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COSMOLOGY IN THEORIES WITH EXTRA DIMENSIONS

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Some possible cosmological effects of the existence of extra compact dimensions are discussed. Particular attention is given to the possibility that extra dimensions might naturally lead to an inflationary Universe scenario.

I. INTRODUCTION

Searches for a consistent quantum theory of gravity and attempts to unify gravity with the strong and electroweak forces have led to the speculation that there may be more than four space-time dimensions, and that the extra spatial dimensions are unseen because they form a compact space of very small physical dimensions. If the physical size of extra compact dimensions is d, to explore the extra dimensions it is necessary to have a probe with wavelength comparable to d, i.e. it is necessary to use a probe with energy comparable to d^{-1}. If the physical size of extra spatial dimensions are smaller than about 10^{-16}cm $\simeq$ (100 GeV)$^{-1}$, direct physical effects of the extra dimensions would have escaped notice. The small size of extra dimensions is expected if gravity is to set the scale, as the only length in gravity is the Planck length, $R_{pl} = 10^{-33}$cm. Therefore the extra dimensions are expected to have a characteristic scale of 10^{-33}cm, and an energy comparable to the Planck energy $E_{pl} = 10^{19}$GeV would be necessary to excite the extra dimensions.

It is by now standard practice to use the early Universe to explore physics at energy scales orders of magnitude greater than those accessible in terrestrial accelerators. Here, I will discuss some possible cosmological effects of the existence of extra dimensions.

For the most part I will consider only simple Kaluza-Klein theories of extra dimensions, where the entire theory is gravity in 4+D dimensions, plus a possible cosmological constant and external matter fields to enforce the compactification. Of course in recent years Supergravity and Superstring theories have provided the motivation for considering theories in more than 4 space-time dimensions.* Such theories are far more complicated than the simple theories I will consider. However, it is reasonable to hope that the toy models I consider, while incomplete, can be used to study the physical cosmological effects of the existence of extra dimensions.

In any theory with extra dimensions there are several general questions which may be asked. The most obvious question has to do with the huge size disparity of the dimensions. If we live in a closed Universe, the three spatial dimensions we observe have the symmetry of the three-sphere, S^3. A simple possibility for the internal space is that of a D-dimensional sphere S^D. The physical size of S^3 must be greater than the Hubble distance $R_3 \geq R_H \simeq 10^{28}$cm, while we expect the physical size of the internal space to be slightly larger than the Planck radius $R_D = 10R_{pl} \simeq 10^{-32}$cm. There is an obvious size disparity of more than 60 orders of magnitude! The usual way this disparity is pointed out is with the question "why is the internal space so small?" From a cosmological point of view this is the wrong question to ask. The Planck length is the only reasonable size for dimensions, since it is the "natural" length scale for gravitation. Therefore the internal space is the "natural" size. I think that the correct question to ask is why is the size of S^3 so large? This, indeed, is a cosmological question that should be addressed whether there are extra dimensions or not. The size of S^3 can be understood in cosmological models

*See the contributions of S. Weinberg and M. Gell-Mann in these proceedings for more details and a comparison of different approaches.[1,2]

that have "inflation."[3] In the second section I will discuss inflation with extra dimensions, and the possibility that the observed three spatial dimensions are large because of the existence of extra dimensions.

A second general question about the existence of extra dimensions concerns the ground state. In Kaluza-Klein models, the ground state does not have all the symmetry of the action, i.e. there is a spontaneous symmetry breaking. What drives this spontaneous symmetry breaking? Is there a unique ground state solution? Why are there three large dimensions? What determines the total number of dimensions? It may be that the answer to all these questions are determined by the microphysics. It may be that a consistent low energy theory can only be constructed for a unique ground state. However if extra-dimensional inflation can operate, it may be that the structure of space-time itself determines the answer to the above questions. In the next section I will discuss this possibility.

In the Kaluza-Klein approach fundamental constants depend upon radii of the extra D-dimensional internal space, Newton's gravitational constant, G_N, is given by $G_N = G\, R_D^{-D}$, where R_D is the geometric mean of radii of the internal space.[4] The gauge coupling constants are given by $g_i = R_{pl}/R_i$ where R_i is a r.m.s. radius in the internal space. Any change in volume of the internal space will in general generate a change in physical constants, for instance, α and G_N. There are a variety of astrophysical arguments to suggest that α and G_N have remained unchanged over cosmological time. In fact as far back as the time of primordial nucleosynthesis, one second after the big bang, any change in α or G_N would have resulted in a change in the yield of primordial helium.[5] If we assume that a single radii, R_D, for the extra dimensions then it is reasonable to parameterize

$$\frac{\alpha}{\alpha_o} = \left(\frac{R_D^o}{R_D}\right)^2 \quad ; \quad \frac{G_N}{G_N^o} = \left(\frac{R_D^o}{R_D}\right)^D \quad ;$$

$$\frac{G_F}{G_F^o} = \left(\frac{R_D^o}{R_D}\right)^2 \qquad (1.1)$$

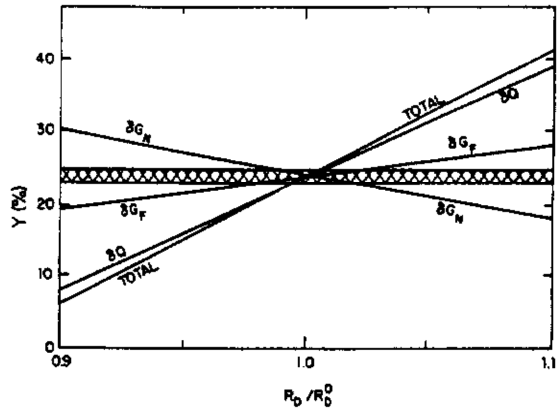

Fig. 1 The mass fraction of ^{4}He produced in the big bang as a function of R_D/R_D^o assuming α, G_N and G_F scale as in Eq. (1.1).

where the "o" sub- and superscript refer to the values today.* In Fig. 1, the primordial helium produced is given as a function of R_D/R_D^o. The line marked $\delta G_N(\delta G_F)$ results from only changing $G_N(G_F)$. A much larger change results if we note that the neutron-proton mass difference is largely electromagnetic in origin, and any change in α will result in a change in $Q = M_n - M_p$. We cannot calculate Q, but it is reasonable to assume $Q/Q_o = \alpha/\alpha_o$. The line marked δQ in Fig. 1 is the primordial helium production only changing Q. The total effect is given by the curve marked "Total." If we assume the primordial helium produced in the big bang is 24±1% (the shaded area in Fig. 1),[6] then $R_D/R_D^o = 1\pm0.005$ at the time of nucleosynthesis (1 second after the big bang).† Obviously something in the theory must keep the extra dimensions static. In models I will discuss in the next section the additional features responsible for keeping the extra dimensions balled up can give a zero four-dimensional cosmological constant at only one value of R_D. If R_D is ever away from this equilibrium value there would result an effective

*The Fermi constant, $G_F = \sqrt{2}\, g_2^2/M_W^2$, depends on M_W in addition to g_2 (the SU_2 coupling constant). In Eq. (1.1) I have only included the dependence upon g_2.

† Of course, we are not sensitive to rapid oscillations of R_D about R_D^o. In the next section it is argued that R_D might oscillate about its equilibrium value with a characteristic frequency of $m_{pl}^{-1} = 10^{-43}$ seconds.

cosmological constant in the dimensionally-reduced four dimensional Universe. This will be used in the next section to derive an inflationary Universe scenario.

Finally, we might ask if there are any low-energy tests of the existence of extra dimensions.* In the final section I will discuss two possible remnants of the early Universe when the extra dimensions were dynamically important.

Some, or all, of the above questions about extra dimensions may have cosmological answers. It is also clear that for a study of the dynamics of the Universe at very early times, when the scale of the three observed dimensions were comparable to the scale of the internal dimensions, it is necessary to take into account the existence of th extra dimensions. Indeed, the evolution of the three large spatial dimensions, and the evolution of the internal dimensions are in general coupled through the Einstein equations. Therefore, from both the particle physics and the cosmology viewpoint it is interesting to study cosmology in theories with extra dimensions.

II. INFLATION AND EXTRA DIMENSIONS

If it is possible to somehow generate a huge amount of entropy, $S \gtrsim 10^{88}$, in a smooth way in a causally connected volume early in the expansion of the Universe, it is possible to understand several outstanding questions about the standard big-bang model.[3] In the standard inflationary approach the entropy is created during a cosmological phase transition, with some dynamical Higgs field responsible for spontaneous symmetry breaking used as an order parameter.[3]

For Higgs field inflation it is usually assumed that the initial value of the Higgs field is $\phi_o = 0$, due to the high-temperature form of $V(\phi)$. For the simple one scale potential model shown in Fig. 2, at some temperature $T \lesssim O(\mu)$ the potential evolves to its zero temperature form, and $\phi = 0$ is no longer the global minimum. Eventually the Higgs field will evolve to the zero temperature minimum.

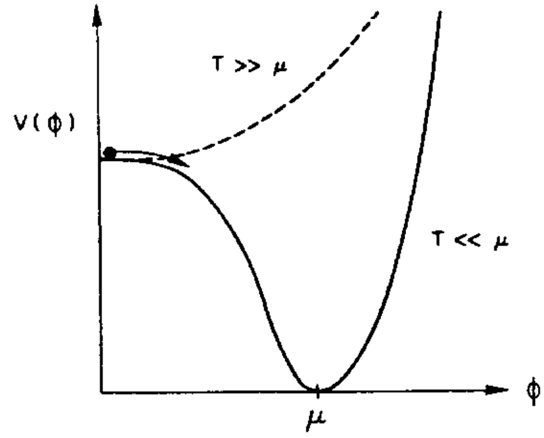

Fig. 2 Higgs potential as a function of temperature. In the new inflation picture ϕ has an initial value of $\phi = 0$ at $T \lesssim \mu$ and the Universe inflates while ϕ evolves to the low temperature minimum at $\phi = \mu$.

The evolution of ϕ during the phase transition may be described by the equation of motion

$$\ddot{\phi} + 3(\dot{a}/a)\dot{\phi} + (\partial V/\partial \phi) + \Gamma_\phi \dot{\phi} = 0 , \qquad (2.1)$$

where Γ_ϕ is the ϕ decay width, and $\dot{a}/a$ is the expansion rate of the Universe, related to the energy density, ρ, by

$$(\dot{a}/a)^2 = 8\pi G_N \rho/3 = (8\pi G_N/3)\ (V(\phi) + 1/2\ \dot{\phi}^2) \qquad (2.2)$$

(where we have ignored for simplicity the contribution of radiation to ρ). If $V(\phi) > 1/2\ \dot{\phi}^2$, the Universe is approximately in a "deSitter phase" and expands exponentially, $a/a_o \propto \exp[(G_N\ V(\phi))^{1/2} t] = \exp[\mu^2 t/M_{pl}]$. If the Universe remains dominated by $V(\phi)$ for a time Δt during its evolution to the minimum, then a physical volume of the Universe originally smooth will increase by a factor of $(a/a_o)^3 = \exp[3\mu^2 \Delta t/M_{pl}]$. If the Higgs field dissipates the potential energy upon reaching the minimum, the Universe can be "re-heated" to a temperature $T = [V(0)]^{1/4} = \mu$, which was comparable to the initial temperature at the start of the phase transition. Therefore the entropy will increase by $\exp[3\mu^2 \Delta t/M_{pl}]$, a potentially huge number.

There have been many attempts to construct a successful inflationary Universe model. We have

*In the Kaluza-Klein viewpoint the fact that we observe gauge interactions is evidence for the existence of extra dimensions.

learned that only a very special class of models for $V(\phi)$ will work. One criterium is that $V(\phi)$ must have a "flat" region where $\partial V/\partial\phi$ is small. There are also many other constraints on $V(\phi)$ to have successful new inflation.[7] Among the imposed constraints on the model is the demand that quantum effects during inflation produce density perturbations that will subsequently grow to become the observed structure in the Universe.

I will now describe two approaches that use extra dimensions to produce a large amount of entropy. The first approach is closely related to new inflation. This general method was first proposed by Shafi and Wetterich[8] in a model with higher derivative terms in the action. Before discussing the Shafi-Wetterich model, I will illustrate the method in two simple models, a six dimensional Einstein-Maxwell model considered by Okada[9] and Wetterich[10] for inflation, and the Casimir model considered by Frieman, Kolb and Rubin[11] for inflation.

The first example is the six-dimensional Einstein-Maxwell theory.[12] The action in this theory is given by $(M,N = 0,1,2,3,4,5)$

$$I = -(16\pi G)^{-1}\int d^6x\,\sqrt{-g}[R + 1/4\ F_{MN}F^{MN} + \Lambda], \qquad (2.3)$$

where g, R are the six-dimensional metric determinant and scalar curvature, $F_{MN} = \partial_M A_N - \partial_N A_M$ is the curl of a six-dimensional abelian gauge field A_M, Λ is a cosmological constant and $\bar{G}$ is a constant with dimension $(\text{length})^4$. A ground state solution $M^4\times S^2$ exists if the gauge field takes the monopole form on S^2

$$\begin{aligned} A_\phi &= (n/2e)(\cos\phi \pm 1) \\ A_M(M \neq \phi) &= 0 \end{aligned} \qquad (2.4)$$

where ϕ is the polar coordinate on S^2, and if we fine tune the condition

$$8e^2 = n^2\Lambda\,(16\pi\bar{G})^{1/2}. \qquad (2.5)$$

We can now solve the Einstein equations

$$R_{MN} - 1/2\ g_{MN}R = -8\pi\bar{G}T_{MN} + 1/2\ \Lambda g_{MN} \qquad (2.6)$$

with T_{MN} from the Maxwell field

$$T_{MN} = F_{MP}F_N{}^P - 1/4\ g_{MN}F^2. \qquad (2.7)$$

We search for cosmological solutions of the form

$$g_{MN} = \begin{pmatrix} -1 & & \\ & a^2(t)\bar{g}_{mn} & \\ & & b^2(t)\bar{g}_{\mu\nu} \end{pmatrix} \qquad (2.8)$$

where a and b are the cosmological scale factors for S^3 and S^2, and $\bar{g}$ is the maximally symmetric 3- and D- metric for S^3 and S^2. The equations of motion are*

$$3(\ddot{a}/a) + 2(\ddot{b}/b) = -(b^{-2}-2\Lambda)(b^{-2}+2\Lambda)/8\Lambda \qquad (2.9a)$$

$$(d/dt)(\dot{a}/a) + [3(\dot{a}/a)+2(\dot{b}/b)]\dot{a}/a = -(b^{-2}-2\Lambda)(b^{-2}+2\Lambda)/8\Lambda \qquad (2.9b)$$

$$(d/dt)(\dot{b}/b) + [3(\dot{a}/a)+2(\dot{b}/b]\dot{b}/b = 3(b^{-2} - 2\Lambda/3)(b^{-2}-2\Lambda)/8\Lambda \qquad (2.9c)$$

The Minkowski solution corresponds to $b = b_o = (2\Lambda)^{-1/2}$, and there is a deSitter solution corresponding to $b = \sqrt{3}(2\Lambda)^{-1/2}$. In inflation with extra dimensions, in place of the Higgs scalar field in usual inflation, a dynamical scalar field

$$\phi = \ln(b/b_o) \qquad (2.10)$$

is used. If we define a variable $x = t/b_o$, then Eq. (2.9c) takes the form $(' = d/dx)$

$$\phi'' + 3(a'/a)\phi' + \phi'^2 + (\partial V/\partial\phi) = 0\ , \qquad (2.11)$$

where V is a dimensionless "potential" given by

$$V(\phi) = 3/4\ [(1/4)e^{-4\phi} - (2/3)e^{-2\phi} - \phi/3 + 5/12]. \qquad (2.12)$$

Note that $V(0) = (\partial V/\partial\phi)_{\phi=0} = 0$. The potential is shown in Fig. 3.

There are two obvious conclusions to be drawn from the figure. The first is that successful inflation is not possible. The potential does not

*Note that we have assumed $\bar{g}_{mn}$ flat. This assumption will be discussed later.

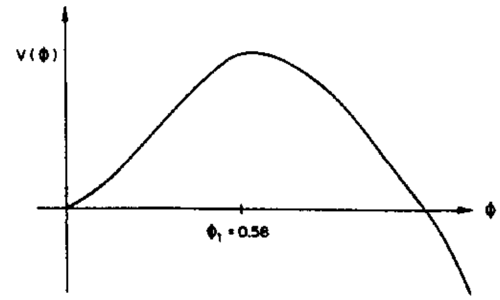

Fig. 3 The inflationary potential for the six-dimensional Einstein-Maxwell Theory.

have a long, flat region. Although $\partial V/\partial\phi$ vanishes at ϕ_1, it is an unstable point, and any thermal or quantum correction would "start the ball rolling" and terminate the transition before sufficient inflation. The second conclusion is that the minimum at $\phi=0$ is only a metastable minimum. If ϕ were ever greater than ϕ_1 (and $\dot\phi$ positive) the extra dimensions would expand forever. This point will be discussed later.

A similar program can be applied to Kaluza-Klein models with Casimir pressure balancing a cosmological constant to keep the internal dimensions static.[13,14] In such models, the relevant action is the Einstein-Hilbert action with a cosmological constant, plus free massless matter fields, and quantum fluctuations in the matter fields responsible for T_{MN}. If we define

$$\rho = g^{oo}T_{oo};\; p_3 = g^{mn}T_{mn};\; p_D = g^{\mu\nu}T_{\mu\nu} \qquad (2.13)$$

$(m,n = 1,2,3;\ \mu,\nu = 4,5\ldots.D)$, then with the assumption of Eq. (2.8) for g_{MN}, the equation of motion for b is

$$\frac{\ddot b}{b} + (D-1)\left(\frac{\dot b}{b}\right)^2 + 3\,\frac{\dot a}{a}\,\frac{\dot b}{b} = \frac{8\pi\bar G}{(D+2)}\,(\rho-3p_3+2p_D)$$

$$+ \frac{\Lambda}{(D+2)} - (D-1)\,\frac{1}{b^2}\,. \qquad (2.14)$$

In general ρ, p_3, and p_D will depend on a, b, $\dot a$, $\dot b$. We will assume that S^3 is flat compared to S^D, so there should be no a dependence. Furthermore, we will ignore the $\dot a$ and $\dot b$ dependence, and simply assume

$$\rho,\ p_3,\ p_D \propto b^{-(4+D)} \qquad (2.15)$$

In that case the right-hand side of (2.14) can be expressed in terms of b_o (the radius of S^D for $M^4\times S^D$) and a dimensionless constant $L = \Lambda b_o^2 \sim 1$.* The equation of motion becomes [again, $\phi = \ln(b/b_o)$]

$$\phi'' + 3(a'/a)\phi' + D\phi'^2 + \partial V/\partial\phi = 0, \qquad (2.16)$$

where, in this case

$$V(\phi) = \left[\frac{D-1}{D+4} - \frac{L}{(D+2)(D+4)}\right] e^{-(D+4)\phi}$$

$$- \frac{D-1}{2}\,e^{-2\phi} - \frac{L\phi}{D+2} + \frac{L}{(D+2)(D+4)}$$

$$+ \frac{(D-1)(D+2)}{2(D+4)}\,. \qquad (2.17)$$

The potential in this case has the same general form shown in Fig. 3, with the same general conclusions.

It is interesting to consider the question of stability of the solution. Obviously the ground-state solution is stable against small perturbations. The figure leads one to believe that for a sufficiently large value of b, the extra dimensions would grow, and not relax to the ground state value $\phi=0$. It should be mentioned that the stability analysis of Candelas and Weinberg[14] is not relevant in the case we consider here. If b is ever different than b_o, then it is impossible to cancel Λ and there would result a four-dimensional cosmological constant. Hence, their assumption of a static metric no longer holds. It must also be admitted that in this case terms proportional to $\dot b$, $\dot a$, etc., may become important in ρ, p_3, p_D, and the simple assumption in Eq. (2.15) might well break down and invalidate the form of the potential.

*If it is possible to calculate ρ, then the constant L can be calculated. However, for present considerations, it is not necessary to know L.

Work on this question is being done by Rubin, Frieman, and Kolb.[11] In any case, it may be possible that if b were ever much greater than b_o in the early Universe, then the extra dimensions may expand, rather than relaxing to b_o.

The final example I mention is the original effort by Shafi and Wetterich.[8] Their model is a gravity theory in $N = 4+D$ dimensions with higher derivative terms in the action, which is given by

$$I \propto \int d^N x \sqrt{g_N} \{\Delta R + \varepsilon + \alpha R^2 + \beta R_{MN} R^{MN} + \gamma R_{MNPQ} R^{MNPQ}\} \qquad (N = D+4) \tag{2.18}$$

where $\alpha, \beta, \gamma, \delta$ are constants. If certain inequalities are satisfied by the constants, a ground state solution $M^4 \times S^D$ is possible. If, in addition, the constants γ and β satisfy

$$\gamma = 1/2\ (12/D-3)[1-12/D(D-1)]^{-1} \beta, \tag{2.19}$$

then the inflationary potential will be of the form shown in Fig. 4. This potential has a long flat region for ϕ large. Such a long flat region allows sufficient inflation to occur. With reasonable values for the parameters the Hubble constant during inflation is several orders of magnitude below the Planck mass. The model also produces reasonably small density perturbations. It seems that this model has all the ingredients for a successful inflationary cosmology.

The success of the Shafi-Wetterich model proves that it is indeed possible to have inflation through extra dimensions. From the previous two examples it is clear that not every extra-dimension model will work, and perhaps cosmology can help us sort through possible models.

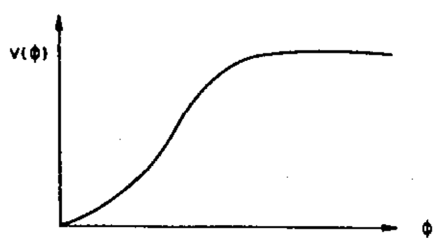

Fig. 4 The inflationary potential for the Shafi-Wetterich higher-dimensional gravity model.

Thus far I have only discussed the exponential expansion phase of inflation. The second crucial phase involves the conversion of the vacuum energy into radiation. The detailed calculations have not been done, however Koikawa and Yoshimura[15] have discussed the general physics of the conversion. If we imagine a free, massless scalar field, s, coupled to gravity, then the action for the field s would be

$$I = \int d^N x\ s \partial_M (\sqrt{-g}\ g^{MN} \partial_N\ s) \qquad (N = D+4). \tag{2.20}$$

The time rate of change of g_{MN} will appear in the equations of motion for s. Therefore as b oscillates about its equilibrium value b_o, terms proportional to $\dot{\phi}$ will appear in the equations of motion for s. The oscillation of ϕ will be damped as entropy is created.

A completely different approach for inflation does not depend upon vacuum energy or a deSitter phase. In this second approach, entropy is created when the Universe goes through an epoch with three expanding dimensions and D contracting dimensions. An increase in temperature results if the mean volume decreases while the three (D) dimensions expand (contract).[16-19] Although the entropy per D+3-dimensional comoving spatial volume remains constant, the entropy per 3-dimensional comoving volume can increase. This approach has been studied by several groups, with the result that no consistent physical model with extra dimensions has been shown to work, although there is no doubt that the method can effectively increase the entropy in the horizon three-volume.*

Let us assume for a moment that some method for inflation with extra dimensions is operative. Then inflation can explain some of the questions discussed above. If we assume that the Universe is closed, then the three spatial dimensions we observe have the geometry of S^3, with physical radius today, $R_3 > R_H$, where R_H is the Hubble radius $R_H = H_o^{-1} \approx 10^{28}$cm. This radius is to be contrasted with the radius of the internal space,

* Note that the second method does not depend upon a cosmological constant. The first method can only work if the internal space has curvature and a cosmological constant. The second method can work with a flat internal space.

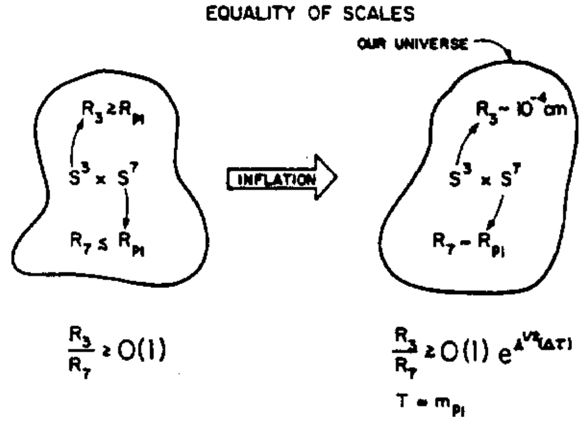

Fig. 5 Inflation can take a volume of space with $R_3 \approx R_7$ and evolve it to a space with $R_3 \ggg R_7$.

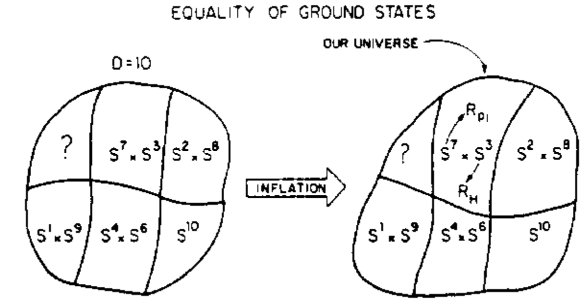

Fig. 6 Different regions of the Universe may have a different split of the original ten spatial dimensions.

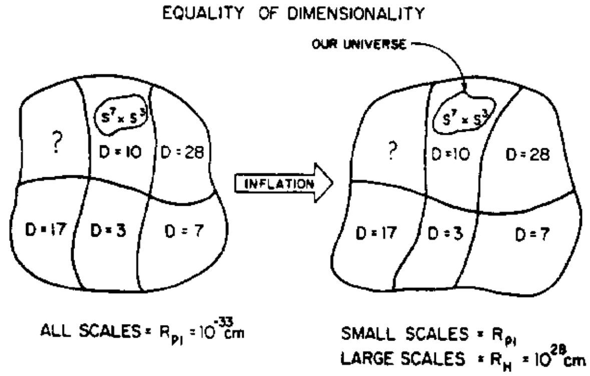

Fig. 7 Different regions of the Universe may have different dimensionality of space-time.

R_D, with a radius today $R_D \approx 10R_{pl} = 10^{-32}$cm. If there is some mechanism for inflating the physical size of S^3 while keeping the size of S^D constant, then the disparity of 10^{60} in scale can be understood. This idea is illustrated (for D=7) in Fig. 5. If in any physical volume with $R_3/R_D > O(1)$, dimensional reduction is possible, and inflation can operate (perhaps in one of the ways outlined above), to increase the R_3/R_D ratio to the exponentially large ratio we see today, although the two scales were once similar.

In the inflation picture, the Universe outside of our horizon might well be a quite different Universe from the one we observe. If at the Planck time, the ten-dimensional space* approximates $S^3 \times S^7$ only locally there may be other ground state geometrics elsewhere. All inflation requires is a region of Planck volume of this ten-dimensional space that is flatter in three directions than in the other seven directions, for the dynamical evolution of this space to become the Universe we observe. Outside of this volume the geometry may be different. This possibility is shown in Fig. 6. It is also possible to imagine that the total dimensionality of space-time varies as shown in Fig. 7.

III. OTHER COSMOLOGICAL EFFECTS

If the Universe was ever at a temperature comparable to R_D^{-1}, or if the scale factors for the large and small spaces were ever of comparable size, then for consideration of microphysical processes, the Universe had more than four space-time dimensions i.e. dimensional reduction is not possible. In such a state it is possible to excite modes that today have masses of the order of the Planck mass.

All theories with extra dimensions, (including superstring theories) have an infinite number of degrees of freedom corresponding, upon compactification, to an infinite tower of massive particles that are quantized excitations of the extra dimensions. This was first pointed out in 1926 by Klein,[20] who studied the five-dimensional

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*Of course, the general scenario will follow with any number of spatial dimensions.

theory of Kaluza. These massive excitations of the extra dimensions have been called pyrgons.[21]* Although the particles are massive in the dimensionally reduced theory, they appear in higher dimensions as massless particles.

In the five-dimensional theory of Kaluza, the equation of motion for excitations of the vacuum metric satisfy the five-dimensional wave equation

$$\Box_5\ \psi(x,y) = 0 \qquad (3.1)$$

where $\Box_5$ is given by

$$\Box_5 = \Box_4 + \partial^2/\partial y^2 \qquad (3.2)$$

with $\Box_4 = -\partial^2/\partial t^2 + \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$. We may take advantage of the symmetry of the extra spatial dimension (y) to expand the field $\psi(x,y)$ in the harmonic series

$$\psi(x,y) = \sum_{n=-\infty}^{\infty} \psi_n(x) e^{iny/R_1} \qquad (3.3)$$

where R_1 is the radius of S^1. Therefore the equation of motion, Eq. (3.1), becomes

$$[\Box_4 - (n/R_1)^2]\psi(x,y) = 0, \qquad (3.4)$$

and upon dimensional reduction the nth mode has mass $m = n/R_1$, although Eq. (3.1) is the equation of motion (in five dimensions) for a massless particle. Particles massless in five dimensions, become upon dimensional reduction either massless or massive with $m \simeq m_{pl}$. Therefore if the Universe was ever at a temperature greater than R_1^{-1}, excitations of the five-dimensional vacuum will be distributed over all values of n, and after dimensional reduction they will be distributed over all masses $m_\psi = nR_1^{-1} \simeq nm_{pl}$.

In the simple five-dimensional model the n=1 massive pyrgons are stable, and ineffective at annihilation. Therefore it might be reasonable to expect as many pyrgons as photons in the Universe. This presents a problem, as $m_\psi \simeq m_{pl}$ and they would contribute about 4×10^{26} times the critical density.[21] Obviously, something must either rid the Universe of massive stable pyrgons, or dilute their abundance through entropy creation.

The most likely mechanism to rid the Universe of pyrgons is decay. In the simple five-dimensional theory they were stable because they carried a charge not present in the zero-mode sector of the theory. More interesting models, such as 11-dimensional supergravity with S^7 for the internal space,[22] have stable pyrgons[21] even though most of the 256 zero modes carry the SO_8 quantum numbers of the symmetry of S^7. In models with the correct low energy theory it is possible to imagine two reasons for stable pyrgons. One example is if all zero modes satisfy the usual electric charge/color relation, but some pyrgon does not. It may also be possible that there is an additional charge under which the zero modes are neutral but some pyrgon is not. If either mechanism leads to stable pyrgons, the Universe is in trouble.

It is also possible that some excitation with Planck mass is stable for topological reasons. Sorkin,[23] and Gross and Perry[24] have shown that topological defects in the geometry of compactification would appear in four space-time dimensions as massive magnetic monopoles.

The cosmological production of GUT monopoles have been studied by many people.[25] Harvey, Perry and I have considered the cosmological production of Kaluza-Klein monopoles.[26] We have found a basic problem in predicting the number of monopoles produced in the big bang. GUT monopoles arise from a topological defect in the orientation of a Higgs field vacuum expectation value responsible for GUT symmetry breaking. At temperatures greater than the scale of symmetry breaking, thermal effects should have restored the symmetry, melting monopoles. The number of GUT monopoles produced in the big bang is independent of initial conditions for $\langle\phi\rangle$, the Higgs field VEV, since at high temperature $\langle\phi\rangle = 0$. However it is not so clear that the topological defects that lead to Kaluza-Klein monopoles disappear above some critical temperature. It may be that the splitting of the spatial dimensions is present as initial conditions, and we cannot predict the production of the Kaluza-Klein monopoles. However if the splitting arises as a result of some dynamical

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*From the Greek word πυργος, for tower.

mechanism, the monopoles would be independent of initial conditions. Detection of Kaluza-Klein monopoles are a potential source of information on the dynamics of dimensional reduction.

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