

LEFT-RIGHT SYMMETRIC MODEL WITH ULTRALIGHT DIRAC NEUTRINOS

J. Oliensis

Argonne National Laboratory, Argonne, IL 60439

and

Carl H. Albright*

Northern Illinois University, DeKalb, IL 60115

and

Fermi National Accelerator Laboratory, Batavia, IL 60510

Abstract

A low energy, left-right symmetric gauge model incorporating mirror fermions and a discrete symmetry yields a skewed Dirac neutrino mass matrix. Some of the Dirac neutrinos can be made ultralight, since a ratio of Higgs vacuum expectation values can be taken to be naturally small, while others become heavy with masses on the order of 100 GeV. To avoid neutrino masses in the cosmologically disfavored range, $100 \text{ eV} < M_\nu < 2 \text{ GeV}$, the numbers of standard and mirror generations must be equal.

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We describe here a left-right model incorporating mirror fermions that produces naturally ultralight Dirac neutrinos (< 100 eV.) Our model is a low energy one, valid for $E \lesssim 1$ TeV, and avoids awkward assumptions about higher energy physics characteristic of the see-saw mechanism,¹ for example. Though Dirac neutrinos appear rather naturally in left-right models, it is difficult to make them sufficiently light in this context.^{2,Fl} Dirac neutrinos would also explain the non-observation³ of neutrinoless double beta decay in the most straightforward way, while allowing the neutrino masses to be in an accessible range.⁴

Our approach has three basic ingredients. First, as mentioned above, we assume the existence of mirror generations. Our major new assumption is that the neutrino mass matrix is skewed as a result of a discrete Z_6 symmetry. It has been made plausible⁵ that such a discrete symmetry can arise from a GUT and cure some of the ills associated with mirror fermions. Lastly, we hypothesize a small hierarchy of Higgs vacuum expectation values (VEV's) on the order of 10^{-3} . Such a hierarchy is, as we show below, natural, and is usually assumed in left-right models to satisfy the phenomenological requirement that W_L - W_R mixing be small.⁶ The ultralight righthanded neutrinos do not violate the cosmological bound⁷ on the number of neutrinos since these extra degrees of freedom do not reach thermal equilibrium during nucleosynthesis in the absence of Majorana contributions.⁸

Although the existence of equal numbers of mirror and standard generations would produce automatic anomaly cancellation, some composite models require that these numbers differ in order to provide anomaly matching at the composite and preon levels.⁹ We find the mass spectrum of neutrinos generated in our model favors an equal number of mirror and standard

generations in order to circumvent the cosmological argument¹⁰ against neutrino masses in the 100 eV - 2 GeV range. If a neutrino does exist with mass in the keV range¹¹, this would signal an unequal number of generations.

We consider here just the lepton sector of our model since our innovations do not affect the quark sector. We present an example of our mechanism with two standard and two mirror generations. The lepton fields are written as left-handed fields and transform under $SU(2)_L \times SU(2)_R \times U(1) \times Z_6$ as follows:

$$\begin{aligned}
 L_{2,0} &= \begin{pmatrix} \nu \\ \ell^- \end{pmatrix}_{2,0} \sim (2,1;-1)_{2,0} & L_{0,1}^c &= \begin{pmatrix} \ell^c \\ -\nu^c \end{pmatrix}_{0,1} \sim (1,2;+1)_{0,1} \\
 M_{1,2} &= \begin{pmatrix} E^+ \\ N \end{pmatrix}_{1,2} \sim (2,1;1)_{1,2} & M_{-2,2}^c &= \begin{pmatrix} N^c \\ -E \end{pmatrix}_{-2,2} \sim (1,2;-1)_{-2,2} \cdot
 \end{aligned} \tag{1}$$

Here L and M are the normal and mirror lepton fields, respectively. The subscript n_f refers to the Z_6 transformations properties: a field f transforms like $f \rightarrow e^{(2\pi i n_f / 6)} f$ under a Z_6 rotation. As indicated, leptons of different generations differ in their Z_6 quantum numbers. We now introduce the following Higgs fields:

$$\begin{aligned}
 \chi_L &= \begin{pmatrix} \chi^+ \\ \chi^0 \\ \chi^- \end{pmatrix} \sim (2,1;1) & \chi_R &= \begin{pmatrix} \chi^- \\ \chi^0 \\ \chi^+ \end{pmatrix} \sim (1,2;1) \\
 \phi_{-1,-2} &= \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}_{-1,-2} \sim (2,2;0)_{-1,-2} & \phi_{-1,1,3} &= \begin{pmatrix} \phi^+ & \phi^{++} \\ \phi^0 & \phi^+ \end{pmatrix}_{-1,1,3} \sim (2,2;2)_{-1,1,3}
 \end{aligned} \tag{2}$$

The χ_R field is needed to break the $SU(2)_R$ symmetry at a scale larger than the

electroweak scale, so as to minimize righthanded weak interactions. Since this is its only role we do not specify its Z_6 quantum numbers. We assume that $\langle \chi_R \rangle \gg \langle \chi_L \rangle$ as is standard in left-right models.⁶ The ϕ and $\tilde{\phi}$ fields, transforming as doublets under both $SU(2)_{L,R}$, generate Dirac mass terms for the leptons by mixing $SU(2)_L$ fields with $SU(2)_R$ fields. No Majorana contributions occur in our model, even when radiative corrections are taken into account.

Although our Higgs sector is more complex than in the standard model, this is not necessarily a disadvantage. The possibility exists that the symmetry breaking is dynamical in origin. It could well be a complex process, which perhaps we can simulate using several Higgs fields in low representations of the gauge group. A positive feature of our Higgs sector is that all Higgs fields are doublets or singlets under $SU(2)_L$, so that the ρ -parameter is unity at the tree level.

The most general Yukawa Lagrangian respecting all symmetries has the skewed form:

$$L_Y = g_{LL} L \phi L^C + \tilde{g}_{MM} M \tilde{\phi} M^C + \tilde{g}_{ML} M \tilde{\phi} L^C + \text{h.c.} \quad (3)$$

where for example

$$g_{LL} L \phi L^C \equiv \sum_{ijk} g_{LL}^{ij;k} L_i^T (-i\sigma_2) \phi_k L_j^C$$

and the several fields of each type are summed over. We have employed the notation $(\tilde{\phi}, \tilde{\phi}) \equiv \sigma_2 (\phi^*, \phi^*) \sigma_2$. $\tilde{\phi}$ transforms under $SU(2)_L \times SU(2)_R$ as $\tilde{\phi} \rightarrow U_L \tilde{\phi} U_R^+$, just as ϕ does. Note that there is no g_{LM} coupling, and that the terms involving ϕ and $\tilde{\phi}$ are now segregated. This segregation is, in any case, necessary in most left-right models in order to avoid flavor-changing

neutral currents (FCNC) due to Higgs exchange.¹²

After symmetry breaking we obtain the lepton mass matrix

$$\mathcal{M}_{\pm,0} = \begin{pmatrix} 0 & M \\ M^T & 0 \end{pmatrix}_{\pm,0} \quad (4)$$

in the basis $[(M,L), (M^c,L^c)]$. The diagonal submatrices are null because there are no Majorana mass contributions in our model. The off-diagonal submatrices in the charged and neutral sectors are

$$M_{\pm} = \begin{array}{c} (E_{\mu}^c)_{-2} \quad (E_e^c)_2 \quad (\mu^c)_0 \quad (e^c)_1 \\ \begin{array}{c} (E_{\mu})_1 \\ (E_e)_2 \\ (\mu)_2 \\ (e)_0 \end{array} \end{array} \left[\begin{array}{cccc} \sim 11 & & & \\ g_{MM} \langle \phi^{o*} \rangle_1 & 0 & 0 & 0 \\ 0 & \sim 22 & & \\ & g_{MM} \langle \phi^{o*} \rangle_2 & 0 & 0 \\ 0 & 0 & g_{LL}^{11} \langle \phi^o \rangle_{-2} & 0 \\ 0 & 0 & 0 & g_{LL}^{22} \langle \phi^o \rangle_{-1} \end{array} \right] \quad (5a)$$

$$M_0 = \begin{array}{c} (N_{\mu}^c)_{-2} \quad (N_e^c)_2 \quad (\nu_{\mu}^c)_0 \quad (\nu_e^c)_1 \\ \begin{array}{c} (N_{\mu})_1 \\ (N_e)_2 \\ (\nu_{\mu})_2 \\ (\nu_e)_0 \end{array} \end{array} \left[\begin{array}{cccc} \sim 11 & & & \\ g_{MM} \langle \phi^{o*} \rangle_1 & 0 & \sim 11 & \\ & & g_{ML} \langle \phi^{o*} \rangle_{-1} & 0 \\ 0 & \sim 22 & & \\ & g_{MM} \langle \phi^{o*} \rangle_2 & 0 & \sim 22 \\ 0 & 0 & g_{LL}^{11} \langle \phi^o \rangle_{-2} & 0 \\ 0 & 0 & 0 & g_{LL}^{22} \langle \phi^o \rangle_{-1} \end{array} \right] \quad (5b)$$

In this simple $N_s = N_m = 2$ case no mixing occurs among the standard generations nor among the mirror generations. The skewed form of the neutrino mass matrix is a direct consequence of the discrete symmetry imposed. We will neglect CP violation and take all vacuum expectation values to be real. Note that the VEV $\langle \phi^0 \rangle$ appears only in the charged matrix M_{\pm} while $\langle \phi'^0 \rangle$ appears only in the neutral matrix, as a result of the segregated form of the Yukawa Lagrangian in Eq. (3). Also, there is no standard/mirror mixing for the charged leptons unlike the situation for the neutral leptons. This allows the possibility that ultralight neutrino masses can be generated by standard/mirror mixing.

To compute the lepton masses, we calculate the eigenvalues of $M_{\pm}M_{\pm}^T$ and $M_0M_0^T$ which appear in

$$\mathcal{M}_{\pm,0}^2 = \begin{bmatrix} MM^T & 0 \\ 0 & M^T M \end{bmatrix}_{\pm,0} \quad (6)$$

For the charged lepton case the result is straightforward, with the standard and mirror masses set by $g_{LL} \langle \phi^0 \rangle$ and $\tilde{g}_{MM} \langle \phi'^0 \rangle$, respectively. Experiment reveals that the former are in the range $.5 \times 10^{-3} - 1.8$ GeV, while the charged mirrors must be heavier than 22 GeV to have avoided detection at PETRA.¹²

The results in the neutral sector are affected additionally by the standard/mirror mixing. We will assume that the scale of this mixing is the natural one, which is just the electroweak breaking scale of approximately 250 GeV. The block diagonal entries in the neutrino mass matrix are proportional to the same Yukawa couplings of the ϕ fields that fix the charged lepton masses, but not the same VEV's, as mentioned above. It turns out that the ratio of the VEV's of the different components, $\langle \phi'^0 \rangle / \langle \phi^0 \rangle$, can be

naturally small, i.e. the ratio is proportional to couplings which when taken to zero increase the symmetry, as will be seen below. We will take $\langle \phi^{\prime 0} \rangle / \langle \phi^0 \rangle \sim 10^{-3}$.

Scaling the block diagonal entries by 10^{-3} relative to M_{\pm} , M_0 has entries of the following order of magnitude:

$$M_0 \equiv \begin{bmatrix} I & B \\ 0 & S \end{bmatrix} \sim \begin{bmatrix} > 22 \times 10^{-3} & 10^2 \\ 0 & 10^{-3} - 10^{-6} \end{bmatrix} \text{ GeV} . \quad (7)$$

Let N_m represent the number of mirror generations and N_s the number of standard generations of leptons. Barring accidental cancellations, there are in general for a matrix of this type N_m eigenvalues on the order of $B^2 \sim (10^2 \text{ GeV})^2$, N_m eigenvalues on the order of $\frac{I^2 S^2}{B^2} \sim (10^{-7} - 10^{-10} \text{ GeV})^2$, and $N-2N_m$ eigenvalues on the order of $S^2 \sim (10^{-3} - 10^{-6} \text{ GeV})^2$.

Masses in the ultralight (1-100 eV) and massive ($> 2 \text{ GeV}$) ranges are cosmologically acceptable, but the intermediate (100 eV - 2 GeV) mass range is cosmologically unacceptable unless very rapid decay modes exist¹⁰. In the absence of very rapid neutrino decay modes for the intermediate mass neutrino, our mechanism in general favors an equal number of standard and mirror generations.

Returning to our 2 generation example, for the sake of numerical illustration we set the masses of E_{μ} , E_e , μ , and e equal to 100, 50, 0.1, and 10^{-3} GeV , respectively. These are just the diagonal entries in (5a). Scaling the corresponding M_0 entries by 10^{-3} , and making an appropriate choice for the off-diagonal entries, we obtain:

$$M_0 = \begin{bmatrix} .1 & 0 & 200 & 0 \\ 0 & .05 & 0 & 50 \\ 0 & 0 & 10^{-4} & 0 \\ 0 & 0 & 0 & 10^{-6} \end{bmatrix} \text{ GeV} \quad (8a)$$

yielding neutrino masses

$$\begin{aligned} m_1 &\approx 200 \text{ GeV} & m_3 &\approx 50 \text{ eV} \\ m_2 &\approx 50 \text{ GeV} & m_4 &\approx 1 \text{ eV} \end{aligned} \quad (8b)$$

On the other hand, if $N_s = 2$ but $N_m = 1$, so that the numbers of standard and mirror generations are unequal, the neutrino mass matrix is of the form

$$M_0 = \begin{bmatrix} .1 & 200 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix} \quad (9)$$

and generates a neutrino of mass 1 keV, which is cosmologically unfavored as noted before.

We discuss briefly the Higgs sector to clarify the origin of the small ratio $\langle \phi'^0 \rangle / \langle \phi^0 \rangle$. As indicated in (2), it includes two ϕ fields and three ϕ fields as well as the χ fields which are unimportant in the following discussion. By introducing the third ϕ_{-1} field, we ensure that the most general Lagrangian consistent with the Z_6 symmetry does not respect any additional continuous global symmetry. The terms in the Lagrangian violating what would otherwise be the $U(1)$ symmetry

$$\phi_{-1} \rightarrow e^{i\theta} \phi_{-1}, \quad \phi_{-2} \rightarrow e^{-i\theta} \phi_{-2}, \quad \phi_i \rightarrow \phi_i \quad (10)$$

are the mixing terms

$$\text{Tr}(\phi_3^\dagger \phi_1 \phi_1^\dagger \tilde{\phi}_1), \text{Tr}(\phi_3^\dagger \phi_1 \tilde{\phi}_2^\dagger \phi_{-2}), \text{Tr}(\phi_3^\dagger \phi_{-1} \tilde{\phi}_{-1}^\dagger \phi_{-1}), \text{ etc.} \quad (11)$$

These are the terms involving both ϕ and $\tilde{\phi}$ fields in a non-trivial way. Since setting the corresponding couplings to zero increases the symmetry from Z_6 to $Z_6 \times U(1)$, it is natural to take these couplings to be small. Furthermore, for a range of the parameters of the Higgs potential, the VEV's $\langle \phi'^0 \rangle_{-1}$ and $\langle \phi'^0 \rangle_{-2}$ would vanish in the absence of these terms. Upon their inclusion these VEV's become non-zero, in magnitude proportional to the above couplings:

$$\langle \phi'^0 \rangle \sim g \frac{\langle \phi \rangle^2}{\langle \phi^0 \rangle} \sim g(250 \text{ GeV}), \quad (12)$$

assuming $\langle \phi^0 \rangle \sim 250 \text{ GeV}$. Thus $\langle \phi'^0 \rangle \ll \langle \phi^0 \rangle$ can be achieved naturally in the sense of 't Hooft.¹⁴ The small explicit breaking of the $U(1)$ due to the terms in Eq.(11) gives the associated would-be Goldstone boson a small mass. This scalar can be made invisible by slightly complicating the Higgs sector, for instance by introducing a second Higgs field transforming like χ_R .

We now turn to the experimental constraints on our model. It turns out that the mixing effects due to the mirror leptons are extremely small, of order $\epsilon_i = g_{LL}^{ii} \langle \phi'^0 \rangle / (g_{ML}^{ii} \langle \phi^0 \rangle) \lesssim \frac{m_\mu}{(250 \text{ GeV})} \frac{\langle \phi'^0 \rangle}{\langle \phi^0 \rangle} \sim 10^{-6}$, or else involve right-handed currents and are suppressed for this reason. Thus our $N_s = N_m = 2$ model is completely consistent with observed limits on rare processes.

In conclusion, we have proposed a new means of naturally obtaining

ultralight Dirac neutrinos, consistent with the nonobservation of neutrinoless double beta decay. Our model is a left-right gauge theory and contains mirror fermions which are prevented from condensing with the standard ones by means of a discrete symmetry that is unbroken down to low energies. This discrete symmetry enforces a skewed mass matrix for the neutral leptons; with an assumed natural hierarchy $\langle \phi'^0 \rangle / \langle \phi^0 \rangle \sim 10^{-3}$ consistent with small $W_L - W_R$ mixing, some of the neutrinos are rendered ultralight (< 100 eV) and others massive (~ 100 GeV) in a manner reminiscent of the see-saw mechanism. The discrete symmetry also avoids the generic problem of left-right theories with FCNC's in the Higgs sector. The smallness of the ratio m_ν/m_ℓ is explained as $m_\nu/m_\ell \sim (\langle \phi'^0 \rangle / \langle \phi^0 \rangle)^2 m_E/m_W \sim 10^{-6} m_E/m_W$.

The cosmological bounds on neutrino masses can also be satisfied if an equal number of mirror and standard generations of leptons appear, so that neutrinos in the dangerous mass range (100 eV - 2 GeV) are avoided. Although the righthanded neutrinos are equally ultralight or massive as their lefthanded counterparts, difficulties with the cosmological bound for four ultralight two-component neutrinos are avoided even for the physically interesting case of 3 standard and 3 mirror generations, since no Majorana contributions are present which could thermalize the righthanded neutrinos during nucleosynthesis. On the other hand the existence of a neutrino with mass in the keV range would indicate that the numbers of mirror and standard generations are unequal.

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Footnotes

- F1. The contrary claim of S. Panda and U. Sarkar, Ref. 2, is negated due to an error in the manuscript.