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Light Composite Supermultiplets

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ABSTRACT

It is argued that in a wide class of massless strongly interacting supersymmetric gauge theories there exist confining phases with unbroken supersymmetry. We explain how to identify and interprete the light composite supermultiplet content of these phases.

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The outstanding question in every strongly interacting supersymmetric gauge theory is how supersymmetry and other global symmetries are realized in the effective theory of gauge-singlet composite supermultiplets. Recently two approaches[1,2,3] has been developed, leading to at least partial answers for the left-right symmetric models. The index analysis of Witten[1] proves that there is no dynamical supersymmetry breaking in theories with matter in real representations. The effective Lagrangian analysis[2,3], less rigorous although more intuitive one, confirms Witten's results, leading however to some unexpected conclusions about the zero mass limit of the supersymmetric QCD (SQCD)[3]. The results of Ref.3 indicate that some supersymmetry preserving biscalar condensates, which break chiral symmetry down to the flavour subgroup blow up to infinite values in the zero mass limit[F1]. One could then expect qualitatevely different physics of massless theories, first of all of the genuinely massless non-left-right symmetric ones, with matter in complex representations. Such a hypothesis is supported by the present ignorance of how to compute the index in the massless cases and by the failure of constructing any simple effective Lagrangians, when limiting oneself to the intuition provided by ordinary gauge theories[4].

In the present paper we argue that in a wide class of massless models there exist confining phases with unbroken supersymmetry. We present a simple prescription, based on the conjectured complementarity[5,6,7] and 't Hooft anomaly conditions[7], which permits the identification and interpretation of the light composite supermultiplet content of these phases. The key observation is that in supersymmetric massless gauge theories with a vanishing f-term (superpotential) and matter in large, in general reducible representations, the scalar potential may have nontrivial zero value degenerate minima, which break the gauge group, while preserving supersymmetry[F2]. If the matter representation is large enough, like for example three families of $\underset{\sim}{3} + \underset{\sim}{\bar{3}}$ in SU(3) or five families of $\underset{\sim}{5} + \underset{\sim}{\overline{10}}$ in SU(5), some supersymmetric vacua exist, which break the gauge group completely. For such models we are able to find completely broken Higgs phases, corresponding to some nontrivial configurations of the vacuum expectation values (vevs) of scalar components of the matter chiral supermultiplets. The perturbation theory in the weak coupling regime (corresponding to small distances in asymptotically free theories under consideration) can be then applied in order to identify the massless fermions. At this point one can invoke complementarity[5] between broken and confining phases and reinterprete massless supermultiplets of the Higgs phase as light gauge-singlet bound states of the confining phase. Several remarks are here in order. First of all, the complementarity has not been established for supersymmetric theories and one should consider it as a conjecture. It is however supported by the examples and nontrivial consistency checks, which will be presented throughout this paper. Another point, which needs clarification, is our use of the Higgs phases with vevs of some elementary scalar fields. This is in contrast with the case of ordinary gauge theories, where the Higgs phases has been usualy related to some fermion-antifermion condensates, indicated according to the most attractive channel criterion[6]. Supersymmetry provides us with elementary scalars and well defined potential, it is then natural to consider their vevs[9] in the presence of degenerate minima. Moreover, the massless scalars by screening may prevent fermions from condensation.

Before proceeding to the examples let us explain some technical points. An elegant group-theoretical analysis of the spontaneous breaking of the gauge group compatible with unbroken supersymmetry has been given by Buccella et al. (BDFS)[10]. Let us consider the case of a semisimple group G, with chiral scalar matter superfields transforming according to some representation R, in general reducible, of G. In the absence of a superpotential the condition for unbroken supersymmetry reads

$$\langle d^a(z,z^*)\rangle = \sum_{\alpha,\beta=1}^{\dim R} \langle z^*_\alpha\rangle (T^a)^\alpha_\beta \langle z^\beta\rangle = 0, \quad \text{all } a, \qquad (1)$$

where z^α denote the (complex) scalar components of chiral supermultiplets, and the matrices T^a stand for the generators of G in the representation R. In Ref.10 a simple algorithm for solving[F3] Eq.(1) has been proposed. The set of Eq.(1) is satisfied by the vev $\langle z \rangle$ if there exists an analytic G-invariant polynomial I(z) such that

$$\left.\frac{\partial I}{\partial z^\alpha}\right|_{z=\langle z\rangle} = c\langle z^*_\alpha\rangle , \tag{2}$$

where c is a complex constant, $c\neq 0$. Indeed, from the gauge invariance of I(z) it follows that

$$\frac{\partial I}{\partial z^\alpha}(T^a)^\alpha_\beta z^\beta = 0 , \quad \text{all } a , \tag{3}$$

therefore Eq.(1) is satisfied whenever Eq.(2) is fulfilled. We will use the BDFS algorithm in order to find and classify some solutions of Eq.(1).

The first example to be studied here is SU(3) SQCD with three massless flavours. The physical degrees of freedom correspond in this case to the chiral superfields

$$\begin{aligned} W^a &= (\lambda^a , F^a_{\mu\nu}) && \textit{gauge multiplet}, \; a = 1 \div 8 \\ \left.\begin{aligned} Q^\alpha_i &= (\varphi^\alpha_i , \psi^\alpha_i) \\ \bar{Q}^i_\alpha &= (\eta^i_\alpha , \chi^i_\alpha) \end{aligned}\right\} && \textit{matter multiplets}, \; \alpha, i = 1 \div 3 \end{aligned} \tag{4}$$

This model contains 18 massless complex scalars φ and η, 26 chiral fermions λ, Ψ and χ, and 8 vector gluons. Classically it possess the global symmetry $SU(3)_L x SU(3)_R x U(1)_V x U(1)_A x U(1)_R$, where the last U(1) factor corresponds to so-called R-symmetry[3]. As explained in Ref.3, both axial $U(1)_A$ and $U(1)_R$ are anomalous, so that only one combination $U(1)_X$ remains unaffected by the Adler-Bell-Jackiw colour anomaly. The quantum numbers of scalars and fermions are

$$\varphi(\underset{\sim}{3},\underset{\sim}{\bar{3}},\underset{\sim}{1},\tfrac{1}{3},0) \qquad \Psi(\underset{\sim}{3},\underset{\sim}{\bar{3}},\underset{\sim}{1},\tfrac{1}{3},\tfrac{3}{2})$$

$$\eta(\underset{\sim}{\bar{3}},\underset{\sim}{1},\underset{\sim}{3},-\tfrac{1}{3},0) \qquad \chi(\underset{\sim}{\bar{3}},\underset{\sim}{1},\underset{\sim}{3},-\tfrac{1}{3},\tfrac{3}{2})$$

$$\lambda(\underset{\sim}{8},\underset{\sim}{1},\underset{\sim}{1},0,-\tfrac{3}{2}), \tag{5}$$

where the first number in a bracket corresponds to the G=SU(3) gauge group representation and the subsequent ones to the global $SU(3)_L x SU(3)_R x U(1)_V x U(1)_X$ transformation properties.

In the weak coupling regime, within the perturbation theory at least three distinct types of solutions of Eq.(1) exist, which break the gauge group completely. They correspond to different global symmetry breaking patterns, implied by configurations of the vevs of scalar fields related to different BDFS polynomials I. We will list one solution after another, in each case identifying the unbroken global subgroup H and the massless fermion spectrum.

(i)
$$I = \epsilon_{\alpha\beta\gamma}\,\epsilon^{ijk}\,\varphi_i^\alpha\,\varphi_j^\beta\,\varphi_k^\gamma \tag{6}$$

$$\langle \varphi_i^\alpha \rangle = \delta_i^\alpha \ , \qquad \langle \eta_\alpha^i \rangle = 0 \tag{7}$$

$$H = SU(3)_D \times SU(3)_R \times U(1)_X \ , \tag{8}$$

where $SU(3)_D$ is the diagonal subgroup of $G \times SU(3)_L$.

Massless fermions:

$$\sum_{\alpha=1}^{3} \Upsilon_\alpha^\alpha \ , \qquad \chi_\beta^j \ . \tag{9}$$

(ii)
$$I = \varphi_i^\alpha\,\eta_\alpha^i \tag{10}$$

$$\langle \varphi_i^\alpha \rangle = \langle \eta_\alpha^i \rangle = \delta_i^\alpha \tag{11}$$

$$H = SU(3)'_D \times U(1)_X \ , \tag{12}$$

where $SU(3)'_D$ is the diagonal subgroup of $SU(3)_D \times SU(3)_R$.

Massless fermions:

$$\sum_{\alpha=1}^{3} \Upsilon_\alpha^\alpha \ , \qquad \sum_{\alpha=1}^{3} \chi_\alpha^\alpha \ ,$$

$$\Upsilon_i^\alpha + \chi_i^\alpha - \tfrac{1}{3}\,\delta_i^\alpha \sum_{\beta=1}^{3}\left(\Upsilon_\beta^\beta + \chi_\beta^\beta\right) . \tag{13}$$

(iii)
$$I = \sum_{i=1}^{2} \varphi_i^\alpha\,\eta_\alpha^i \tag{14}$$

$$\langle \varphi_i^\alpha \rangle = \langle \eta_\alpha^i \rangle = \delta_i^\alpha \qquad \text{for } i \leq 2 \ ,$$

$$\langle \varphi_3^\alpha \rangle = \langle \eta_\alpha^3 \rangle = 0 \tag{15}$$

$$H = SU(2)'_D \times U(1)_X \times [U(1)]^2 \ , \tag{16}$$

where $SU(2)'_D$ is an obvious subgroup of $SU(3)'_D$ and the $[U(1)]^2$ factor corresponds to two conserved combinations of the $U(1)_V$ baryon number with the λ_8 generators of G and $SU(3)_D$.

Massless fermions:

$$\Psi_i^\alpha + \chi_i^\alpha - \tfrac{1}{2}\,\delta_\alpha^i \sum_{\beta=1}^{2} (\Psi_\beta^\beta + \chi_\beta^\beta)\ ,\qquad \Psi_3^\alpha\ ,\quad \chi_\alpha^3\ ,$$

$$\sum_{\beta=1}^{2} (\Psi_\beta^\beta + \chi_\beta^\beta)\ ,\quad \Psi_3^3\ ,\quad \chi_3^3\ ,\qquad \text{with } \alpha, i \leq 2\ . \tag{17}$$

In all three cases eight of matter fermions pair with gauge fermions to form massive Dirac particles. The remaining ten massless fermions form representations of the unbroken global subgroups.

At this point we will invoke complementarity and continue our massless spectra to the strong coupling regime in the confining phase of unbroken gauge symmetry. The light fermions have to be recovered there as gauge-singlet composites. We find that the complementary light composite fermions can always be expressed as some linear combinations of the fermionic components of the following chiral composite superfields:

$$\begin{aligned} T_j^i &= \bar{Q}_\alpha^i\, Q_j^\alpha \\ B &= \epsilon^{ijk}\epsilon_{\alpha\beta\gamma}\, Q_i^\alpha\, Q_j^\beta\, Q_k^\gamma \\ \bar{B} &= \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}\, \bar{Q}_\alpha^i\, \bar{Q}_\beta^j\, \bar{Q}_\gamma^k\ . \end{aligned} \tag{18}$$

In either of the cases they form representations of different subgroups K of the global symmetry group. The light fermions of Eqs.(9), (13) and (17) correspond to the fermionic components of the following composite supermultiplets:

(i) one linear combination[F4] of B and $\bar{B}$, T^i_j (19)

$$K = SU(3)_L \times SU(3)_R \times U(1)_X \tag{20}$$

(ii) B , $\bar{B}$,

$$T^i_j - \frac{1}{3}\delta^i_j \sum_{k=1}^{3} T^k_k \tag{21}$$

$$K = SU(3)_V \times U(1)_V \times U(1)_X \ , \tag{22}$$

where $SU(3)_V$ is the diagonal subgroup of $SU(3)_L \times SU(3)_R$.

(iii) $T^i_j - \frac{1}{2}\delta^i_j \sum_{k=1}^{2} T^k_k$, T^i_3 , T^3_j with $i,j \leq 2$,

$$B \ , \ \bar{B} \ , \ \sum_{k=1}^{2} T^k_k \tag{23}$$

$$K = SU(2)_V \times U(1) \times U(1)_V \times U(1)_X \ , \tag{24}$$

where $SU(2)_V \times U(1)$ is an obvious subgroup of $SU(3)_V$.

This can be easily verified by replacing the scalar constituents of the fermions of Eqs.(19), (21) and (23) by their vevs of Eqs.(7), (11) and (15), respectively.

The first nontrivial check of our results comes from the 't Hooft anomaly conditions[7]. Indeed, in all three cases there is anomaly matching between elementary and composite fermions, with respect to the appropriate subgroups K. An obvious question is what condensates could be responsible for dynamical breakings of the global group down to these subgroups. Not surprisingly, the BDFS polynomials I [see Eqs.(6), (10) and (14)] turn out to be good candidates. Their vevs provide correct symmetry breaking patterns, and furthermore, being vevs of the scalar components of the superfields B, $\bar{B}$ or T, they do not break supersymmetry. In such a way complementarity supported by anomaly matching lead to an interesting conclusion. We find the degeneracy of supersymmetric vacua in the weak coupling regime replicated in the effective potential of the strongly interacting effective theory of composite superfields.

From the discussion of SQCD it is clear that the reality property of the matter representations is irrelevant to our considerations. A similar analysis can be repeated for other examples, in particular for the SU(5) model with five families of $\underset{\sim}{5} + \overline{\underset{\sim}{10}}$ matter representations. This model contains the following chiral superfields

$$\begin{aligned} W^a &= (\lambda^a, F^a_{\mu\nu}) && \text{gauge multiplet}, \; a = 1 \div 24 \\ \left.\begin{aligned} P^\alpha_i &= (\varphi^\alpha_i, \psi^\alpha_i) \\ M^i_{\alpha\beta} &= (\eta^i_{\alpha\beta}, \chi^i_{\alpha\beta}) \end{aligned}\right\} && \text{matter multiplets}, \; \alpha, \beta, i = 1 \div 5 \end{aligned} \tag{25}$$

where $M^i_{\alpha\beta}$ is totally antisymmetric in α and β. Classically it possess the global symmetry $SU(5)_P \times SU(5)_M \times U(1)_P \times U(1)_M \times U(1)_R$. As in SQCD, only two combinations of three U(1) symmetries remain unaffected by the gauge anomaly. The quantum numbers of scalars and fermions are

$$\varphi(\underset{\sim}{5}, \underset{\sim}{\bar{5}}, \underset{\sim}{1}, 3, 0) \qquad \Psi(\underset{\sim}{5}, \underset{\sim}{\bar{5}}, \underset{\sim}{1}, 3, \tfrac{3}{2})$$

$$\eta(\underset{\sim}{\overline{10}}, \underset{\sim}{1}, \underset{\sim}{5}, -1, -1) \qquad \chi(\underset{\sim}{\overline{10}}, \underset{\sim}{1}, \underset{\sim}{5}, -1, \tfrac{1}{2})$$

$$\lambda(\underset{\sim}{24}, \underset{\sim}{1}, \underset{\sim}{1}, 0, -\tfrac{3}{2}), \tag{26}$$

where the first number in a bracket corresponds to the G=SU(5) gauge group representation and the subsequent ones to the global $SU(5)_P \times SU(5)_M \times U(1)_V \times U(1)_X$ transformation properties. From the plethora of supersymmetric phases, for the purpose of illustration, we quote here only those related to

$$I = \epsilon_{\alpha\beta\gamma\delta\rho}\,\epsilon^{ijklm}\,\varphi_i^\alpha\,\varphi_j^\beta\,\varphi_k^\gamma\,\varphi_l^\delta\,\varphi_m^\rho \tag{27}$$

and

$$I = \varphi_1^\alpha\varphi_2^\beta\eta^1_{\alpha\beta} + \varphi_3^\alpha\varphi_4^\beta\eta^2_{\alpha\beta}\,. \tag{28}$$

The 51 complementary light composite fermions can then be expressed as some fermionic components of the superfields

$$E = \epsilon_{\alpha\beta\gamma\delta\rho}\,\epsilon^{ijklm}\,P_i^\alpha\,P_j^\beta\,P_k^\gamma\,P_l^\delta\,P_m^\rho$$

$$F^i_{jk} = M^i_{\alpha\beta}\,P_j^\alpha\,P_k^\beta$$

$$G^{jkl}_i = \epsilon^{\alpha\beta\gamma\delta\rho}\,M^j_{\alpha\beta}\,M^k_{\gamma\delta}\,M^l_{\rho\lambda}\,P^\lambda_i\,. \tag{29}$$

They form representations of the little groups of the appropriate BDFS polynomials of Eqs.(27) and (28), i.e. of $SU(5)_P \times SU(5)_M \times U(1)_X$ and $SU(3)_M \times [SU(2)_P]^2 \times [U(1)]^4$, respectively. As in the previous example, the light particle content is consistent with the 't Hooft anomaly conditions. Again, some multiscalar BDFS condensates are expected to form, in a one-to-one correspondence with the classical supersymmetric solutions, which break the gauge group completely.

Many other examples can be studied in a similar manner. The only limitations are coming from the dimensions of matter representations. They have to be large enough, so that completely broken Higgs phases exist, however not too large, so that asymptotic freedom still holds. Such models have several common features. For a given representation the number of massless fermions is by construction the same for all completely broken Higgs and confining phases, and equal to the number of elementary fermions minus the number of the gauge group generators. The light composite fermions belong to the simplest gauge-singlet composite chiral superfields and are built of one elementary fermion plus a number of scalars. Finally, the global symmetries, like the chiral ones are always found to be at least partially broken by multiscalar condensates corresponding to some BDFS polynomials.

Our analysis clearly indicates for the existence of supersymmetric phases, neverthless we do not regard it yet as a proof of the absence of dynamical supersymmetry breaking in this type of models. We are not able to exclude the possibility that some strange nonperturbative phenomena occur, which create some other, more energetically favourable, perhaps nonsupersymmetric, vacua.

In summary, the use of complementarity between Higgs and confining phases leads to the definite conclusions about the realization of supersymmetry and other global symmetries in a wide class of strongly interacting supersymmetric gauge theories. It enables to identify the light supermultiplet content consistent with the anomaly conditions and to recognize the origin and patterns of the dynamical symmetry breaking in the confining supersymmetric phases. We hope that our approach, maybe supplemented by other arguments, could also give some insight into the physics of the models not covered by the present investigation.

As a possible application of our results we would like to point out supersymmetric preon models[11], where the multiscalar preon condensations could lead to some physically interesting symmetry breaking patterns.

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FOOTNOTES

F1 A possible explanation of such a singular behaviour has been recently found by G. Veneziano, to be published.

F2 These are so-called ambiguous solutions of supersymmetric theories, for which the scale of breaking remains undetermined to all orders of perturbation theory[8]. In the following we will set this scale equal to 1.

F3 The trivial solution $\langle z^{\alpha} \rangle = 0$, for all α, does not break any symmetry.

F4 At this point B itself would fit too [see Eq. (7)], however as we will see later, $U(1)_V$ is expected to be broken because of the anomaly non-matching, therefore this state should not carry a definite $U(1)_V$ quantum number.

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