Fermi National Accelerator Laboratory

FERMILAB-Pub-82/38-THY
May, 1982

# Composite Models Without Spectators

A.J. Buras,* S. Dawson and A.N. Schellekens
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

(Received

ABSTRACT

We discuss those composite models of quarks and leptons with an SU(N) metacolor group and preons in the complex, anomaly free and asymptotically free representations, which do not have spectators with SU(3) ⊗ SU(2) ⊗ U(1) quantum numbers. We find that many of these models can be ruled out independently of the way the chiral (metaflavor) symmetries are realized and the bound states are constructed. The remaining models contain a large number of preons; 0(100) or more.

*Permanent address after September 1, 1982:
Max-Planck Institute fur Physik und Astrophysik,
Fohringer Ring 6, 8000 Munchen 40.

## 1. Introduction

During the last two years there has been much interest[1] in constructing "composite models" in which the quarks and leptons are supposed to be the bound states of more elementary objects--preons. The main motivations for considering such models are:

i) the proliferation of the observed quarks and leptons, which in composite models could hopefully be built out of a small number of preons;

ii) the generation structure observed in the fermion spectrum;

iii) the proliferation of parameters, of which the quark and lepton masses and the Kobayashi-Maskawa mixing angles are the prime examples; and

iv) the gauge hierarchy problem.

These problems have been addressed by many authors in the past in the context of weak interaction models with gauge symmetries larger than the standard $SU(2)_L \otimes U(1)$; Grand Unification models and the Extended Technicolor models among others.[2] It is however fair to say that in spite of many efforts and many interesting suggestions no convincing solutions to the problems above have been found. There was a hope then that progress could be made by making the quarks and leptons composite. The studies of the last two years have shown however that it is very difficult to find a composite model which could solve simultaneously all the problems listed above.[1]

Since it is so difficult to find a successful composite model it seems useful to change the strategy and first eliminate in a systematic way the models which are not able to solve the problems i)-iii). A lot of care has to be taken however in such an approach for the following reasons.

It is often very easy to rule out composite models by making specific assumptions about the preon dynamics (metacolor), and in particular about the realization of metaflavor (chiral) symmetries, and the way the bound states are constructed.[3-5] For instance, many authors[4,5] assume that the metaflavor symmetries do not undergo a spontaneous breakdown. This assumption when combined with 't Hooft's anomaly matching equations[6] and specific assumptions about the way the bound states are constructed (Fermi statistics, spin rules, etc.) may lead to an immediate elimination of some models. These models might otherwise survive the test had we changed our assumptions about the bound states and allowed a part of the metaflavor symmetry to be spontaneously broken. Therefore in view of the fact that our understanding of preon dynamics is very limited at present, we think that in this "destructive" approach to composite models detailed assumptions about the realization of metaflavor symmetries and about the way the bound states are constructed should be preceded by other assumptions which have much firmer basis.

With these ideas in mind we have analyzed all possible composite models characterized by the following three properties:

a) the metacolor gauge group is SU(N), where N is arbitrary,

b) the preons are in a complex, anomaly free, reducible representation of SU(N), which satisfies asymptotic freedom, and

c) there are no spectators (elementary metacolor singlet fermions) which carry $SU(3) \otimes SU(2) \otimes U(1)$ quantum numbers.

The last property implies in particular that in this class of composite models all the observed quarks and leptons are preon bound states.

Our main result, which does not depend on the realization of metaflavor symmetries nor on the assumptions about the bound states, but is a direct consequence of properties b) and c), is that this class of composite models cannot be simple. The point is that as usual we do not want to have $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ triangle anomalies. In the absence of spectators these anomalies have to be cancelled among the preons themselves. This restriction when combined with the requirement of asymptotic freedom and anomaly freedom for the metacolor group (and a reasonable restriction on the quantum numbers of preons) severly limits the number of possible models.

We find in particular that either

i) the number of preons in each of these models is at least 135, or

ii) the metaflavor symmetry must be larger than SU(15) in which case the number of preons could be slightly less than 100.

Consequently the problem of proliferation of particle species found on the level of quarks and leptons cannot be solved in these models. On the contrary, the minimum number of preons allowed in these models is substantially larger than the number of observed quarks and leptons.

Without making detailed assumptions about the preon dynamics we cannot however exclude the possibility that some of these models may provide clues to the generation puzzle and the fermion spectrum. Therefore as a preparation for possible future investigations, some of which we make in this paper, we give a list of all models of this type which

a) Allow an anomaly free embedding of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ into the metaflavor group, and

b) have metaflavor symmetries which do not contain factors larger than SU(14).

We also give a useful classification of all these embeddings.

Our paper is organized as follows. In Section II we define the class of models considered in this paper in more detail, we list the basic formulae and we outline our strategy. In Section III we present a list of models which cannot be ruled out without making detailed assumptions about the preon dynamics. In Section IV we discuss these models (including those which have a metaflavor symmetry with a factor which is larger than SU(14)) with respect to fermion generations. We find several models which can accommodate the known generations of quarks and leptons, but a full investigation of this question would require more detailed assumptions about the realization of metaflavor symmetries, which is beyond the scope of this paper. We defer such analysis for the future. Section V contains the summary and conclusions.

## 2. The Models and the Strategy

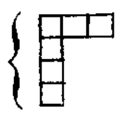
### 2.1 Defining the Models

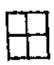
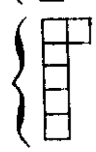
The models which we will consider here have an SU(N) metacolor gauge group and preons in complex, anomaly free and asymptotically free representations of this group.

The first task in our study is then to find all anomaly free and asymptotically free representations of all SU(N) groups. Fortunately this has been already done by the authors of Ref. 7, and we shall use here their results and their notations. The full preon content of each model is then represented by a direct sum

$$R = \sum_{i=1}^{9} n_i R_i \ , \tag{2.1}$$

where $R_i$ are irreducible representations of SU(N). In terms of Young Tableaux, the $R_i$'s are given as follows

$R_1$ =   $R_4$ = N-1   $R_7$ = 

$R_2$ =   $R_5$ = N-2   $R_8$ =   (2.2)

$R_3$ =   $R_6$ =   $R_9$ = 

All preons in Eq. (2.1) are assumed to be left-handed, spin-1/2 particles. The multiplicities $n_i$ can be positive, negative or zero, where negative values correspond to complex conjugate representations. For instance $n_9$=5 means that a given model contains five fundamental representations whereas if $n_9=-5$ there are five complex conjugate fundamental representations.

We should stress however that we only consider here models with fully complex representations, i.e., we do not study models which contain m representations of a given type and some number of the corresponding complex conjugate representations. (Models of this class have been considered by Bars.[5]) Consequently in our models all preons are protected from acquiring a mass by the metacolor symmetry itself.

In the absence of other than metacolor interactions the metaflavor group $G_f$ corresponding to (2.1) is

$$G_f = \prod_{\otimes i} SU(n_i) \otimes U(1)^{p-1} \otimes Z \qquad (2.3)$$

where p is the number of representations with nonzero $n_i$, and Z denotes the U(1) group broken by the metacolor instantons to a discrete symmetry. Suppressing the U(1) quantum numbers, the preons in the $R_i$ representations under metacolor transform under $G_f$ as follows

$$(1,1,..n_i,..1,1) \qquad (2.4)$$

i.e. they are singlets under all non-abelian factors in Eq. (2.3) except for one $SU(n_i)$ with respect to which they transform as the fundamental representation.

In summary each of our models is characterized by a set of numbers $(N,\{n_i\})$ and the corresponding metaflavor group $G_f$. All sets $(N,\{n_i\})$ which are allowed by asymptotic freedom and anomaly freedom are tabulated in Ref. 7.

To proceed further we have to say something about the metacolor singlet particles, of which the standard quarks and leptons are the prime examples. We shall assume that in all models considered by us there are no spectators (with respect to the metacolor group) which carry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers which implies that <u>all</u> the known quarks and leptons are not elementary but are preon bound states. If metacolor is the only new interaction, and if there are no fundamental scalars it is hard to generate masses for fundamental metacolor singlet fermions. This problem is obscured (but not necessarily solved) if other new gauge interactions are present in addition to metacolor, but in any case, it provides sufficient motivation to consider spectatorless models first.

Having defined our models we could now proceed along the standard route[1], i.e.

a) make assumptions about the bound states,

b) make assumptions about the realization of metaflavor symmetries, i.e. assume which part of the group $G_f$ remains unbroken,

and finally

c) find the massless bound states by solving 't Hooft's anomaly matching equations.[6]

As we have emphazised in the Introduction this is not the route we will follow in the first part of our investigations. We first attempt to eliminate as many models as possible without having to make assumptions a) and b), and only at the end shall we discuss the implementation of the standard machinery into our program.

## 2.2 Embedding the Standard Model into $G_f$

The number of allowed models can be substantially restricted through the study of possible embeddings of the Standard ($SU(3)_c \otimes SU(2)_L \otimes U(1)$) Model into the group $G_f$ of Eq. (2.3).

The embedding of the Standard Model in $G_f$ can be quite generally specified by assigning the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers to the preons. In a given model characterized by the set $(N,\{n_i\})$ this can be done in many ways, and consequently to each model there corresponds a set of embeddings of $SU(3) \otimes SU(2) \otimes U(1)$ into $G_f$. For instance if in a given model there are seven $(n_i=7)$ representations $R_i$ we can group them into the following $SU(3) \otimes SU(2)$ representations

$$\begin{aligned} (7) &= (3,1) + (3,1) + (1,1), \\ (7) &= (3,2) + (1,1), \\ (7) &= 3(1,2) + (1,1), \text{ etc.} \end{aligned} \tag{2.5}$$

This, at first sight very arbitrary, assignment of $SU(3) \otimes SU(2) \otimes U(1)$ quantum numbers to preons is severly restricted by the fact that as usual we do not want to have $SU(3) \otimes SU(2) \otimes U(1)$ anomalies. In the absence of spectators these anomalies have to be cancelled among the preons themselves. There are five types of triangle anomalies to be considered. These are

$$[SU(3)]^3,\ [SU(N)]^2 \otimes U(1),\ [SU(3)]^2 \otimes U(1),$$
$$[SU(2)]^2 \otimes U(1),\ [U(1)]^3\ . \tag{2.6}$$

Consequently only embeddings are allowed for which all these anomalies vanish simultaneously. The relevant set of anomaly equations is as follows

$$[SU(3)]^3: \qquad \sum_i A(i) = 0 \tag{2.7}$$

$$\begin{matrix}[SU(m)]^2 \otimes U(1): \\ m = 2,3,N\end{matrix} \qquad \sum_i Y_i K_m(r_i) = 0 \tag{2.8}$$

$$[U(1)]^3: \qquad \sum_i Y_i^3 = 0 \tag{2.9}$$

where the sums are over all preons grouped appropriately in representations of $SU(N) \otimes SU(3)_c \otimes SU(2) \otimes U(1)_Y$, and $Y$ stands for the weak hypercharge which in our normalization is related to the electromagnetic charge by

$$Q = T_3 + \frac{Y}{2}\ . \tag{2.10}$$

Furthermore A(i) is the SU(3) anomaly of the i-th representation of SU(3) (e.g. $A(3)=-A(\bar{3})=1$) and $K_n(r_i)$ denotes the second index of a representation $r_i$ of the SU(n) group. The $K_n$ $(R_i)$'s are given in Ref. 7. As we shall see in Section 3 in many models there are no embeddings free of the anomalies (2.6). These models are then immediately ruled out.

## 2.3 Restricting the Quantum Numbers of Preons

Clearly in a given model the $SU(3) \otimes SU(2) \otimes U(1)$ quantum numbers of preons are restricted by the anomaly equations (2.7)-(2.9). We shall further restrict these quantum numbers by demanding that all charges of unconfined particles (i.e. particles which are metacolor and color singlets) are integer. To formulate this more precisely, consider a preon with an $SU(N) \otimes SU(3) \otimes SU(2)$ representation with 'N-alites' $A_N$, $A_3$ and $A_2$. The most general charge assignment which guarantees integer observable charges is given by

$$\frac{Y}{2} = \ell_N \frac{A_N}{N} + \ell_3 \frac{A_3}{3} + \frac{A_2}{2} + Q_0 \tag{2.11}$$

where $Q_0$ is an integer. The factors $\ell_N$ and $\ell_3$ are integers defined modulo N and 3 respectively, and have to be the same for all preons. $\ell_3 \neq 0$ since quarks have fractional charges. In the standard model $\ell_3$ is by convention chosen to be -1.

We will restrict the range of $Q_0$ in such a way that $|Q_{em}|\leq 1$, where $Q_{em}$ is the electric charge of the preon. Furthermore, we will only give explicit results for preons in the singlet and fundamental representation of SU(3) and SU(2). If $\ell_N=0$ (which turns out to be the case in all models except one) the preons can then only appear in the following SU(3) $\otimes$ SU(2) $\otimes$ U(1) representations:

$$
\begin{aligned}
Q &= (3,2,\tfrac{1}{3}) & L &= (1,2,-1) \\
D &= (\bar{3},1,\tfrac{2}{3}) & U &= (\bar{3},1,-\tfrac{4}{3}) \\
E &= (1,1,2) & V &= (1,1,0)
\end{aligned}
\qquad (2.12)
$$

and in the corresponding complex conjugate representations. For instance $D^*=(3,1,-2/3)$.

In other words, in looking for solutions to Eqs. (2.7)-(2.9), we shall only allow those which are contained in the set (2.12).

We shall later comment on what happens when this restriction is relaxed.

## 2.4 Two Classes of Models

In what follows it will be useful to divide the models considered here into two classes

$$\text{Class I}: \quad |n_i| < 15 \quad \text{for all } i$$

and

$$\text{Class II}: \quad |n_i| \geq 15 \quad \text{for at least one } i,$$

where $n_i$ is the multiplicity which enters Eqs. (2.1), (2.3) and (2.4).

It turns out that the anomaly Eqs. (2.7)-(2.9) can much easier be satisfied for the models of Class II. The reason is that in these models the preons which belong to the fundamental representation of an $SU(n_i)$ group (see Eq. (2.4)) with $n_i \geq 15$ can be put in the anomaly free representation $\bar{5} + 10$ of SU(5). Consequently if the remaining preons are singlets under $SU(3) \otimes SU(2) \otimes U(1)$ (e.g. V in (2.12)) the anomaly Eqs. (2.7)-(2.9) are automatically satisfied. Furthermore, as we shall discuss in Section 4, in these type of models the preon bound states can also be put into $\bar{5} + 10$ representations of SU(5) and consequently fermion generations with the right structure appear automatically. For our purpose the SU(5) group in question does not have to be gauged completely; we are only interested in its $SU(3) \otimes SU(2) \otimes U(1)$ subgroup.

### 2.5 Strategy

Having discussed various aspects of the models in question, we can now set up a program for our investigations. This program is as follows.

1. We shall first solve the anomaly Eqs. (2.7)-(2.9) for the models of Class I taking into account the restrictions on the quantum numbers of preons as given in Eq. (2.12)

2. Next we shall eliminate the models which do not contain any SU(2) doublets and/or any SU(3) triplets or antitriplets. In such models we obviously cannot construct all the known quarks and leptons.

3. We eliminate all models in which $SU(2)_W$ appears only in the following combination

$$(R_i,1,2,1) + (R_i,1,2,-1) \qquad (2.13)$$

Such models are undesirable for the following reason. In such models, the composite electron and neutrino $SU(2)_L$ doublet <u>must</u> contain at least one of the preon representations of Eq. (2.13). But then using the other preon representation of Eq. (2.13), it is always possible to construct a composite electron and neutrino with the <u>wrong</u> electromagnetic charge. Since the "right" and the "wrong" doublet are constructed in exactly the same way it is not reasonable to assume that the latter is very heavy.

4. Next we shall introduce a useful classification of those models of Class I which pass the above tests. This classification will facilitate our subsequent discussion. This discussion, which will also include the models of Class II, will primarly

concentrate on the question of quark and lepton generations.

## 3. First Results

Using the Tables of Ref. 7 we have analyzed all the models of Class I, i.e. the sets $(N,\{n_i\})$ with $|n_i|<15$. All the models of this class which pass the first three tests of our program are collected in Tables I and II. A few SU(8) and SU(9) models have been omitted from Table II for reasons to be discussed in Section IV. Before going to the fourth step of our program, let us make the following observations on the basis of these Tables.

i) The smallest allowed metacolor group is SU(5).

ii) Many values of N are excluded. In particular SU(6), SU(11) and SU(12) metacolor groups are not allowed.

iii) The number of the preons in each model is larger than the number of quarks and leptons (counting color, weak isospin, helicities, etc.) presently observed.

The smallest model (with a metacolor group SU(5)) contains 135 preons. The numbers of preons in SU(5), SU(7) and SU(8) models are 0(150), 0(300) and 0(400) respectively. Larger numbers are found for larger metacolor groups.

We conclude therefore that none of these models solves the problems of proliferation of particle species found on the level of quarks and leptons. This conclusion is not changed when the restriction (2.12) is relaxed and the models of Class II are considered. We would like to emphasize that we have reached this conclusion without making any assumptions about the realization of metaflavor symmetries and the nature of the bound state.

In principle then if economy were the only motivation for the construction of composite models our study of spectatorless models would be already completed. We have not found any economical model of this type among the SU(N) metacolor groups with preons in complex representations.

It is however possible that nature is more complicated than we first thought. We shall therefore continue our study paying attention to the ability of the simplest of our models (those with metacolor gauge groups SU(5) and SU(7)) to reproduce the generation pattern observed in the data.

First however, let us discuss various embeddings which in the Tables I and II have been denoted by capital letters A,B,...T. We encounter two distinct classes of embeddings:

a) Those in which the trace of the $U(1)_Y$ generator over each of the metacolor representations separately is zero, $TrY_r=0$, and

b) Embeddings in which the trace of the $U(1)_Y$ generator is non-zero for at least one of the metacolor representations, $TrY_r \neq 0$.[8]

The traceless and non-traceless embeddings are listed in Tables III and IV respectively. The traceless combinations $T_i$ which appear there are related to the representations of Eq. (2.12) as follows

$$T_1 = D + L; \quad T_2 = Q + U + E; \quad T_3 = U + L^* + E$$

$$T_4 = Q + L; \quad T_5 = U + 2E; \quad T_6 = D + E^* \tag{3.1}$$

$$T_7 = Q + U + D; \quad T_8 = L + E$$

Note that the Tables III and IV correspond to the Tables I and II respectively.

We note first that the cases A,B, and C in Table III can be conveniently classified according to SU(5) and SU(6) representations. Cases A and B of Table III are described by their SU(5) representations, where the [5] of SU(5) transforms as $D^* + L^*$ under $SU(3) \times SU(2) \times U(1)_Y$. Case A corresponds to a $[\bar{5}]$ and two [5]'s embedded in the fundamental representations of $SU(n_8)$ and $SU(n_9)$, respectively, while Case B has a $[\overline{10}]$ and two [5]'s in these same representations. Case C has a similar embedding with the fundamental representations of $SU(n_8)$ and $SU(_9)$ containing a $[\bar{6}]$ and two [6]'s of SU(6) respectively. In this case the [6] is $U^* + L^* + E^*$ under the standard model.

The remaining embeddings of Table III are more involved. In Case D, the $[SU(3)]^3$ anomalies are cancelled trivially by embedding two [3]'s and two $[\bar{3}]$'s of SU(3) in $SU(n_8)$, while $SU(n_9)$ contains only color singlets. The next three cases (E,F, and G), all have the same SU(3) embedding: $SU(n_8)$ contains a $(\bar{3},2)+(3,1)$ of $SU(3) \times SU(2)$, while $SU(n_9)$ has two (3,1)'s. In all the cases discussed so far the color charge and the SU(2) doublets are contained in only two different hyperflavor groups, $SU(n_8)$ and $SU(n_9)$. In the remaining cases in Table III (H,I,J) color triplets (or anti-triplets) and SU(2) doublets can also be found in the fundamental representations of $SU(n_3)$ and $SU(n_7)$ groups. Note that in both models H and I, the embeddings in $SU(n_7)$ and $SU(n_8)$ are complex conjugates of each other, while in J, the embedding in $SU(n_8)$ is twice the complex conjugate of the embedding in $SU(n_3)$. Finally there is precisely one model with $\ell_N \neq 0$ ($\ell_N$ is defined in Eq. (2.11)). This model has an SU(8) metacolor group and consists of the following $SU(8) \times SU(3) \times SU(2) \times U(1)$ representations:

$$(R_3^*,1,2,-1/2); \qquad (R_8,1,2,0); \qquad (R_9,3,2,7/6);$$

$$2(R_3^*,1,1,1/2); \qquad 2(R_9,3^*,1,-7/6).$$

We have considered the possibility of extending the range of preon quantum numbers from those given in Eq. (2.12). When we allow the preons to be in arbitrary

SU(3) representations, we find an additional twelve models which satisfy the criteria of Section 2.5. The smallest number of preons in any of these models is 195, (corresponding to SU(5) metacolor and $R=-13R_8 + 13R_9$). We list only the metacolor and metaflavor groups of these models in Table V. (There is more than one anomaly free embedding of the standard model in some of the flavor groups.)

Relaxing all restrictions on the SU(2) × U(1) quantum numbers would not reduce the number of preons significantly: by looking only at arbitrary embeddings of SU(3) we find that the smallest model has at least 90 preons.

## 4. Search for a Realistic Model

In this section we shall determine those models of Tables I and II in which it is possible to construct the observed quarks and leptons. We shall also briefly discuss the models of Class II.

### 4.1. Models of Class I

4.1.1. Three Preon Bound States. Until this point, we have been able to avoid making detailed assumptions about the nature of the composite states. However, we are unable to proceed further without some general assumptions about the dynamics of the bound states. In the models considered the preons do not have any $SU(3)\otimes SU(2)\otimes U(1)_Y$ anomalies and it is clear that these anomalies must also vanish for the bound states. This is equivalent to satisfying 't Hooft's anomaly matching conditions with respect to the $SU(3)\otimes SU(2)\otimes U(1)_Y$ subgroup of $G_f$ but otherwise it does not require any assumptions about the breakdown, (or conservation) of the metaflavor symmetry (2.3).

We first look for three preon bound states making only the assumption that they are left-handed metacolor singlets. Our findings are as follows:

1. Anomaly free three preon bound states exist for only ten of the hyperflavor groups of Tables I and II. These models are distinguished by an (*) in the Tables.
2. Five of these models have an SU(5) metacolor group and preons with the quantum numbers of embedding A. All of these models have bound states in the $[\bar{5}]$ and [10] representations of a flavor group SU(5) and so they all contain at least one standard generation of fermions. Since we satsify 't Hooft's conditions in a trivial way [the bound states are anomaly free with respect to $SU(3)\otimes SU(2)\otimes U(1)$] we cannot determine the allowed number of generations. Enlarging [beyond $SU(3)\otimes SU(2)\otimes U(1)$] the flavor group with respect to which we should satisfy 't Hooft's conditions would give us

restrictions on the number of generations but this is beyond the scope of this paper. Finally, although all these models can in principle accommodate the known generations, except for the $R = -R_5 - 5R_8 + 11\,R_9$ model they all contain composite particles with exotic $SU(3)_L \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers.

3. The models with N>5 either:

   a) do not contain a standard generation of quarks and leptons, or

   b) have more than 240 preons.

   Consequently they are not interesting.

In summary the only model with three preon bound states which has a chance of being realistic is the model with an SU(5) metacolor group and the preons in the $R = -R_5 - 5R_8 + 11R_9$ representation.

4.1.2. Multipreon bound states. We now turn our attention to models where the number of preons in the bound states can be arbitrarily large. Let us first enumerate conditions which are necessary (but by no means sufficient) for a model to have a chance to be realistic.

a) It must be possible to construct the observed quarks and leptons as metacolor singlet states. In practice we will only require that the composites have vanishing "N-ality" for metacolor.

b) The bound state spectrum should be classified according to representations of the full metaflavor group. This requirement does not imply that we do not allow spontaneous symmetry breaking, but only that all components of a broken multiplet should be present in the low-energy spectrum if one of the components is identified with a quark or lepton. Only when some composites can pair off according to their $SU(3)\otimes SU(2)\otimes U(1)$ representations we might assume that they become heavy, although there may still be unbroken chiral symmetries forbidding that.

c) When composites with unusual electroweak quantum numbers, which are singlets or triplets of $SU(3)_c$, are present, the model is unacceptable unless these states are in a real $SU(3)\otimes SU(2)\otimes U(1)$ representation.

d) The $SU(3)_c\otimes U(1)_{EM}$ representation of the composites must be real.

With these conditions in mind we have analyzed the models of Tables I and II. Our findings are as follows:

1. In all models with traceless embeddings (see Table I) except those of class C, the embedding of $SU(3)\otimes SU(2)\otimes U(1)$ in $G_f$ is simply one generation, split into two parts which are

embedded in different factors of the flavor groups. Therefore it is always possible to reconstruct a generation of composite fermions in these models. It should be remarked that it is sufficient to construct first only one generation since it is always possible to add a cluster of preons ("generation counter") to the bound state in order to make a state with the same quantum numbers but a different preon content. It turns out however that in most of the models in Tables I and II a generation counter can only be constructed out of 10 or more preons. Consequently the reproduction of standard generations will generally require bound states with a large number of preons. As we already remarked in section 4.1.1. the number of generations can only be restricted once further assumptions about the realization of metaflavor symmetries are made.

2. The models of class C can be ruled out on the following grounds. In models of this type, SU(3)xSU(2)xU(1) can be "unified" in an SU(6) group (see Section III), and therefore all composites can be classified according to this symmetry. In order to satisfy condition d), the $SU(3)_C \otimes U(1)_{EM}$ subgroup must be in a real representation. In particular this implies

that one needs an anomaly-free representation of SU(6). We have considered all anomaly-free reducible SU(6) representations with Young diagrams with five boxes or less which break down to a real representation of $SU(3)xU(1)_{em}$ and found only two such cases, (with dimension 286 and 436), neither of which contained a standard generation. It is clear that these models have no chance of being realistic.

3. If SU(3) is embedded according to $T_7$, the color triplet preons appear in the fundamental representation of SU(12). The simplest way to obtain quarks as composites is to construct a bound state in which the preon containing $T_7$ appears only once and all other preons appear only as flavor singlets. Consider models of class H with bound states constructed out of $\ell_i$ preons in representations $R_i$ (i=7,8,9). To get a metacolor singlet, it is necessary to have $2\ell_7 + 2\ell_8 + \ell_9 = 0 (\mathrm{mod}\ N)$. If $\ell_9 = 1$ (mod $n_9$), as argued above, this equation can not be solved if both N and $n_9$ are even. Thus we find that all models in Table I with even N in class H are ruled out. In the same way, six "non-traceless" models with N=8 can also be ruled out and we have omitted them from Table II.

4. All of the "non-traceless" SU(9) models have a global SU(4) flavor symmetry. This symmetry can not be gauged since that would introduce a triangle anomaly with a $U(1)_Y$ gauge boson and two SU(4) gauge bosons. Moreover it has to be realized in the spectrum because of the anomaly matching conditions. Therefore this symmetry will also lead to unwanted symmetries in the fermion mass matrix or a plethora of Goldstone bosons. We omit these models from Table II.

In this section we have shown that in most of the models of Tables I and II, it is possible to construct the known quarks and leptons if the bound states are allowed to be arbitrarily complicated. The only class of models we unambiguously rule out is that of Class C.

### 4.2. Models of Class II

Up to now we have limited the dimension of the largest flavor group to 14. When a larger group is present one can simply embed into it a complete generation. It is also usually very easy to construct several generations of quarks and leptons in such a model.

As an example we will discuss the simplest model of this kind. It is based on the metacolor group SU(4) with the following preon representation

$$R_6^* + R_7^* + 15R_9$$

and the corresponding metaflavor group $G_f$ = $SU(15)\otimes U(1)\otimes U(1)\otimes Z$.

We denote the preon fields as $\phi$, $\chi$ and $\psi$ respectively. Notice that there are only 90 preons in this model, less than in any model of Tables I and II. In this model it is simple to construct several generations because of the existence of a simple "generation counter", $\chi^2$. By adding this two preon cluster to a given bound state one can construct another bound state with the same SU(15) representation but different U(1) and Z charges[9]. We recall that in most of the models of Class I such a generation counter can only be constructed out of 10 or more preons.

The bound states in our example are $\phi\chi^3\psi$, $\phi\chi^5\psi$, $\phi\chi^7\psi$ and $\phi\chi^9\psi$. These composites, which are metacolor singlets, form four standard generations of quarks and leptons and satisfy 't Hooft's anomaly matching conditions with respect to SU(15). They also satisfy the Pauli principle with a ground state spatial wave function[10]. The anomaly equations for the two U(1) factors are not satisfied for any linear combination of these U(1)'s and therefore these U(1)'s have to be broken spontaneously by the metacolor forces. The different generations are then distinguished only by different charges of the discrete symmetry $Z^9$.

Up to this point, the model meets all the constraints we have used in this paper. To make the model realistic however we have to break the remaining chiral symmetries and $SU(2)\otimes U(1)$ to give masses to fermions and the W and Z bosons (One can either gauge the $SU(3)\otimes SU(2)\otimes U(1)$ subgroup of SU(15) or an SU(5) subgroup. In the latter case additional symmetry breaking is required at the unification scale). The metacolor forces alone cannot do this since one would expect all fermion masses to be $O(\Lambda_{MC})$ in that case. One may hope that the color or electroweak interactions could be responsible for some spontaneous or explicit symmetry breaking[11] but unfortunately we have not been able to realize such a mechanism in our example. Therefore we have to conclude that additional interactions are required (e.g. technicolor or fundamental scalar bosons) in order to generate realistic fermion and W and Z boson masses.

A second problem for models based on SU(15) is lepton number violation. Baryon number can be assigned trivially to the $\psi$ preon, and it is an SU(15) generator, which is not broken by metacolor. Lepton number however is not traceless, and therefore it has to be embedded partially in the U(1)- factors of the flavor group, which are either broken by metacolor instantons or have to be broken spontanously because of anomaly matching. In either case lepton number is broken, but it is possible to leave a discrete symmetry unbroken to prevent most of the lepton number violating processes. This requires however a rather artificial arrangement of condensates. A much less artificial solution is to consider SU(16) instead of SU(15) as the metaflavor group in which a generation is embedded. Then lepton number is an SU(16) generator and is not necessarily broken by metacolor.

A complete list of models of Class II can easily be extracted from ref. 7 and will not be reproduced here.

## 5. Summary

In this paper we have discussed those composite models with an SU(N) metacolor group and preons in the complex, anomaly free and asymptotically free representations, which do not have spectators with $SU(3)\otimes SU(2)\otimes U(1)$ quantum numbers. As we have shown, many of these models can be ruled out without making detailed assumptions about the preon dynamics, such as realization of metaflavor symmetries or the structure of the bound states. In particular we have found that all the models of this type contain a large number of preons and consequently they do not solve the problem of proliferation of particle species found on the level of quarks and leptons. Many of these models could also be ruled out on the basis that they could not accommodate even a single generation. We have found however several models which could in principle accommodate all the known generations of quarks and leptons, although generally this would require bound states with a large number of preons. We have discussed these models very briefly in

Section IV. Further study of these models would require detailed assumptions about the realization of metaflavor symmetries and the structure of the bound states, to which assumptions we do not want to commit ourselves in this paper. If such a study would lead to a negative result we might conclude that in order to find a realistic composite model one has to look elsewhere, e.g. look for models with spectators or preons with partly or fully real representations.

References

1. For a recent review of composite models see M.E.Peskin, proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies.

2. For a recent review see J. Ellis, LAPP-TH-48, TH-3174-CERN; lectures presented at the les Houches Summer School, August 1981.

3. Y. Tosa and R. E. Marshak, VPI preprint, VPI-HEP-81/9.

4. See for instance: T. Banks, S. Yankielowicz, and A. Schwimmer, Phys. Lett. 96B, (1980), 67; I. Bars and S. Yankielowicz, Phys. Lett. 101B, (1981) 159.

5. I. Bars, Phys. Lett. 106B, (1981) 105; Yale preprints, YTP 82-04, YTP82-10 (1982).

6. G. 't Hooft, in Recent Developments in Gauge Theories, ed. G. 't Hooft et al. (Plenum Press, New York, 1980.)

7. E. Eichten, K. Kang and I. G. Koh, Fermilab-Pub-81/83-THY, BROWN-HET-464.

8. In all models the trace of Y, summed over all preons, turns out to vanish.

9. H. Harari and N. Seiberg, Phys. Lett. 102B, (1981) 263. J. Preskill, Harvard preprint, HUTP-81/A051.

10. A. N. Schellekens, I. G. Koh and K. Kang, FERMILAB-Pub-82/23-THY to be published in Phys. Rev. D.; A.N. Schellekens, I.G. Koh, and K. Kang, FERMILAB-Pub-82/24-THY, to be published in J. Math. Phys.

11. S. Weinberg, Phys. Lett. 102B, (1981) 401.

TABLE I. Models with Traceless Embeddings

| SU(N) Metacolor | Hyperflavor Group $R_3$ | $R_5$ | $R_6$ | $R_7$ | $R_8$ | $R_9$ | Embeddings (P) |
|---|---|---|---|---|---|---|---|
| 5 | 0 | 0 | -1 | 0 | P | (16-P) | A (P=5,6) (P=6)$^{(*)}$ |
| 5 | 0 | -1 | 0 | 0 | -P | (6+P) | A (P=5,6) (P=5)$^{(*)}$<br>C (P=6) |
| 5 | 0 | 0 | 0 | 2 | -P | P-18 | A ($5 \le P \le 8$) (P=5,7,8)$^{(*)}$<br>C (P=6) |
| 5 | 0 | 0 | 0 | -1 | -5 | 14 | A |
| 5 | 0 | 0 | 0 | 0 | P | -P | A ($10 \le P \le 13$)<br>C (P=12,13)<br>B ($10 \le P \le 13$)<br>D (P=12,13)<br>E (P=12,13)<br>F (P=13)<br>G (P=13) |
| 7,13<br>8,16<br>9,10,22,23<br>14,15,16,40,41,42 | 0 | 0 | 0 | 3 | P | -3(N+4)-P(N-4) | H (P=-7)<br>H (P=-6)<br>H (P=-5)<br>H (P=-4) (N=14)$^{(*)}$ |
| 13,14,26,27<br>22,23,24,48,49,50<br>13,27<br>22,23,49,50 | 0 | 0 | 0 | 4 | P | -4(N+4)-P(N-4) | H (P=-6) (N=14)$^{(*)}$<br>H (P=-5)<br>I (P=-6)<br>I (P=-6) |
| 7 | -3 | 0 | 0 | 0 | 6 | -12 | J * |

$^{(*)}$Models with anomaly free three preon bound states.

TABLE II. Models with Non-Traceless Embeddings

| SU(n) Metacolor | Hyperflavor Group | | | | Embeddings (P) |
|---|---|---|---|---|---|
| | $R_3$ | $R_7$ | $R_8$ | $R_9$ | |
| 5 | 0 | -2 | P | (18-P) | K (P=6,7,8)<br>L (P=6,7)<br>M (P=8)<br>N (P=7)<br>O ($7\leq P\leq 10$) |
| 7 | -2 | -2 | 4 | 14 | P,Q,R,S,T |
| 7 | 1 | -2 | 2 | 14 | P(*) |
| 7 | -3 | -1 | 1 | 14 | P(*) |
| 7 | 3 | -2 | 1 | 13 | Q |
| 7 | 3 | -1 | -3 | 14 | P,R(*) |
| 7 | 4 | -2 | 0 | 14 | P,Q |

(*)Models with anomaly free three preon bound states.

TABLE III. "Traceless" Embeddings of
$SU(3) \times SU(2) \times U(1)$

| Class | $SU(3) \times SU(2) \times U(1)$ Embeddings in | | | |
|---|---|---|---|---|
| | $SU(n_3)$ | $SU(n_7)$ | $SU(n_8)$ | $SU(n_9)$ |
| A | – | – | $T_1$ | $2T_1^*$ |
| B | – | – | $T_2$ | $2T_1$ |
| C | – | – | $T_3$ | $2T_3^*$ |
| D | – | – | $T_7$ | $2T_8$ |
| E | – | – | $T_4+T_6$ | $2T_5$ |
| F | – | – | $T_4+T_5$ | $2T_6$ |
| G | – | – | $T_1+T_4$ | $2T_3$ |
| H | – | $T_8$ | $T_8^*$ | $T_7$ |
| I | – | $T_6$ | $T_6^*$ | $T_4+T_5$ |
| J | $T_8$ | – | $2T_8^*$ | $T_7$ |

TABLE IV. Non-Traceless Embeddings of $SU(3) \times SU(2) \times U(1)$

| Class | $SU(3) \times SU(2) \times U(1)$ Embeddings in | | | |
|---|---|---|---|---|
| | $SU(n_3)$ | $SU(n_7)$ | $SU(n_8)$ | $SU(n_9)$ |
| K | - | $L^*$ | $U+E^*+L$ | $2(U^*+E)+L$ |
| L | - | E | $U+L+E^*$ | $2(U^*+L^*)+E^*$ |
| M | - | L | $D+E+2L^*$ | $2(D^*+E^*)+L$ |
| N | - | $E^*$ | $D+L^*+2E$ | $2(D^*+L)+E^*$ |
| O | - | $L^*$ | $U+2L$ | $2U^*+L^*$ |
| P | E | $E^*$ | - | $T_7+L$ |
| Q | L | $L^*$ | - | $T_7+E$ |
| R | L | E | $L^*+E^*$ | $T_7+L^*$ |
| S' | E | L | $L^*+E^*$ | $T_7+E^*$ |
| T | E | $E^*$ | $T_1$ | $3T_1^*+E^*$ (†) |

(†) E can be replaced by $E^*$

TABLE V. Models in Which the Preons are in Exotic SU(3) Representations

| N | R | Number of Preons |
|---|---|---|
| 5 | $-13R_8+13R_9$ | 195 |
| 7 | $-5R_3-R_8+13R_9$ | 287 |
| 7 | $-3R_7+7R_9+12R_9$ | 315 |
| 7 | $-3R_7+8R_8+9R_9$ | 315 |
| 7 | $R_7-8R_8+13R_9$ | 287 |
| 7 | $R_3+R_7-8R_8+11R_9$ | 308 |
| 9 | $2R_7-8R_9+14R_9$ | 504 |