



Naturally light Higgs doublet in  
supersymmetric  $E_6$  grand unified theory

Ashoke Sen  
Fermi National Accelerator Laboratory  
P.O.Box 500, Batavia, IL 60510

ABSTRACT

We show how some supersymmetric  $E_6$  grand unified theories, which may arise as the low energy limit of superstring theory, provide a natural solution of the fine tuning problem.

Although supersymmetry may protect large mass hierarchy in a theory against radiative corrections<sup>1</sup>, it does not explain why such hierarchy is present in the first place. In particular, it does not explain why the weak doublet Higgs field is so light compared to its color triplet partner. In this paper we propose a simple mechanism for obtaining light Higgs doublets in supersymmetric  $E_6$  grand unified theories. Our model is motivated by the recent observation that supersymmetric  $E_6$  grand unified theories may appear as low energy limit of superstring theories<sup>2,3,4</sup>

The model that we shall consider contains chiral superfields  $\phi(78)$ ,  $H(27)$  and  $\tilde{H}(\overline{27})$ , with the superpotential,

$$W = M_1 \text{Tr}_{27} \cos(\phi/M) + \alpha \phi H \tilde{H} \quad (1)$$

where, for simplicity, we have ignored all the  $E_6$  group indices. Here  $\text{Tr}_{27}$  denotes the trace in the 27 representation of  $E_6$ . Although the superpotential given in (1) seems ad hoc, its general features that we shall use may come naturally in the low energy limit of superstring theories. For example, this model has supersymmetric minima at values of  $\phi$  which are integral multiples of some minimum mass  $\pi M$ . This is a general feature of theories in which the breaking of  $E_6$  occurs due to twisted gauge field configuration in certain higher dimensions<sup>3,5</sup>. Also, we have omitted mass terms of the form  $MH\tilde{H}$  from  $W$ , thereby assuming



$$\langle \phi \rangle_{(3,1)} = \pi M \begin{pmatrix} n_3 \\ -n_3 \end{pmatrix} \quad (4.c)$$

$$\langle \phi \rangle_{(2,2^0)} = 0 \quad (4.d)$$

where  $n_i$ 's are integers.<sup>6</sup> This is the most general minimum consistent with an unbroken symmetry group that contains  $SU(3) \times SU(2) \times U(1)$  as its subgroup. The fields belonging to the  $(2, \bar{6})$  representation may now be split into a color anti triplet ( $\bar{3}$ ), a weak doublet (2) and a color and weak  $SU(2)$  singlet (1) part, each part also carrying an index  $\uparrow$  or  $\downarrow$  showing the  $SU(2)$  quantum numbers. The masses of various fields are given by,

$$\begin{aligned} M_{\bar{3}\uparrow} &= \alpha \pi M (-n_1 + n_3), & M_{\bar{3}\downarrow} &= \alpha \pi M (-n_1 - n_3) \\ M_{2\uparrow} &= \alpha \pi M (-n_2 + n_3), & M_{2\downarrow} &= \alpha \pi M (-n_2 - n_3) \\ M_{1\uparrow} &= \alpha \pi M (3n_1 + 2n_2 + n_3), & M_{1\downarrow} &= \alpha \pi M (3n_1 + 2n_2 - n_3) \end{aligned} \quad (5)$$

Similarly, the fields belonging to the  $(1, 15)$  representation of  $SU(2) \times SU(6)$  has a color anti triplet component ( $\bar{3}'$ ), a color triplet component ( $3'$ ), a weak doublet part ( $2'$ ), a component which transforms as weak

doublet and color triplet  $(2', 3')$ , and a component  $(1')$  which is a singlet under the color  $SU(3)$  and weak  $SU(2)$  group. The masses of these particles may again be read directly from their quantum numbers, and are as follows,

$$\begin{aligned}
 M_{\bar{3}'} &= \alpha \pi M (2\eta_1), & M_{(2', 3')} &= \alpha \pi M (\eta_2 + \eta_1) \\
 M_{3'} &= \alpha \pi M (-2\eta_1 - 2\eta_2), & M_{2'} &= (-3\eta_1 - \eta_2) \alpha \pi M \\
 M_{1'} &= \alpha \pi M (2\eta_2) & & (6)
 \end{aligned}$$

Let us now consider a particular minimum where,

$$\eta_3 = \eta_2 = -3\eta_1 \quad (7)$$

Then,

$$M_{2\uparrow} = M_{1\downarrow} = M_{2'} = 0 \quad (8)$$

whereas all the other fields acquire masses of order  $M$ . Thus from the 27 representation, we get a massless chiral superfield which is a singlet of  $SU(3)^C \times SU(2)^W$ , and two massless chiral fields which transform as doublets of  $SU(2)^W$ . The  $\bar{27}$  representation gives rise to massless chiral superfields belonging to complex conjugate representations.

The vev of these light fields break the remaining gauge group to  $SU(3)^C \times U(1)^{e.m.}$  at some scale below the GUT scale. We expect that after the effect of supersymmetry breaking is taken into account, the  $2^\dagger$  and  $2'$  fields will acquire vev of order  $m_w$ , whereas the  $1^\dagger$  component should acquire a vev of order  $10^{10}$  GeV, at least in the case of four generations of quark-lepton fields, for reasons to be discussed shortly.<sup>7</sup>

The quark fields in one generation may be taken to belong to a single 27 representation of  $E_6$ , with a coupling to the Higgs field of the form,

$$\lambda Q Q H \quad (9)$$

To see how the quark fields get mass, we may decompose the 27 representation in terms of its transformation properties under the  $SU(2) \times SU(6)$  subgroup. Thus, for example, the field  $Q$  may be taken to consist of the fields  $Q_{\bar{6},i}$  and  $Q_{15}$ , where  $i$  is the  $SU(2)$  index. (9) may then be written as,

$$\lambda \{ a \epsilon_{ij} Q_{\bar{6},i} Q_{\bar{6},j} H_{15} + b \epsilon_{ij} Q_{\bar{6},i} Q_{15} H_{\bar{6},j} + c Q_{15} Q_{15} H_{15} \} \quad (10)$$

where  $a$ ,  $b$  and  $c$  are constants. We may now further decompose the fields in terms of their  $SU(5)$  quantum numbers, under which  $\bar{6}$  decomposes as a  $\bar{5}$  and a singlet, whereas 15 decomposes as a 5 and a 10. The vev of  $1^\dagger$  component coming from  $H_{\bar{6},\dagger}$  then produces a mass term of the

form  $Q_{\bar{5},\uparrow}Q_5$ . We assume this mass to be at least of order  $10^{10}$  GeV, in order that there are not too many light fields in the theory, and the gauge coupling constants do not blow up to infinity before reaching the grand unification scale. We are then left with light fields  $Q_{\bar{5},\downarrow}$  and  $Q_{10}$  belonging to the 5 and 10 representations of SU(5), as well as two light fields  $Q_{1,\uparrow}$  and  $Q_{1,\downarrow}$  belonging to the singlet representation. (Although SU(5) is not a good symmetry of the theory at low energy, the light quark lepton fields belong to full multiplets of SU(5), hence it is more convenient to describe the low mass fields in terms of their SU(5) transformation properties). The vev of  $2\uparrow$  component (of order  $m_w$ ) which comes from  $H_{\bar{6},\uparrow}$ , then produces a mass term of the form  $Q_{\bar{5},\downarrow}Q_{10}$ , giving mass to the d quark and the electron, whereas the vev of the  $2'$  component, which comes from the  $H_{15}$ , produces a mass term of the form  $Q_{10}Q_{10}$ , giving mass to the u quark.

The vev of the  $2'$  component, however, also gives mass to the neutrino contained in  $Q_{\bar{5},\downarrow}$  by coupling it to the light singlet field  $Q_{1,\uparrow}$ . Another way to see the existence of neutrino mass is to note that the unbroken subgroup below the GUT scale is  $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)\times U(1)$ , so that light right handed neutrinos arise naturally in this model. Although this is not obvious by looking at the decomposition of various fields according to their transformation properties under the  $SU(2)\times SU(6)$  subgroup, this becomes

clear if we look at the decomposition of  $\phi$  according to irreducible representations of the  $SU(3) \times SU(3) \times SU(3)$  subgroup of  $E_6$ . Under this decomposition, the vev of  $\phi$  may be written as,

$$\langle \phi \rangle = 2\pi M \left[ \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \end{pmatrix} \oplus \begin{pmatrix} n_1 & & \\ & n_1 & \\ & & -2n_1 \end{pmatrix} \oplus \begin{pmatrix} n_1 & & \\ & n_1 & \\ & & -2n_1 \end{pmatrix} \right] \quad (11)$$

where the three matrices refer to the generators of the three  $SU(3)$  subgroups. The vev of the  $1\uparrow$  field will break the symmetry to  $SU(3)^C \times SU(2)_L \times SU(2)_R \times U(1)$ , which, in turn, breaks down to  $SU(3)^C \times U(1)^{em}$  by the vev of the  $2\uparrow$  and  $2'$  fields.

Besides solving the problem of having a massive neutrino, we also have to find a way of breaking the symmetry between the  $SU(2)_L$  and  $SU(2)_R$  groups. Both these problems, however, may be solved, if we assume that there exists another Higgs field belonging to the adjoint representation of the theory, whose vev breaks the  $SU(2)_R$  group at the GUT scale, thereby also opening up the possibility of having a large mass for the right handed neutrino. This keeps the left handed neutrino almost massless. This new Higgs field, however, must not couple to the  $H, \tilde{H}$  fields, so that the mass hierarchy is preserved.

Note added: After completion of this work we found some work by Witten<sup>8</sup> and Breit, Ovrut and Segre<sup>9</sup> who have also noted the existence of naturally massless Higgs doublets in superstring theories. I wish to thank H. Tye and M. Dine for drawing my attention to these papers. The possibility of the existence of an intermediate scale symmetry breaking has also been discussed by Dine et. al.<sup>10</sup>.

## REFERENCES

- <sup>1</sup>E. Witten, Nucl. Phys. B188, 513 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981); N. Sakai, Z. Phys. C11, 153 (1981); R. K. Kaul, Phys. Lett. 109B, 19 (1982).
- <sup>2</sup>M. B. Green and J. H. Schwarz, Phys. Lett. 149B, 117 (1984), Caltech report No. Calt-68-1194; D. J. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54, (1985) 502. See also, P. G. O. Freund, *Phys Lett. 151B, 387(1985)*, and J. Thierry-Mieg, Berkeley preprint.
- <sup>3</sup>P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, Santa Barbara report No. NSF-ITP-84-170.
- <sup>4</sup>Original proposal for considering  $E_6$  as a possible unifying gauge group was made by F. Gursev, P. Ramond and P. Sikivie, Phys. Lett. 60B, 177 (1976); F. Gursev and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976); P. Ramond, Nucl. Phys. B110, 214 (1976).
- <sup>5</sup>The symmetry breaking in these models is obtained by giving a vev to  $P \exp(\oint_{\gamma} A_m dy^m) \equiv e^{i\phi/M}$  while keeping the  $E_6$  field strength to be zero. Here  $y^m$  are the co-ordinates of the internal manifold, and  $\gamma$  is a non-contractible loop. In the manifolds considered in Ref.3, although  $\gamma$  is non-contractible, if we travel along  $\gamma$  a certain number ( $n$ ) of times, we get a contractible loop. Thus  $(e^{i\phi/M})^n = 1$ , which shows that  $\phi$  is quantized. Similar mechanisms for gauge symmetry breaking has also been considered by

Y. Hosotani, Phys. Lett. 126B, 309 (1983), Phys. Lett. 129B, 193 (1984).

<sup>6</sup>Actually, the value of  $\phi$  where all the  $n_i$ 's are half-integral also gives a supersymmetric minimum.

<sup>7</sup>This may occur naturally in theories with a light singlet field coupled to  $H, \tilde{H}$ . If supersymmetry is broken in the hidden sector at a scale of order  $10^{10}$  GeV, then, after the inclusion of radiative corrections, some light components of  $H, \tilde{H}$  may acquire vev of order  $10^{10}$  GeV. For a discussion of the instability of mass hierarchy in the presence of a light singlet field, see J. Polchinski and L. Susskind, Phys. Rev. D26, 3661 (1982); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. 124B, 337 (1982); A. B. Lahanas, *ibid.* 124B, 341 (1982); M. Dine, in Lattice Gauge Theories, Supersymmetry and Grand Unification, proceedings of the 6th Johns Hopkins Workshop on current problems in Particle Theory, Florence, 1982.

<sup>8</sup>E. Witten, 'Symmetry breaking patterns in superstring models', Princeton preprint (1985).

<sup>9</sup>J. D. Breit, B. A. Ovrut, and G. C. Segre, Univ. of Pennsylvania report No. UPR-0279T.

<sup>10</sup>M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi and N. Seiberg, 'Superstring model building', Inst. for Advanced Study preprint (1985).