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SCALE–INVARIANT DENSITY PERTURBATIONS, ANISOTROPY OF THE COSMIC MICROWAVE BACKGROUND AND LARGE–SCALE PECULIAR VELOCITY FIELD

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ABSTRACT

The large scale peculiar velocity field and the large and intermediate angular scale anisotropy of the cosmic microwave background are studied in inflationary cosmological models of critical density and containing primordial scale–invariant adiabatic density perturbations. Comparison with observations supports a cold dark matter scenario in which the dark matter is not appreciably less clustered than the luminous galaxy distribution.

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According to the standard scenario of galaxy formation,the observed structure in the universe is determined by the growth of small density fluctuations, commonly assumed to be adiabatic, present "ab initio", and with an initially scale–free power spectrum. The presence of these irregularities necessarily produces fluctuations in the cosmic microwave background (CMB), thereby providing a unique and powerful technique for the direct comparison of observations and linear theory that bypasses most of the uncertainties connected with the non–linear evolutionary stages of galaxy formation and clustering.

The CMB anisotropies are described by considering, in the synchronous gauge, the perturbed Boltzmann equation for the radiation fractional brightness i_γ: for a spatially flat, optically thin universe, in the subhorizon limit, there exists an analytical solution of this equation (Peebles and Yu, 1970; Wilson and Silk, 1981; Vittorio and Silk, 1984):

$$i_\gamma (\mu, k, t_0) = |i_{\gamma_{\substack{small \\ scale}}} (\mu, k)| e^{-i\mu k c r_0} + \frac{8i}{r_0}\mu \frac{\delta (k, t_0)}{k} - \frac{8}{r_0^2} \frac{\delta (k, t_0)}{k^2} , \qquad (1)$$

where μ is the angle between the line of sight and wavenumber $\vec{k}, r_o = 2c/H_o$ is the present horizon radius and $\delta(k, t_o)$ is the amplitude of the Fourier components of the matter density fluctuations. Eq (1) enables one to identify three different sources of anisotropies: i) fine–scale anisotropies, ii) dipole anisotropy, and iii) large-scale anisotropies associated with gravitational potential fluctuations [the Sachs–Wolfe (1967) effect].

Study of the fine scale CMB anisotropy has been performed in a complete numerical treatment by the aforementioned authors, and more recently by Vittorio and Silk (1984) and Bond and Efstathiou (1984). The complexity arises from the need to quantitatively evaluate the growth of fluctuations through the epoch of matter–radiation decoupling when the photon mean free path increases by many orders of magnitude. On large angular scales, however, the problem simplifies and the radiation fluctuations are directly connected with the primordial matter density fluctuation spectrum (e.g., Peebles 1982). Specific predictions of the primordial fluctuation spectrum are made in the inflationary scenario (Guth, 1982, and references therein), which suggests that the universe is flat and that the primordial density fluctuation spectrum is scale–invariant, $|\delta(k, t_i)|^2 = A k$, generated by quantum fluctuations in the early universe (Hawking 1982; Turner 1983).

If the initial fluctuations are adiabatic, pure baryonic models are unacceptable: primordial nucleosynthesis studies constrain the present baryonic density parameter to be $0.01 < \Omega_b h^2 < 0.04$ (Yang et al., 1984), and in a low density pure baryonic universe the fine–scale CMB anisotropies exceed (Wilson 1983) the present observational limit of 3.10^{-5} on an angular scale of 4'.5 (Uson and Wilkinson, 1984). One infers that the universe is most probably dynamically dominated by weakly interacting relics of the early universe: popular candidates include axions ($m_x \equiv 10^{-5} eV$), massive photinos ($m_x \equiv 1\, GeV$), or massive neutrinos ($m_\nu \equiv 30\, eV$) (Bond and Szalay, 1983; Blumenthal et al., 1984). One usually refers to massive neutrinos as "hot" matter, because of the importance of free streaming in damping out small scale power, and to axions or photinos as "cold" matter, because no significant damping occurs and small scale power is present down to subgalactic scales. Hot models are presently out of favour, because a flat massive neutrino–dominated universe appear to lead to excessive clustering at the present epoch (White, Frenk and Davis,1984), and to an excessive large scale–peculiar velocity field (Kaiser, 1983). It has been found that a flat cold dark matter dominated universe reproduces the observations of galaxy clustering and small–scale galaxy peculiar velocities only if galaxies are not a good tracer of the overall mass distribution (Davis et al., 1985) However, the large–scale peculiar velocity field has not been evaluated in such models. The purpose of this Letter is to evaluate in the context of an inflationary model the large scale peculiar velocity field and to describe some observational implications of the hypothesis that the large–scale mass and light distributions are uncorrelated.

We will model the present power spectrum using the following expressions for massive neutrinos and axions (or photinos) respectively:

$$|\delta(k,t_0)|^2 = A\, k\, exp[-4.61(\frac{k}{k_\nu})^{1.5}] \tag{2.a}$$

and

$$|\delta\,(k,t_0)\,|^2 = A\,\frac{k}{(1+\alpha k+\beta k^{1.5}+\gamma k^2)^2} \tag{2.b}$$

where $k_\nu = 0.49\Omega_0 h^2$ is the comoving wavenumber corresponding to a present neutrino damping length of $13\,(\Omega_0 h^2)^{-1}\, Mpc$; $\alpha = 1.7h^{-2} Mpc$; $\beta = 9h^{-3} Mpc^{1.5}$; $\gamma = h^{-4} Mpc^2$; $\Omega_0 = 8\pi G\rho_0/\left(3H_0^2\right)$; $h =$

$H_0 / (100 km\, s^{-1} Mpc^{-1})$ (White,Frenk,Davis, 1983; Davis, et al., 1985). The normalization of the spectra is performed differently for hot and cold matter. In the former case, we require nonlinearity (defined as the moment at which the rms density contrast is 0.6) to occur at a redshift $z_{nl} = 3$, chosen in order to explain the most remote observed quasars. In the latter case we require that the r.m.s. density fluctuation averaged inside a randomly–placed sphere of radius $r = 8h^{-1} Mpc$ is unity (Peebles 1982).

De Vaucouleurs and Peters (1984) have evaluated the velocity of the Local Group with respect to different cosmological reference frames, defined by shells of galaxies at systematically increasing redshift. One can immediately infer from their data (no error bars are given, unfortunately) the peculiar velocity of these reference frames relative to the CMB. Hart and Davies (1982) performed a similar measurement for only one shell of galaxies of radius $25h^{-1} Mpc$, and found the shell to be essentially at rest relative to the CMB:$v_p\,(25h^{-1} Mpc) = 130 \pm 70\, km\, s^{-1}$. A least–square parabolic fit, in the range $(10 - 35)\, h^{-1} Mpc$,to the de Vaucouleurs and Peters data, including some of the Hart and Davies subsample, gives $v_p\,(r) = (690 - 20\, r\, +\, 0.19\, r^2)\; km\, s^{-1}$ where r is expressed in units of $(h^{-1} Mpc)$: this implies $v_p\,(25h^{-1} Mpc) = 300\, km\, s^{-1}$. These kinds of measures are extremely important because, at least in principle, a way is provided of constraining not only the amplitude but also the slope of the power spectrum on large scales.

The second term in (1) describes a present dipole–like anisotropy due to a peculiar velocity $v\,(k,\, t_0) = H_0\, \delta\,(k,\, t_0) / k$. The rms peculiar velocity of a shell of radius r relative to the CMB is obtained by a convolution with a gaussian spherically symmetric window function (Clutton–Brock and Peebles, 1981; Kaiser, 1983):

$$v_p^2\,(r) = \frac{H_0^2}{2\pi^2} \int_0^\infty dk |\delta\,(k, t_0)\,|^2\, exp\,(-k^2 r^2)\ , \tag{3}$$

and is induced by perturbations of wavenumbers $k \leq 1 / r$. the theoretical calculations are well fitted, again in the range $(10 - 35)\, h^{-1} Mpc$, by the law $v_p\,(r) = (a - br + c\, r^2)\; km\, s^{-1}$ with the values of the coefficients given in table 1. For large enough $r\,(> 35h^{-1} Mpc)$, where the power spectrum is well approximated by its initial power law, we find $v_p\,(r) = \frac{A^{1/2} H_0}{2^{3/2}\pi}\, \frac{1}{r}$ where A is defined in eq. (2).

Fig.1 shows a comparison of the numerical results with the observational data: in the cold matter scenario, with the dark matter tracer of the luminous galaxy distribution, there is complete consistency with the observed values, independently of h, at the 1 σ level. However, in a universe dominated by massive neutrinos there is excessive peculiar velocity if $h = 0.5$, although the prediction may be marginally consistent with the observations if h=1. We recover Kaiser's (1983) result when only the Hart and Davies sample is modeled. The dipole anisotropy is calculated with an identical technique, incorporating contributions from all scales that are still linear today: this is a reliable procedure, because, at least for a scale–invariant primordial spectral index, most of the anisotropy comes from linear scales which enter the horizon at the epoch of matter–radiation equality. We find that a cold dark matter dominated universe is, independently of h, reasonably compatible with the observed motion of the Local Group relative to the CMB, namely 640 km s^{-1}. Once the infall velocity to Virgo is subtracted, the residual motion of the Local Group is $v_{LG} = 300 - 450$ km s^{-1} (Sandage and Tammann, 1984; Aaronson and Mould, 1985): it is this (the "corrected dipole" anisotropy) that we compare with the model predictions, shown in Table 1.

The third term in eq (1) describes the Sachs–Wolfe effect, which is dominated by contributions from perturbations of wavenumbers $k \sim r_0^{-1}$. For these scales, which have experienced uninterrupted growth up to the present, the power spectrum is well approximated by its initial power law and the rms value for the temperature fluctuations is found to be $\triangle_{rms} = \frac{2^{1/2}A^{1/2}}{\pi^{1/2}r_0^2}$. Numerical values of $\triangle_{rms}$ are shown in Table 1. However, to perform a more detailed comparison with the observations, it is useful to consider the anisotropy measured in a specific experimental configuration, with an antenna of beam width σ, which measures the difference in temperature between two points in the sky spaced an angle α apart. The expected anisotropy is $\triangle^2_{rms}(\alpha\sigma) = 2\,(C(0,\sigma) - C(\alpha,\sigma))$, where $C(\alpha,\sigma)$ is the angular correlation function (see, e.g.,Vittorio and Silk, 1984). On angular scales $2^0 < \alpha < 10^0$, where it is possible to approximate the present power spectrum with its initial power law, we find, for $\alpha \sim \sigma$,

$$\triangle^2_{rms}(\alpha,\sigma) = \frac{2A}{\pi^2 r_0^4} \sum_{\iota=1}^{\iota=\infty} \frac{(-1)^{(\iota-1)}}{\iota^2\,(\iota-1)!} \left(\frac{\alpha}{2\sigma}\right)^{2\iota} . \tag{4}$$

This result is, by hypothesis, independent of the detailed slope of the power spectrum on small scales. Hence, the only dependence on the dominant particle species (hot or cold) we are considering is via the normalization factor. Moreover, the value of $\triangle_{rms}(\alpha,\sigma)$ is independent of α and σ at constant α/σ: this is a consequence of the scale–invariant nature of the spectrum (2). Fig. 2 shows the dependence of $\triangle^2_{rms}(\alpha,\sigma)$ on the quantity α/σ. We infer that cold matter is consistent, either for h=1 and h=0.5, with the upper limit of 4.10^{-5} set by the experiment of Melchiorri et al.(1981)($\alpha = 6^0; \sigma = 2^0.2$).

A key point in the evaluation of the CMB anisotropies is the normalization of the power spectrum. Alternative approaches imply normalization of the spectrum (2) to the observed two–point galaxy–galaxy correlation function $\xi(r)$, or to the integral quantity $J_3(R) = \int_0^R r^2\, dr\, \xi(r)$. Moreover, if galaxies are not a good tracer of the mass distribution (Kaiser, 1984; Bardeen, 1984; Davis et al. 1984), we can try to normalize the power spectrum to the observed value of the dipole anisotropy, or, preferably, in order to be safe from local contamination due to the Virgo cluster, to the value of the large–scale peculiar velocity field on scales of $25h^{-1}Mpc$: this should determine the minimum value of the CMB anisotropy. Table 2 displays the appropriate correction factors by which all previous linear theory calculations, including the fine scale anisotropy predictions, have to be multiplied if the various alternative normalization criteria are applied. For example, in the case of cold matter and h=1, the correction factor is close to unity if we normalize the spectrum (1) to the observed value of either $J_3(30h^{-1}Mpc)$, the dipole anisotropy, or $\xi(5h^{-1}Mpc)$, but is almost two if we normalize to $v_p(25h^{-1}Mpc)$. This provides an indication of the uncertainty in our predictions.

To summarize, we have found that a cold dark matter scenario gives acceptable CBR anisotropy and large–scale velocity field if light traces mass. However such a model has been found to yield unacceptably large small–scale galaxy–galaxy velocity correlations. Davis et al. (1985) reconcile a flat cold dark matter dominated universe with the observations by arguing that galaxies formed only in high density regions. Peaks in the primordial fluctuation spectrum which lie above some threshold τ (in units of the rms density fluctuations) are more correlated by a factor of order τ^2 (Kaiser, 1984). This biasing hypothesis implies that the dark matter is substantially less clustered than the luminous matter and that the amplitude of the overall density fluctuation field is lower, relatively to the luminous matter, by a factor of roughly τ. This definitely helps in matching the amplitude of the small scale pairwise galaxy velocity dispersion with the CFA data (Davis and Peebles, 1983). However, this prescription also leads to a reduction of the dipole anisotropy and the large scale peculiar velocity field, relative to the values given above. In particular, taking $\tau = 2.5$, as considered by Davis et al. (1985), and normalizing the primeval fluctuation spectrum by requiring the two point correlation function of the peaks above the threshold τ to be unity on scales of $5h^{-1}\,Mpc$, we predict a dipole anisotropy of $200\,km\,s^{-1}$ and $160\,km\,s^{-1}$ for $h = 0.5$ and $h = 1$,respectively, at the 1σ level. This implies that there is a probability of only 15% (if $h = 0.5$) of measuring a velocity of $450\,km\,s^{-1}$ (or bigger), although there is a probability of 50% of measuring a velocity of $300\,km\,s^{-1}$ (or bigger), It should be noted that the fraction of v_{LG} determined by the local matter distribution is very uncertain, and so v_{LG} must be considered as an upper limit on the velocity induced by long wave-length density perturbations. However, the large scale peculiar velocity field provides a more reliable and direct way of measuring the amplitude of fluctuations on large scales.

If galaxies are indeed not a fair tracer of the mass, applying the same normalization previously used, we predict, at the 1σ level,$v_p\,(25h^{-1}\,Mpc) = 76\,km\,s^{-1}$ and $46\,km\,s^{-1}$ for h=0.5 and 1, respectively: that is, matter and radiation are basically at rest already on scales of $25h^{-1}Mpc$ and no appreciable density fluctuation are expected on scales $> 25h^{-1}\,Mpc$. Now the only measures available for $v_p\,(25h^{-1}\,Mpc)$ are those of Hart and Davies (1982) and de Vaucouleurs and Peters (1984). While there is a probability of 40% of measuring $v_p\,(25h^{-1}\,Mpc) \geq 130\,km\,s^{-1}$, there is a probability less than 0.01% of measuring $v_p \geq 350\,km\,s^{-1}$. In other words, confirmation of the de Vaucouleurs and Peters result would provide strong evidence against the more uniform large–scale dark matter distribution (relative to that of the luminous matter) implied by the biasing hypothesis.

Finally, we cannot resist speculating on what theoreticians might do should the higher large–scale peculiar velocities be confirmed. One could question the large small–scale peculiar velocities predicted by the N–body simulations if mass traces light: for example, if massive dark halos are attached to galaxies, galaxy interactions would be considerably softened, and smaller peculiar velocities would arise (Barnes 1984). Another possibility is that galaxies preferentially form in regions of inwardly-directed peculiar velocities which are subsequently reduced by dissipation: Oort (1970) has advocated such a scheme in a slightly different context, and Schaeffer and Silk (1984) have stressed the role of dissipation in reducing small-scale velocities in pancake collapse and fragmentation. However, the current measurements of the large scale peculiar velocity field and of the corrected dipole anisotropy are too uncertain to provide a discriminant between the various theoretical models at present, but we wish to stress the crucial significance of more sensitive observations. If peculiar velocities as high as $350\,km\,s^{-1}$ on scales of $25h^{-1}\,Mpc$ can be confirmed, the biased galaxy formation hypothesis (and perhaps the cold dark matter scenario itself) could be in considerable difficulty.

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Figure Captions

Fig.1 : Peculiar velocities of different cosmological reference frames (associated with shells of galaxies of radius r) relative to the CMB vs. r. The continuous lines (heavy, $h = 0.5$; light, $h = 1$) are the predictions at the 1σ level for a cold matter–dominated universe. The hatched line is the lower limit set by the theory ($h = 0.5$) at the 95% level of confidence. The dotted lines (heavy, $h = 0.5$; light, $h = 1$) are the 1σ predictions for a neutrino–dominated universe and the cross–hatched line is, similarly, a lower limit ($h = 0.5$) at the 95% level of confidence. The dots refer to de Vaucouleurs and Peters (1984) data (squares) and to some of the Hart and Davies (1982) subsamples (circles).

Fig.2 : Anisotropy of the cosmic microwave background as a function of α/σ (see text). The arrow refers to the experimental upper limit of Melchiorri et al. (1981) obtained with $\alpha = 6°$ and $\sigma = 2°.2$. The continuous lines (heavy, $h = 0.5$; light, $h = 1$) are the predictions at the 1σ level for a cold matter–dominated universe. The dotted lines (heavy, $h = 0.5$; light, $h = 1$) are the 1σ predictions for a neutrino–dominated universe.

TABLE 1

LARGE SCALE VELOCITY FIELD AND CMB ANISOTROPY

	h	a (km s^{-1})	b(km s^{-1} h Mpc^{-1})	c (km $s^{-1} h^2 Mpc^{-2}$	D (km s^{-1})	Δ_{rms}
ν's	.5	1585	35	0.23	1882	2.95 10^{-5}
	1.	488	12	0.08	774	7.38 10^{-6}
c's	.5	534	11	0.07	690	1.16 10^{-5}
	1.	357	8	0.06	554	6.09 10^{-6}

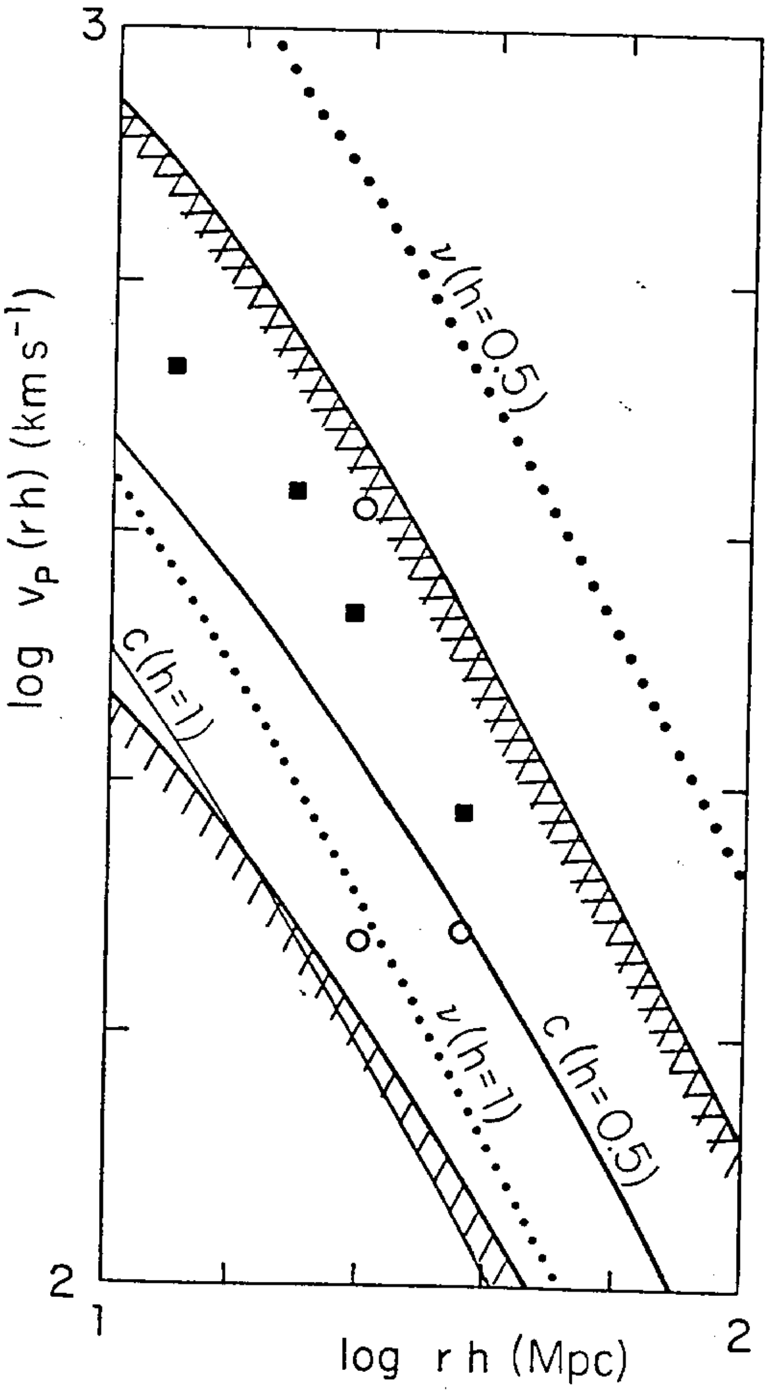
Notes to Table 1 : Values of the parameters a,b,c used in the fit to the theoretical calculations, for hot and cold matter and for different values of h. Values of the velocity associated with the dipole anisotropy (column 5) and of the rms temperature fluctuations (column 6) are also given.

TABLE 2

NORMALIZATION PARAMETERS

		observed	correction factor h=0.5	correction factor h=1
$J_3(30h^{-1}Mpc)$	ν's	$600\ h^{-3}\ Mpc^3$	0.28	0.72
	c's		0.78	1.00
D	ν's	$640\ km\ s^{-1}$	0.34	0.81
	c's		0.89	1.09
$\bar{v}_p(25h^{-1}Mpc)$	ν's	$300\ km\ s^{-1}$	0.40	1.48
	c's		1.13	1.86
$\xi(5h^{-1}Mpc)$	ν's	1	0.45	0.54
	c's		0.72	0.89

Notes to table 2: Correction factors to be applied to the results obtained when the amplitude of the primordial spectrum is normalized either to the observed value of ξ, of J_3, of the dipole anisotropy (D), or of the large scale peculiar velocity field (v_p) in neutrino (ν) or cold particle (c) - dominated cosmological models.



3
2
1
2
log v_P (r h) (km s^{-1})
log r h (Mpc)
ν(h=0.5)
ν(h=1)
c(h=1)
c(h=0.5)