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Dynamical Generation of Fermion Masses

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ABSTRACT

A model is presented in which Lagrangian masses of quarks and leptons arise dynamically through radiative corrections. An estimate of the t-quark mass is made. Possible mechanisms for generation mixings are discussed.

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It is now recognized by many people that the standard $SU(2)_L \otimes U(1)$ model,[1] while being very successful phenomenologically,[2] has a few undesirable features due to the existence of a large number of adjustable parameters. Inevitably, the model is unable to predict fundamental quantities like quark and lepton masses. As long as we have fundamental scalars in the theory, at least in the standard $SU(2)_L \otimes U(1)$ model, there seems to be little hope of ever understanding how fermion masses arise. We would expect that if spontaneous symmetry breakdown were realized dynamically,[3] we would be able to reduce significantly the number of free parameters and thus increase our predictive power.

In this note, a model is proposed in which Lagrangian masses of quarks and leptons arise dynamically through radiative corrections. This feature necessitates the introduction of a weak doublet of color sextet as well as the concept of quark-lepton unification at an intermediate mass scale of the TeV's order. Baryon number is absolutely conserved at this stage and there would be no problems with proton decay. An interesting relationship between quark and lepton masses arises leading to an estimate of the t-quark mass. The model can only contain up to three families of "light" quarks. A suggestion is made on how mixings can occur between different families.

In this note, the following basic assumptions are made:

(a) the existence of a weak $SU(2)_L$ doublet of $SU(3)_C$ $\underline{6}$-plet (color sextet), $(U(6), D(6))_L$,

(b) $SU(3)_C$ confines and is asymptotically free,

(c) the condensation $\langle 0|\bar{6}_L 6_R|0\rangle \neq 0$, breaks the sextet $SU(2)_L \otimes SU(2)_R$ chiral symmetry down to diagonal $SU(2)$ with a strength characterized by $F_\pi \simeq 250$ GeV.

Assumptions (a) and (b) are quite reasonable. Assumption (c) ensures that $M_W = (½)gF_\pi$ and $M_W/M_Z = \cos\theta_W$. This feature has been extensively discussed in the literature.[4] As yet there is no proof for assumption (c) but it is consistent with Marciano's conjecture.[5] The statement of the conjecture is the following: there is a universal critical value of the product $\alpha_3(\mu_R)C_2(R)$ = constant, $C_2(R)$ being the quadratic Casimir invariant of the R-representation, for which condensation occurs, independent of any specific representation R of $SU(3)_C$. This product is related to the strength of the quark-antiquark binding potential. Taking the conjecture at face values means that the scales of 3-plet and 6-plet condensations are related by

$$\mu_6/\mu_3 = \exp\left[\int_{\alpha_3(\mu_3)}^{C_2(3)\alpha_3(\mu_3)/C_2(6)} \frac{d\alpha_s}{\beta(\alpha_s)}\right] , \tag{1}$$

by the use of the renormalization group equation, $(\mu\partial/\partial\mu)\alpha_3(\mu) = \beta(\alpha_3)$. With a reasonable range of $\alpha_3(\mu_3)$ between 0.15 and 0.2 and with six flavors of 3-plet the ratio μ_6/μ_3 can be made consistent with $F_\pi/f_\pi \simeq 2700$ where f_π (= 94 MeV) is the strength characteristic of light quark (3-plet) chiral symmetry breaking. Here, $C_2(3) = 4/3$ and $C_2(6) = 10/3$.

Spontaneous chiral symmetry breaking of 6-plet and 3-plet is expected to produce dynamical masses of the order $O(\mu_6)$ and $O(\mu_3)$ for color sextet and triplet respectively. Perturbatively their self-energies are expected to behave like[6] $\Sigma_R(p) \simeq 3C_2(R)\mu_R^3/p^2$ for $p^2 \gg \mu_R^2$, giving $\Sigma_6(p) \simeq 10\mu_6^3/p^2$ and $\Sigma_3(p) \simeq 4\mu_3^3/p^2$.

It is assumed that $\mu_6 \simeq 1$ TeV in order for $F_\pi/f_\pi \simeq 2700$. The justification of this assumption is beyond the scope of this paper. As it turns out, in order for the $\underline{3}$-plet quarks acquire current algebra masses, one needs to unify $\underline{6}$-plet and $\underline{3}$-plet fermions into a single irreducible representation of an enlarged gauge group $G_s \supset SU(3)_C$. This unification needs to be achieved at an intermediate mass scale of the TeV's order, the reasons of which are given in the subsequent sections. As we shall see shortly, the $\underline{6}$-plet and $\underline{3}$-plet unification inevitably also lead to quark-lepton unification.

Before discussing the mass problem, let us see how many $\underline{6}$-plet and $\underline{3}$-plet flavors we can accommodate without destroying the asymptotic freedom (A.F.) of $SU(3)_C$. The coefficient in front of the g^3 term of the β-function is given by

$b_3 = (-33 + 4\sum_i T(R_i))/48\pi^2$, where the sum is over all fermion representations of $SU(3)_C$ which can contribute to b_3. Above all mass thresholds of fermions, $b_3 = (-33 + 2n_3 - 10n_6 + 12n_8 + ...)/48\pi^2$, where n_i is the number of flavors of the ith representation. As can be seen from b_3, we can accommodate at most two flavors (one family) of $\underline{6}$-plet and six flavors (three families) of $\underline{3}$-plet and nothing else. This restriction comes from our requirement that $SU(3)_C$ continues to be asymptotically free until it merges into a larger gauge group G_s. Any other color non-singlet fermions which can contribute to b_3 have to have masses of at least the order of the mass scale where $SU(3)_C$ merges into G_s.

The simplest extension of $SU(3)_C$ is SU(4). The representation with the lowest dimension containing both a $\underline{6}$-plet and a $\underline{3}$-plet is a symmetric second-rank tensor of SU(4), the ten dimensional representation denoted by $\{10\}$. Under $SU(3)_C$, $\{10\}$ decomposes as $\{10\} = \underline{6} + \underline{3} + \underline{1}$. Therefore in this scheme we, inevitably, see the appearance of a color-singlet object which is identified with a lepton. Consequently, quarks and leptons are unified at a mass scale where $SU(3)_C$ merges into SU(4).[8] As can be seen from the decomposition of $\{10\}$, at low energies, the Lagrangian mass of quarks and leptons are determined by the one-loop and two-loop graphs respectively as shown in Figs. (1a) and (1b). The 15 gauge bosons of SU(4) decompose under $SU(3)_C$ as $\{15\} = \underline{8} + \underline{3} + \underline{\bar{3}} + \underline{1}$. It is then recognized that the $\underline{8}$-plet are the usual $SU(3)_C$ gluons and the $\underline{3}$ and $\underline{\bar{3}}$ gauge bosons are the ones which connect the $\underline{1}$ to $\underline{3}$ and $\underline{3}$ to $\underline{6}$ fermions. These leptoquark gauge bosons are the ones responsible for the radiative masses of quarks and leptons and acquire large masses in the breakdown of SU(4) to $SU(3)_C$.

One should notice that $\{10\}$ comes as a doublet under $SU(2)_L$, i.e. it transforms under $SU(4) \otimes SU(2)_L$ as: $\{10\} = (10, 2)$. If ones identifies the $\{10\}$ as the first family, can the other families be other $\{10\}$'s also? The answer is no if one really wants to keep SU(4) asymptotically free as can be seen from $b_4 = (-44 + 2n_4 + 12n_{10} + \ldots)/48\pi^2$. One is allowed to have at most one $\{10\}$ family. The other families would have to transform as $\{4\} = (4, 2)$ under $SU(4) \otimes SU(2)_L$ or as $(3, 2) + (1, 2)$ under the $SU(3)_C \otimes SU(2)_L$ subgroup. This restriction turns out to be beneficial. If we were able to duplicate the $\{10\}$ family, there would be no connection between different families and all fermions would get the same mass. As we shall see shortly, the basic $\{10\}$ family plays a privileged role in the mass generation mechanism. How is it so? If SU(4) were the only story then the quarks in $\{4\}$'s would obtain no Lagrangian masses since they are not connected to the 6-plet condensation. The leptons in $\{4\}$'s would get a tiny Lagrangian mass because they are connected to the 3-plet condensates. This tiny mass is due to the very small ratio μ_3^3/μ_6^3. One could safely say that, at this stage, only the 3-plet and 1-plet in $\{10\}$ acquire Lagrangian mass. Therefore a new mechanism is needed in order for the $\{4\}$'s to obtain masses, probably by being connected to the basic $\{10\}$.

Let us specify the particle content and the charge structure of $\{10\}$ and $\{4\}$'s. The $U(1)_{e.m.}$ charge operator is defined as $Q = T_{3L} + T_0$, where T_{3L} is the $SU(2)_L$ neutral generator and T_o is the U(1) hypercharge generator which in our case is a linear combination of the 15th generator of SU(4) and other, as yet, unspecified generators. This possibility is considered in Ref. 8. One can then write T_0 as $T_0 = T_S + T_H$ where $T_S = C_S T_{15}$, $T_{15} \in SU(4)$, and T_H is a linear combinatic

, (3)

of, as yet, unspecified generators. The T_S-eigenvalues of the $\{4\}$'s are denoted by: t_s for $\underline{3}$ and t'_s for $\underline{1}$, where due to the nature of T_{15}, $t'_S = -3t_S$. The T_S-eigenvalues of $\{10\}$ are then: ($2t_S$ for $\underline{6}$; $t + t'_S$ for $\underline{3}$; $2t'_S$ for $\underline{1}$). The T_H-eigenvalues for $\{10\}$ and $\{4\}$'s are denoted by t_H and t'_H respectively. It turns out that, in order to accommodate conventional quark ($\underline{3}$-plet) and lepton ($\underline{1}$-plet) charges in the $\{10\}$, one is restricted to two possibilities;

I) $t'_S = -1/4$; $t_S = 1/12$; $t_H = 0$, $t'_H = -1/4$. $\{10\} = [(u(6),\ d(6))_L;\ (d^c(3),\ -u^c(3))_L;\ (\ell^o, \ell^-)_L]$, $\{4\} = [(d^c(3),\ -u^c(3))_L;\ (\ell^o,\ \ell^-)_L]$, where $(u(6), d(6)) \in \underline{6}$, $(d^c(3), -u^c(3)) \in \underline{3}$, $(\ell^o, \ell^-) \in \underline{1}$ and $Q(u) = 2/3$, $Q(d) = -1/3$, $Q(\ell^o) = 0$, $Q(\ell^-) = -1$. The superscript "c" denotes "charge conjugate."

(II) $t'_S = -1/2$, $t_S = 1/6$, $t_H = 1/2$, $t'_H = 0$. $\{10\} = [(U(6),\ D(6))_L;\ (u(3),\ d(3))_L;\ (\ell^o,\ \ell^0)_L]$, $\{4\} = [(u(3),\ d(3))_L;\ (\ell^o, \ell^-)_L]$, where now $Q(U(6)) = 4/3$, $Q(D(6)) = 1/3$. In this paper, case (I) is preferred over case (II) for the following reasons:

a) if $t_H \neq 0$ and $t'_H = 0$ (case (II)) then it means that $\{10\}$ would have other partners (other $\{10\}$'s) so that $\{10\}$ would transform non-trivially under a group G_H which is orthogonal to $SU(4) \otimes SU(2)_L$. The possibility of having more than one family of $\{10\}$ has been refuted earlier.

b) if $\{4\}$ were to be connected to $\{10\}$ by enlarging $SU(4) \subset G_S$, then only case (I) is acceptable. From here on, families are classified by: $\{10\}$, $\{4\}^a$, $\{4\}^b$, etc. Notice that in case (I), the leptoquark gauge bosons denoted by $S^{\pm}_{\mu,i}$ where $i = 1,2,3$, are the color indices have charges $Q(S^{\pm}) = \pm 1/3$.

Let us now compute the masses of quarks and leptons in the basic $\{10\}$ family. In the diagrams shown in Figs. (1a) and (1b), the coupling at the vertices is $(g_S/\sqrt{2})\sqrt{3/2}$. The definition of $S^{\pm}_{\mu,i}$ gives a factor $(g_S/\sqrt{2})$ and the relation between the normalizations of SU(4) generators for $\{10\}$ and $\{4\}$ which is

Tr $T_\alpha^2(10) = 6\mathrm{Tr}\ T_\alpha^2(4)$, gives a factor $\sqrt{3/2}$ with Tr $T_\alpha^2(4) = 1/2$. In computing the self-energy diagrams we have to sum over the three colors of $S^{\pm}_{\mu,i}$. We assume that SU(4) is broken down to $SU(3)_C$ such that the leptoquark gauge bosons acquire a mass m_S. The momentum-dependent quark mass denoted by $m_q(p^2)$ is calculated from Fig. (1a), in the Landau gauge, to be

$$-im_q(p^2) = -3(\sqrt{3}g_S/2)^2 \times 3 \times \int \frac{d^4k}{(2\pi)^4}\left[k^2 - \Sigma_6^2(k^2)\right]^{-1} \Sigma_6(k^2)\left[(p-k)^2 - m_S{}^2\right]^{-1} \quad . \quad (2)$$

As an approximation, let us use the asymptotic form of $\Sigma_6(k^2)$. In the limit $p^2 << \mu_6^2 << m_S^2$, the quark mass is approximately given by ($p^2 \simeq 0$)

$$m_q \simeq -(135/2)(\alpha_S/4\pi)(\mu_6^3/m_S^2)\ln\left[m_S{}^2/(10)^{2/3}\mu_6^2\right] \quad , \quad (3)$$

where $\alpha_S \equiv g_S^2/4\pi$. Here $\alpha_S = \alpha_3(m_S)$ with $\alpha_3(m_S)$ being the $SU(3)_C$ coupling determined to be $\alpha_3(m_S) \simeq 0.07$ for $m_S \simeq$ 40-100 TeV by the use of the α_3-evolution equation. Notice that the integral in Eq. (2) is convergent and there is no mass counterterm. The lepton mass is calculated from Fig. (1b) with the approximation $p^2 \simeq 0$ and in the Landau gauge, to be

$$-im_\ell(0) = i3(\sqrt{3}g_S/2)^4 \times 3 \times 3\int \frac{d^4q\,d^4k}{(2\pi)^8}\left[q^2(q^2 - m_S^2)((q-k)^2 - m_S^2)\right]^{-1}$$

$$\times\ 3\Sigma_6(k^2)\left[k^2 - \Sigma_6^2(k^2)\right]^{-1}\left\{1 + q^2\left[k^4 - \Sigma_6^4(k^2)\right]\left[3(k-q)^2(k^2 - \Sigma_6^2(k^2))^2\right]^{-1}\right\} . \quad (4)$$

The result can be expressed in terms of the quark mass as

$$m_\ell \simeq (9/8)(\alpha_S/\pi)m_q \quad , \quad (5)$$

where again $\alpha_S = \alpha_3(m_S)$. Notice that the quark-lepton mass relation (5) is not sensitive to the exact value of μ_6 or the form of $\Sigma_6(k^2)$. With our particle classification within the basic $\{10\}$, the relation (5) is actually between a charged lepton and a charge 2/3 quark or a neutral lepton and a charge -1/3 quark. This paper has nothing to say about mass differences between members of the same weak doublet.

Since one can only accommodate one light ($m < m_S$) $\{10\}$ and two light $\{4\}$ families, it may be appropriate to think that it is the $\{10\}$ which is the heaviest one since in the absence of gauge bosons which connect $\{4\}$ to $\{10\}$, the $\{4\}$'s would be massless. Could it be that the charge 2/3 quark in the $\{10\}$ is the t-quark? If so, what would be its mass? Could it be that the charged lepton in $\{10\}$ is in fact the τ^- lepton? If so, then the relation (5) actually relates the t-quark mass to the τ-lepton mass. Taking $m_\tau = 1.8$ GeV, the t-quark mass would be

$$m_t \simeq 72 \text{ GeV} \quad . \tag{6}$$

The above estimate could very well be wrong but is interesting in its own right.

What is the nature of family mixings? How would the other two $\{4\}$ families get their masses? In this paper, it is proposed to generalize SU(4) to SU(7). According to case (I), $t_H = 0$ and $t'_H = -1/4$. It is natural to have two $\{4\}$'s with $t'_H = -1/4$ and one $\{4\}$ with $t'_H = 1/2$ such that $G_H \equiv SU(3)_H$. It is then easy to see that the $\{4\}$ family with $t'_H = 1/2$ has fermions with weird charges. According to our earlier discussion, this special $\{4\}$ family denoted by $\{4\}^C$ would have to be quite massive ($m > m_S$) in order not to destroy the asymptotic freedom of $SU(3)_C$. Since one needs the two $\{4\}$'s with $t'_H = -1/4$ to be connected to the basic $\{10\}$, one can take a 28-dimensional representation of SU(7) which decomposes under the

$SU(4) \otimes SU(3)_H$ subgroup as: $\{28\} = (10, 1) + (4, 3) + (1, 6)$. The price one has to pay is four extra families of fractionally charged leptons, three extra families of ordinary leptons and one extra family of weird charged quarks. If the extra $\{4\}^c$ gets a dynamical mass and if a mechanism could be found such that $\{4\}^a$ and $\{4\}^b$ (the ordinary ones) only obtain non-diagonal small mass terms by being connected to the $\{4\}^c$ condensate, then one might expect to discuss various things like Cabibbo or Kobayashi-Maskawa mixing angles. Notice that in the breakdown of $SU(7)$ to $SU(4) \otimes SU(3)_H$, $\{4\}^a$ and $\{4\}^b$ would obtain small diagonal masses by being connected to the basic $\{10\}$. Surely, in this paper a number of important questions remain untouched like the nature of neutral lepton (neutrino) masses, mass hierarchies between members of the same weak doublet, the nature of CP-violation. This paper has nothing to say about these fundamental problems and only scratches on the surface of mass hierarchies between different families.

It is hoped that the model considered in this paper provides some stimulation and food for thought on the nature of fermion masses. Whether or not the estimate of the t-quark mass is correct or even reasonable can only be decided by future high energy machines.

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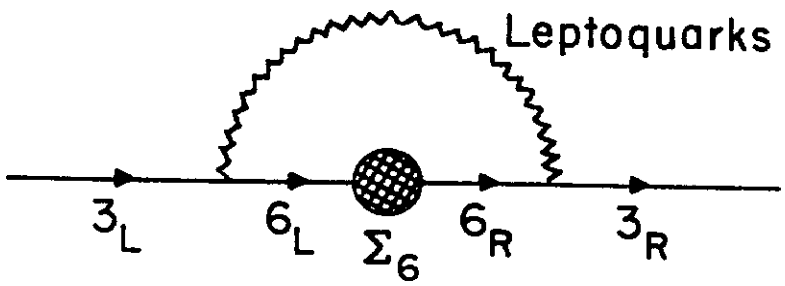
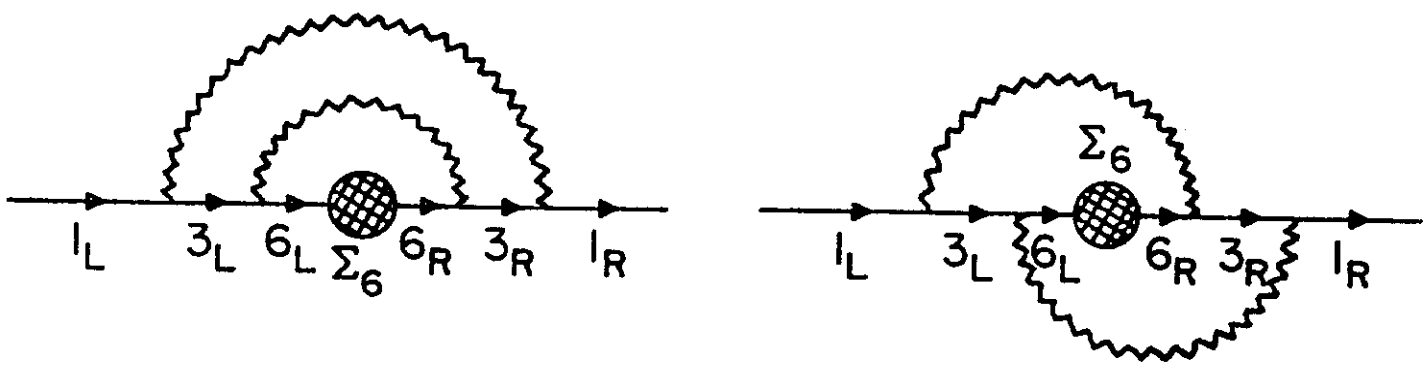
FIGURE CAPTIONS

Fig. (1a): Quark mass with 3 ≡ color triplet, 6 ≡ color sextet; the wavy lines denote leptoquark gauge bosons.

Fig. (1b): Lepton mass.


Leptoquarks
3_L
6_L
Σ_6
6_R
3_R

(a)


l_L
3_L
6_L
Σ_6
6_R
3_R
l_R
Σ_6
l_L
3_L
6_L
6_R
3_R
l_R

(b)