \partial z'_\beta} = \frac{\partial^2 W}{\partial z'_\alpha \partial z'_K} = \frac{\partial^2 W}{\partial z'_K \partial z'_L} = \frac{\partial^2 W}{\partial z'_\alpha \partial z'_A} = \frac{\partial^2 W}{\partial z'_K \partial z'_A} = 0$$
$$\text{at} \quad z_\lambda = \tilde{z}^{(0)}_\lambda \tag{4.11}$$

(These equations have been derived in Ref.12). The matrix U_{ij} is a singlet under the unbroken subgroup H, hence the fields z'_α transform non-trivially under H if z_α s do. Using this and Eqs.(4.10), (4.11) we may show that,

$$\hat{F}'_\alpha = \frac{\partial \tilde{W}}{\partial z'_\alpha} + m_g g'_\alpha \sim m_g^2 \quad \text{for} \quad \hat{z}'_i \sim m_g \qquad (4.12)$$

Next, let us turn to $\hat{F}'_K$, which may be divided into two classes, $\hat{F}'_N$ and $\hat{F}'_S$. Since the generators T_N transform non-trivially under the gauge group H, g'_N as well as z'_N transform non-trivially under H, and are at most of order m_g. $\partial\tilde{W}/\partial z'_S$ may be shown to be $O(m_g^2)$, using Eq.(3.7), together with the property of V discussed below Eq.(3.7). However g'_S may, in general, be of order M. As a result, F'_S is, in general, of order $m_g M$ at the minimum we are considering. The stability of mass hierarchy in the presence of such terms will be the subject of discussion in the next section. In this section, however, we shall consider only those theories where none of the broken generators of the group transform as singlet under the unbroken subgroup of the gauge group, so that,

$$\hat{F}'_K \lesssim m_g^2 \quad \forall\, K \qquad (4.13)$$

Finally, Since both, $z^\dagger T_\rho z$ and $P_\rho + m_g P_\rho$ are of order m_g^2, $\hat{D}_\rho$ must be of order m_g^2.

Let us now turn to the various derivative terms. We have,

$$\frac{\partial \hat{F}'_B}{\partial z'_C} = \frac{\partial^2 W}{\partial z'_B \partial z'_C} + O(m_g) = M_{BC} + O(m_g) \qquad (4.14)$$

All other $\partial F'_i/\partial z'_j$ are, in general, also bounded by terms of order M. $(\partial \hat{D}_\rho/\partial z'_i)$ is of order m_g, since $\hat{D}_\rho$ is of order $m_g{}^2$ for arbitrary values of $\hat{z}'_i$ of order m_g. $(\partial \hat{D}_K/\partial z'_C)$ may be written as,

$$\left[\frac{\partial}{\partial z'_C}(z^\dagger T_K z + P_K)\right] + m_g \frac{\partial}{\partial z'_C}(\hat{P}_K) \qquad (4.15)$$

Since all the z' fields are defined in such a way that z'_C does not receive any contribution to its mass from the D term of the potential in the limit of unbroken supersymmetry, the term inside the bracket in Eq.(4.15) must vanish at $z = \tilde{z}^{(0)}$.[12] Hence (4.15) is at most of order m_g for arbitrary $\hat{z}'_i \sim m_g$. Similar arguments show that $\partial \hat{D}_K/\partial z'_\alpha$ is also of order m_g. $\partial \hat{D}_K/\partial z'_L$, on the other hand, is a non-singular matrix with eigenvalues of order M, whose first term in the perturbation expansion is given by μ_{KL}.

Using the ansatz (4.9), and the estimate for various functions given above, we can show that the right hand sides of Eqs.(4.5) and (4.6) are of order $m_g{}^2 M$ at most. Since

$\partial\hat{F}'_B/\partial z'_C$ and $\partial\hat{D}_L/\partial z'_K$ are both non-singular matrices with eigenvalues of order M, the solutions of Eqs.(4.5) and (4.6) give $\hat{F}'_C \sim m_g{}^2$ and $\hat{D}_K \sim m_g{}^2$. These satisfy the ansatz (4.9), showing that self consistent solutions of the equations of motion may be obtained which satisfy (4.9).

The solution of the first of the Eqs.(4.9) may be obtained as in I. This gives,

$$\hat{z}'_A = -(M^{-1})_{AB}\; m_g \{ z'^{(0)}_B + g'_B (z'_\lambda = z^{(0)}_\lambda) \} + O(m_g^2/M) \tag{4.16}$$

Thus we see that $z'^{(1)}_A$ is independent of z'_α. The second equation of (4.9) gives,

$$D_K(z'^{(0)}) + \left(\frac{\partial D_K}{\partial z'_\alpha}\right)_0 z'_\alpha + \left(\frac{\partial D_K}{\partial z'_A}\right)_0 z'^{(1)}_A + \left(\frac{\partial D_K}{\partial z'_L}\right)_0 z'^{(1)}_L = O(m_g^2) \tag{4.17}$$

As has been argued before, both $(\partial\hat{D}_K/\partial z'_C)$ and $(\partial\hat{D}_K/\partial z'_\alpha)$ are of order m_g. Also, since $\hat{D}_K(z'^{(0)})$ vanishes in the limit of unbroken supersymmetry, we have,

$$\hat{D}_K(z' = z'^{(0)}) = m_g \hat{P}_K (z' = z'^{(0)}) \tag{4.18}$$

Thus,

$$\left(\frac{\partial D_K}{\partial z'_L}\right)_0 \hat{z}'_L = -m_g \hat{P}_K (z' = z'^{(0)}) + O(m_g^2) \tag{4.19}$$

$\hat{P}_K(z'^{(0)})$ is of order m_g for K=N, and is of order M for K=S. This O(M) contribution, however, is independent of z_α. As a result, we get,

$$z'^{(1)}_N = 0 \ , \qquad z'^{(1)}_S = \text{constant} \tag{4.20}$$

We may now proceed to evaluate the effective potential involving z'_α. Contribution from the first two terms in (3.23) is at most of order m_g^4. The contribution from the term in the square bracket may be analyzed by using Taylor series expansion of these functions about the point $z'=z'^{(0)}$. The contribution to order $m_g^3 M$ is given by,

$$m_g \left[z'^{(1)}_C \left(\frac{\partial^2 W(z)}{\partial z'_A \partial z'_B} \right)_0 z'^{(1)}_B + (A-3)\, W(z'^{(0)}) + \frac{A-3}{2} \left(\frac{\partial^2 W(z)}{\partial z'_A \partial z'_B} \right)_0 z'^{(1)}_A \, z'^{(1)}_B \right] + H.c. \tag{4.21}$$

This contribution, however, is independent of z'_α, since $z'^{(1)}_A$ is independent of z'_α. In other words, the z'_α dependent contribution from these terms is at most of order m_g^4.

Finally, let us turn to the last two terms in (3.23). Contribution from the $m_g^2 f$ term may be written as,

$$m_g^2 f(z_\lambda = \tilde{z}_\lambda^{(0)}) + m_g^2 \left\{ \left(\frac{\partial f}{\partial z'_\alpha} \right)_0 z'_\alpha + \left(\frac{\partial f}{\partial z'_A} \right)_0 z'^{(1)}_A + \left(\frac{\partial f}{\partial z'_K} \right)_0 z'^{(1)}_K \right\} + O(m_g^4) \tag{4.22}$$

The term proportional to $(\partial f/\partial z'_\alpha)_0$ vanishes, whereas the other terms are independent of z'_α to order $m_g{}^3M$. The $m_g{}^2(UU^\dagger)^{-1}_{ij}z'_i{}^*z'_j$ term may be analyzed in the same way, and shown to be independent of z'_α to order $m_g{}^3M$.

This shows that the net effective potential involving the fields z'_α is of order $m_g{}^4$ for arbitrary valies of z'_α of order m_g. As a result, the massless particles of the unbroken supersymmetric theory acquire masses of order m_g or less, after we add the supersymmetry breaking terms, and include the effect of radiative corrections.

Finally, we shall discuss the effect of the terms with two or more powers of the auxiliary fields. We may analyze the effect of these terms by using an iterative procedure, in which we first calculate the effective potential, as well as F_i and D_a without the terms with two or more powers of the auxiliary fields; then substitute these values of F and D in the new terms with two or more powers of the auxiliary fields to get a new value of F and D, and a new effective potential. Using the fact that $\hat{F}'_i$ and $\hat{D}_a$ are all of order $m_g{}^2$ at the minimum of the potential with respect to the heavy fields, we see that the new terms will contribute at most a term of order $m_g{}^4$ to the effective potential. Also since $\partial\hat{F}'_i/\partial z'_j$ and $\partial\hat{D}_a/\partial z'_j$ are both at most of order M, the effect of these new terms on Eqs. (4.5) and (4.6) would be to add terms of order $m_g{}^2M$ on the right hand side of these equations. Since such terms did not have any effect on our analysis, we conclude that the terms involving two or more powers of the auxiliary fields do not change the results of this section.

V. EFFECT OF RADIATIVE CORRECTIONS IN THE PRESENCE OF BROKEN GENERATORS WHICH COMMUTE WITH THE UNBROKEN SUBGROUP:

In this section we shall consider theories in which some of the broken generators T_S of the gauge group transform as singlets under the unbroken subgroup of the gauge group. If we minimize the potential with respect to the heavy fields, then, as was pointed out in Sec. IV, the values of F_S at the minimum of the potential may be of order $m_g M$. Hence, in analyzing the effect of radiative corrections in such theories, we not only have to include terms linear in the auxiliary fields, but must also include terms quadratic in F, F^*. (We shall argue later that terms involving cubic and higher powers of F, F^* do not change any of the results that we shall derive). The important radiative corrections may then be written as,

$$-\sum_{a} D_a \{ P_a(z,z^\dagger) + m_g \hat{P}_a(z,z^\dagger) \} - \{ m_g F_i g_i(z,z^\dagger) + H.c. \}$$
$$+ m_g^2 f(z,z^\dagger) + F_i^* h_{ij} F_j \tag{5.1}$$

Let us define,

$$F_i' = U_{ij} F_j \tag{5.2}$$

Adding (5.1) to (3.12) and using Eqs.(3.7), (4.1) and (5.2), we may express the full potential as,

$$- F_i'^{*} \{ (U^{\dagger})^{-1} (I+h) U^{-1} \}_{ij} F_j' - \left(F_i' \frac{\partial \tilde{W}}{\partial z_i'} + H.c. \right)$$

$$-(m_g F_i' g_i' + H.c.) + m_g^2 f(z, z^{\dagger})$$

$$+ \{ m_g (z_i - \tilde{z}_i^{(0)}) \frac{\partial W}{\partial z_i} + m_g (A-3) W(z) + H.c. \} + m_g^2 z_i^* z_i$$

$$- \frac{1}{2} D_a D_a - D_a (P_a + m_g \hat{P}_a) \qquad (5.3)$$

Eliminating the F_i' and D_a fields, we get,

$$V = \hat{F}_i'^{*} \{ U (I+h)^{-1} U^{\dagger} \}_{ij} \hat{F}_j' + m_g^2 f(z, z^{\dagger}) + \frac{1}{2} \hat{D}_a \hat{D}_a$$

$$+ \{ m_g (z_i' - U_{ij} \tilde{z}_j^{(0)}) \frac{\partial W(z)}{\partial z_i'} + m_g (A-3) W(z) + H.c. \}$$

$$+ m_g^2 z_i^* z_i \qquad (5.4)$$

with $\hat{F}_i'$ and $\hat{D}_a$ as defined in Secs.III and IV.

Let us define,

$$N_{ij} = \{ U (I+h)^{-1} U^{\dagger} \}_{ij} \qquad (5.5)$$

We shall now study some properties of the functions N_{ij}. At $z=\tilde{z}^{(0)}$, N_{ij} connects fields belonging to the same representation of the unbroken subgroup H. For $z_\alpha' \sim m_g$, N_{ij}

may connect fields belonging to the different representations of H. However, as we shall show now, the contribution to $(N^{-1})_{Sj}$ for $j=\alpha$, K, or A depends on z'_α only at order m_g/M, and as a result, $(N^{-1})_{Sj}$ is of order m_g/M unless the field z_j transforms as a singlet under H. As a corollary of the above result, we also get $(N^{-1})_{S\alpha} \sim m_g/M$ for $z_\alpha \sim m_g$.

In order to show that $(N^{-1})_{Sj}$ depends on z'_α only at order m_g/M, we must first study the coupling of F'_i to the various fields of the theory. The tree level coupling of F'_i is given by the $F'_i(\partial W/\partial z'_i)$ term in (5.3). Let us define,

$$z''_i = \left(e^{-\lambda_S T_S}\right)_{ij} z_j = \left(V^{-1}\right)_{ij} z'_j \tag{5.6}$$

using Eqs. (3.7). Then,

$$F'_i \frac{\partial W}{\partial z'_i} = F'_i \left(V^{-1}\right)_{ji} \frac{\partial W}{\partial z''_j} \tag{5.7}$$

Using the gauge invariance of W under complex gauge transformation we may show that $W(z)=W(z'')$. Also, since $\tilde{z}^{(0)}=\exp(\lambda_S T_S)\, z^{(0)}$, $z''=z^{(0)}$ at $z=\tilde{z}^{(0)}$. Thus we may write,

$$\begin{aligned}\frac{\partial W}{\partial z''_j} &= \left(\frac{\partial W}{\partial z_j}\right)_{z=z^{(0)}} + \left(\frac{\partial^2 W}{\partial z_j\, \partial z_i}\right)_{z=z^{(0)}} \left(z''_i - z^{(0)}_i\right) \\ &+ \frac{1}{2}\left(\frac{\partial^3 W}{\partial z_j\, \partial z_i\, \partial z_k}\right)_{z=z^{(0)}} \left(z''_i - z^{(0)}_i\right)\left(z''_k - z^{(0)}_k\right)\end{aligned} \tag{5.8}$$

assuming that W is a cubic polynomial. As was shown in Ref.7,

$$\frac{\partial^3 W}{\partial z_K \partial z_\alpha \partial z_\beta} = \frac{\partial^3 W}{\partial z_K \partial z_L \partial z_\alpha} = \frac{\partial^3 W}{\partial z_K \partial z_L \partial z_M} = 0$$

$$\forall\ \alpha, K, L, M \tag{5.9}$$

On the other hand, $\partial W/\partial z_K$ as well as $(\partial^2 W/\partial z_K \partial z_i)$ vanish at $z=z^{(0)}$. Using Eqs.(5.6) and the fact that the only non-vanishing off-diagonal elements of V^{-1} are $(V^{-1})_{KA}$ and $(V^{-1})_{K\alpha}$, we may conclude that $(\partial W/\partial z''_S)$, when expressed in terms of the z'_i fields, do not contain any term with more than one power of the light field z'_α. Since $(V^{-1})_{jS}=0$ except for $j=S$, we see from Eq.(5.7) that the term involving F'_S has the form $F'_S(\partial W/\partial z''_S)$. Hence we may conclude that there is no $F'_S z'_\alpha z'_\beta$ coupling in the Lagrangian, in other words, any radiative correction involving F'_S has at least one heavy line in the loop.

This, in turn, shows that the radiatively generated terms in the effective action of the form $F'^*_S (N^{-1})_{Sj} F'_j$ are free from infrared divergence, and their first derivative with respect to some field z_i is at most logarithmically divergent in the $m_g \to 0$ limit. Thus N^{-1}_{Sj} depends on the light fields z'_α only at order m_g/M. Similarly, one can show that g'_S and $\partial g'_S/\partial z'_i$ are free from infrared divergences, and $(\partial^2 g'_S/\partial z'_i \partial z'_j)$ is at most logarithmically divergent in the

$m_g \to 0$ limit. This shows that g'_S depends on z'_α only at order $m_g{}^2/M$. This, in turn, shows that $\hat{F}'_S$ depends on z'_α only at order m_g^2/M.

We may now proceed to analyze the potential given in (5.4). Let us define a matrix $\tilde{N}$ in the AB space such that $\tilde{N}_{AB}=N_{AB}$. Then V may be written as,

$$V = \hat{F}'^*_A \tilde{N}_{AB} \hat{F}'_B + \{ \hat{F}'^*_A (N_{AK} \hat{F}'_K + N_{A\alpha} \hat{F}'_\alpha) + H.c. \}$$
$$+ (\hat{F}'^*_\alpha N_{\alpha\beta} \hat{F}'_\beta + \hat{F}'^*_\alpha N_{\alpha K} \hat{F}'_K + \hat{F}'^*_K N_{K\alpha} \hat{F}'_\alpha + \hat{F}'^*_K N_{KL} \hat{F}'_L)$$
$$+ \frac{1}{2} \hat{D}_a \hat{D}_a + m_g{}^2 f(z,z^\dagger) + \{ m_g (z'_i - U_{ij} \tilde{z}^{(0)}_j) \frac{\partial W}{\partial z'_i}$$
$$+ m_g (A-3) W(z) + H.c. \} + m_g{}^2 z^*_i z_i \qquad (5.10)$$

Let us define,

$$\hat{F}''_A = \hat{F}'_A - (\tilde{N}^{-1})_{AA'} (N_{A'K} \hat{F}'_K + N_{A'\alpha} \hat{F}'_\alpha) \qquad (5.11)$$

V may then be written as,

$$V = \hat{F}''_A \tilde{N}_{AB} \hat{F}''_B - (N_{AK} \hat{F}'_K + N_{A\alpha} \hat{F}'_\alpha)^* (\tilde{N}^{-1})_{AB}$$
$$(N_{BL} \hat{F}'_L + N_{B\beta} \hat{F}'_\beta) + (\hat{F}'^*_\alpha N_{\alpha\beta} \hat{F}'_\beta + \hat{F}'^*_\alpha N_{\alpha K} \hat{F}'_K$$
$$+ \hat{F}'^*_K N_{K\alpha} \hat{F}'_\alpha + \hat{F}'^*_K N_{KL} \hat{F}'_L) + \frac{1}{2} \hat{D}_a \hat{D}_a + m_g{}^2 f(z,z^\dagger)$$

$$+\{m_g(z_i' - U_{ij}\tilde{z}_j^{(0)})\frac{\partial W}{\partial z_i'} + m_g(A-3)W(z) + H.c.\} + m_g^2 z_i^* z_i$$

$$(5.12)$$

We may now minimize the potential with respect to the heavy fields for arbitrary values of the light fields z_α'. Following the same steps as in Sec. IV, we may show that a self-consistent solution may be found for,

$$\hat{F}_A'' \sim m_g^2 \quad \forall A \, , \quad \hat{D}_K \sim m_g^2 \quad \forall K \qquad (5.13)$$

At this minimum, $\hat{D}_\rho$, $\hat{F}_\alpha'$, $\hat{F}_N'$ are all of order m_g^2, whereas $\hat{F}_S'$ is, in general, of order $m_g M$. The value of $z_A' - z_A'^{(0)}$ at this minimum is independent of z_α' to order m_g. Using these results, we may show that the only possible z_α' dependent term in V, which may be of order $m_g^3 M$ for $z_\alpha' \sim m_g$ is,

$$\{-(N^\dagger)_{SA}(\tilde{N}^{-1})_{AB}(N)_{B\beta} + N_{S\beta}\}\hat{F}_S'^* \hat{F}_\beta' + H.c.$$

$$(5.14)$$

However, using the relations $NN^{-1}=I$, and $(N^{-1})_{\alpha S} \sim (N^{-1})_{S\alpha} \sim (m_g/M)$, we get,

$$N_{AB}(N^{-1})_{BS} + N_{AK}(N^{-1})_{KS} = O(m_g/M) \qquad (5.15)$$

$$N_{\alpha B}(N^{-1})_{BS} + N_{\alpha K}(N^{-1})_{KS} = O(m_g/M) \qquad (5.16)$$

Identifying N_{AB} with $\tilde{N}_{AB}$, we get,

$$\{N_{\alpha B}(\tilde{N}^{-1})_{BA} N_{AK} + N_{\alpha K}\}(N^{-1})_{KS} = O(m_g/M)$$
$$(5.17)$$

Since S refers to generators which transform as singlets of the unbroken subgroup H, $(N^{-1})_{KS}$ is of order (m_g/M) unless K also refers to a generator which is a singlet of the unbroken subgroup. Since $(N^{-1})_{SS'}$ is a non-singular square matrix, we get from (5.17),

$$N_{\alpha B}(\tilde{N}^{-1})_{BA} N_{AS} + N_{\alpha S} = O(m_g/M) \qquad (5.18)$$

Using Eq. (5.18) and that $\hat{F}'_S \lesssim m_g M$, we see that (5.14) is at most of order $m_g{}^4$.

Finally, we must discuss the effect of terms of order F^3. The only potentially dangerous contribution comes from terms involving F'_S. However, since F'_S does not have any coupling of the form $F'_S z'_\alpha z'_\beta$, terms involving F'_S must receive contribution from loops involving heavy fields. As a result, terms of order F^3, which involve one or more F'_S, are suppressed by powers of M, and these contributions are completely harmless.

VI. CONCLUSION

In this paper we have studied the stability of mass hierarchy in a general supersymmetric grand unified theory with explicit soft supersymmetry breaking terms in the Lagrangian, induced by supersymmetry breaking in the hidden sector. We have shown that any mass hierarchy present in the model in the limit of unbroken supersymmetry, is stable under the addition of soft supersymmetry breaking terms induced by the hidden sector, and radiative corrections, provided that the model does not contain any light field that transforms as a singlet under the unbroken subgroup of the theory above the scale m_g.

Before concluding, we wish to make the following comments.

i) In our analysis we have only considered the effect of radiative corrections involving the observable sector superfields, but have ignored the radiative corrections due to the hidden sector fields, as well as the graviton, gravitino, and the other fields in the supergravity multiplet.

ii) We have used background field method in order to analyze the effect of radiative corrections, and hence have assumed that the effective background field action correctly reproduces all the physical results. Although the result is plausible, there is no rigorous proof to this result.

iii) In estimating the order of magnitude of various terms generated by radiative corrections, we have assumed that the maximum infrared divergence in any graph is given by (the total mass dimension carried by external lines - 4). However, a supersymmetric gauge theory in a supersymmetric gauge usually suffers from severe infrared divergences. In Feynman like gauge, such divergences do not occur at the one loop level, but appears at the two loop level. We have assumed that such singularities are gauge artifacts, and cancel in the sum over all graphs. In order to make our results rigorous, however, we must find a gauge in which the power law infrared divergences are absent in all orders in perturbation theory. A class of non-local gauges, proposed recently[15], may provide a solution to this problem.

iv) Finally, we must mention that although the absence of light singlet fields is a sufficient condition for the stability of mass hierarchy, it is, by no means, a necessary condition. An example of a model with a light singlet in the supersymmetric limit, and which has a stable mass hierarchy, is given in Ref.17.

REFERENCES

[1]A. Sen, Phys. Rev. D30, 2608 (1984).

[2]For a complete list of references see H. P. Nilles, Universite de Geneve Report No. UGVA-DPT-1983/12-412.

[3]E. Witten, Nucl. Phys. B188, 513 (1981); S. Dimopoulos and H. Georgi, ibid. B193, 150 (1981); N. Sakai, Z. Phys. C11, 153 (1981); R. K. Kaul, Phys. Lett. 109B, 19 (1982).

[4]E. Gildener and S. Weinberg, Phys. Rev. D13, 3333 (1976); E. Gildener, ibid. 14, 1667 (1976).

[5]R. Arnowitt, A. H. Chamseddine and P. Nath, Phys. Lett. 120B, 145 (1983); S. Ferrara, D. V. Nanopoulos and C. A. Savoy, ibid. 123B, 214 (1983); R. Berbieri and S. Cecotti, Z. Phys. C17, 183 (1983); J. Lykken and F. Quevedo, Phys. Rev. D29, 293 (1984); P.Moxhay and Y. Yamamoto, Univ. of NOrth Carolina report No.

B. Gato, L. Leon, J. Perez-Mercader and M. Quiros, Inst. Estructura Materia report No. IEM-HE-2; I. Affleck and M. Dine, Institute for Advanced Studies report.

[6]J. Polchinski and L. Susskind, Phys. Rev. D26, 3661 (1982); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. 124B, 337 (1982); A. B. Lahanas, ibid. 124B, 341 (1982); M. Dine, in Lattice Gauge Theories, Supersymmetry and Grand Unification, proceedings of the Sixth Johns Hopkins Workshop on current problems in Particle Theory, Florence, 1982.

[7]L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27, 2359

(1983). See also R. Arnowitt, A. H. Chamseddine and P. Nath, Nucl. Phys. B227, 121 (1983).

[8]M. T. Grisaru, M. Rocek and W. Siegel, Nucl. Phys. B159, 429 (1979).

[9]L. Girardello and M. T. Grisaru, Nucl. Phys. B194, 65 (1982).

[10]B. Zumino, Nucl. Phys. B89, 535 (1975); P. West, ibid. B106, 219 (1976); D. M. Capper and M. Ramon-Medrano, J. Phys. G2, 269 (1976); W. Lang, Nucl. Phys. B114, 123 (1976); S. Weinberg, Phys. Lett. 62B, 111 (1976); E. Witten, Nucl. Phys. B188, 513 (1981); S. Ferrara and O. Piguet, Nucl. Phys. B93, 261 (1975); M. T. Grisaru, M. Rocek and W. Siegel, Ref.8; B. Ovrut and J. Wess, Phys. Rev. D25, 409 (1982).

[11]S. Gates, M. T. Grisaru, M. Rocek and W. Siegel, 'Superspace', Benjamin Pub. 1983.

[12]A. Sen, Fermilab report No. FERMILAB-PUB-84/118-T, Phys. Rev. D. (to appear).

[13]This is true in a general renormalizable field theory, so long as the cubic coupling between the low mass scalar fields is of the order of the masses of this fields. Such power law infrared divergences, however, do exist in individual graphs in supersymmetric gauge theories in a general gauge that preserves supersymmetry. In supersymmetric Feynman like gauge the one loop diagrams are free from such divergences, but graphs involving two or more

loops are not. Recently, a new class of non-local gauges have been proposed which may be free from such divergences to all orders in perturbation theory[15]. In any case, we shall assume in our analysis that such singularities are gauge artifacts, and cancel in the sum over all graphs.

[14]M. T. Grisaru and D. Zanon, Brandeis Univ. report No. BRX-TH-163.

[15]L. F. Abbott, M. T. Grisaru and D. Zanon, Nucl. Phys. B244, 454 (1984).

[16]Contribution to P_ρ from one loop graphs involving only background gauge fields as external lines was shown to be harmless in Ref. 12. All the supersymmetry breaking terms in the Lagrangian, on the other hand, may be treated as 'interaction' terms in the convention of Ref.11, and the contribution from any graph, with one or more supersymmetry breaking vertices may be shown to depend on $V_\rho^{(b)}$ only through $\tilde{\phi}^{(b)}$, $\Gamma_{\alpha\dot{\alpha}}^{(b)}$, $W_\alpha^{(b)}$ or their complex conjugate fields. As a result, the net contribution to $P_\rho+m_g\hat{P}_\rho$ from one loop graphs may also be shown to be at most of order $m_g{}^2$ for $z_\alpha \sim m_g$.

[17]A. Sen, Phys. Lett. 148B, 65 (1984), Phys. Rev. D31, 900 (1985).