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Large Distance Effects in CP-Violation and the $K^0-\bar{K}^0$ Mass Matrix

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ABSTRACT

Large distance contributions to the $K^0-\bar{K}^0$ mass matrix are often neglected but have important consequences for CP-violation. If "penguins" dominate $|\Delta S|=1$ weak decays then these large distance contributions correspond to "(penguin)²" diagrams and receive potentially large CP-violating phases. We argue that this effect reduces the size of $|\epsilon'/\epsilon|$ by $\approx 1/2.5$ in the KM model. In the extreme penguin limit, $f=1$ and $\epsilon_m=0$, we have $|\epsilon'/\epsilon| \approx 1/50$.

Interest in the CP-violation of the $K^0-\bar{K}^0$ system has been recently aroused by the prospect of a refined measurement of $|\epsilon'/\epsilon|$ ^{1,2} which can distinguish between an effective milliweak vs. superweak underlying mechanism. Furthermore, it has been suggested that in the Kobayashi-Maskawa³ (KM) scheme this quantity receives contributions principally from "penguin" diagrams^{4,5} and may be close to the experimental upper bound of $\approx 1/50$.⁴ Previously it had been argued that in the absence of penguin contributions the KM scheme is essentially a superweak model with $|\epsilon'/\epsilon|$ expected to be of order $\approx 1/450$. We mention that the case for penguins is still open⁷ and that the KM model may indeed be effectively superweak (as in ref. 5) if such effects are small.

Here we refer to any operator induced by W-boson radiative corrections to gluon emissions as a penguin,³ (Fig. 1), which includes the operator $\bar{s} \sigma_{\mu\nu} (\lambda^A/2) d G^{\mu\nu A}$ first occurring in two loops with an $\mathcal{O}(G_F)$ coefficient.⁹ This operator receives the same CP violation phase as the usual penguin operators (\mathcal{O}_5 and \mathcal{O}_6 of ref. 8) but we expect per the analysis of ref. 9 that its net contribution is small. Also, competing with the KM scheme are possible Higgs boson induced CP-violating interactions.¹⁰ Hence a measurement of $|\epsilon'/\epsilon|$ in the milliweak range does not necessarily imply the veracity of the penguin mechanism.

The expected CP-violation in almost any model at the quark level within the context of the standard $(SU(2) \times U(1))_{\text{electroweak}} \times SU(3)_{\text{color}}$ gauge theory will be a mixture of both superweak (local $\Delta S=2$ operators) and milliweak (local $\Delta S=1$ operators) contributions. The novel feature we wish to discuss here is that the full $\Delta S=2$ amplitudes ($K^0-\bar{K}^0$ mass matrix) will contain both contributions from the local $\Delta S=2$ operators and bilocal " $(\Delta S=1)^2$ " operators occurring in second order perturbation theory. Whereas the local $\Delta S=2$ operators are associated primarily with short-distance QCD (the usual box diagrams + corrections), the $(\Delta S=1)^2$ bilocals have predominantly large distance contributions which can be estimated by insertion of low lying meson and 2π intermediate states and use of PCAC. In the present letter we will not discuss the QCD radiative corrections nor the sizes of the KM parameters.

The $K^0-\bar{K}^0$ mass matrix is usually computed by way of the box diagrams of Fig. 2. Of course, we expect this perturbative calculation to be subject to large corrections for loop momenta less than a GeV or so. Hence, it is conventional to cut-off the loop integrals below some $\mu^2 \approx 1 \text{ GeV}^2$ (as usual, the resulting physics must be independent of μ^2). The box diagrams are U.V. finite and, due to the GIM cancellation, we find that they are rather insensitive to μ^2 for $\mu^2/m_c^2 \ll 1$, e.g., $\mu^2 \sim .75$ and $m_c^2 \sim 4 \text{ GeV}^2$. The real part of the mass matrix is controlled by the (u,c) GIM cancellation whereas the imaginary part involves only the

(t,c) cancellation. The box calculation gives the coefficient function of the local $\Delta S=2$ operator $\bar{s}\gamma_{\mu}d_L\bar{s}\gamma^{\mu}d_L$. To estimate this short-distance contribution to the $K_L K_S$ mass difference one must next estimate the matrix element, $\langle K^0 | \bar{s}\gamma_{\mu}d_L\bar{s}\gamma^{\mu}d_L | \bar{K}^0 \rangle$, which we expect to be subject to errors within a factor of ≈ 2 to 3. In what follows we will circumvent the need to estimate this matrix element directly.

It is important to realize that there are also independent large distance contributions to the $K^0 - \bar{K}^0$ mass matrix as well as the box contribution. These correspond to effective "bilocal operators" of the form:

$$\sum_n \frac{\langle K^0 | \mathcal{O}_{\Delta S=1} | n \rangle \langle n | \mathcal{O}_{\Delta S=1} | \bar{K}^0 \rangle}{m_K - E_n} \quad (1)$$

where the intermediate states will be constrained to have energies $E_n \leq \mu$. Exactly how many states to include in the summation and the μ dependence of the result are delicate questions. Fortunately, however, the predominant contributions are from the low lying states with energies of order m_K . For example, in the $K_L K_S$ mass difference one conventionally discusses the K_L and K_S self energies separately.¹¹ K_S receives primarily 2π intermediate state and K_L receives π^0, η^0 , etc. contributions. The 2π states give both negative ($E_{2\pi} < m_K$) and positive ($E_{2\pi} > m_K$) contributions to the dispersion integral and might be expected to roughly cancel for a cutoff of $\mu \approx 1 \text{ GeV}^2$.

Similarly the ρ, ω , etc. resonance contributions are expected to be suppressed $\approx \mathcal{O}(m_K^2/m_\rho^2)$ (possibly summable in a short-distance computation¹²). Hence, we shall assume the predominant contributions to the mass matrix in eq. (1) come from the π^0 and η^0 poles in the K_L self energy. With the aid of PCAC this is given by:¹³

$$\begin{aligned} \tau_S \Delta m_{K_L K_S} \Big|_{\pi^0, \eta^0} &= \frac{-32\pi}{9} f_\pi^2 \frac{m_K}{\sqrt{m_K^2 - 4m_\pi^2}} \frac{4m_K^2 - 3m_\eta^2 - m_\pi^2}{(m_K^2 - m_\pi^2)(m_\eta^2 - m_K^2)} \\ &= -.75 \quad (\text{cf. } .48 \pm .02 \text{ expt.}) \end{aligned} \quad (2)$$

Note that this result is both negative and of roughly the same magnitude as the experimental result. The purely short-distance box diagrams give the correct sign to $\tau_S \Delta m_{K_L K_S}$ and we argue that the size of this contribution must be sufficiently large to overcome the negative large distance piece of eq. (2).

In the approximation of "penguin dominance" in which we assume the penguin operators, $\mathcal{O}_5, \mathcal{O}_6$ and \mathcal{O}_7 are a large fraction, f (in the notation of ref. 4) of the $\Delta S=1$ nonleptonic weak decays then these large distance contributions are predominantly the "(penguin)²" diagrams of Fig. 3a with small corrections as in Fig. 3b. For large $f \approx 1$ we see that there is no overlap between the process of Fig. 3a and that of Fig. 2 which is why these are independent additive contributions. For small $f \approx 0$ the large distance

contributions of Fig. 3c are more delicately separated from the short distance pieces of Fig. 2 with which they merge for $E_n \gg \mu$. Hence our separation into (large distance)+(short distance) is relatively clear in the large f limit.

Consider now the mass matrix element $\mathcal{M}_{12} = \langle K^0 | T | \bar{K}^0 \rangle$ where we write:

$$\mathcal{M}_{12} = \mathcal{M}_{12}^{\text{box}} + \mathcal{M}_{12}^{\pi^0, \eta^0}, \dots \quad (3)$$

The real part of \mathcal{M}_{12} is just the $K_L K_S$ mass difference:

$$\frac{1}{2} \Delta m_{K_L K_S} = \text{Re} \mathcal{M}_{12}^{\text{box}} + \text{Re} \mathcal{M}_{12}^{\pi^0, \eta^0}, \dots \quad (4)$$

The mass matrix also contains small imaginary parts in the (KM) model. For $\mathcal{M}_{12}^{\text{box}}$ these are just the usual "superweak" phases as discussed in ref. 6:

$$\text{Im} \mathcal{M}_{12}^{\text{box}} = 2s_2 c_2 s_3 \sin \delta P_1 \text{Re} \mathcal{M}_{12}^{\text{box}} \equiv \epsilon_M \text{Re} \mathcal{M}_{12}^{\text{box}} \quad (5)$$

P_1 is subject to calculable QCD corrections which have been studied extensively elsewhere,¹⁴ but in the rough approximation of neglecting these effects we can obtain P_1 directly from Fig. 2⁶ (this is very insensitive to the μ^2 cutoff since it involves the (t,c) GIM pair):

$$P_1 = \frac{\sin^2 \theta_2 (1 + \eta \log \eta) - \eta \cos^2 \theta_2 (1 + \log \eta)}{\eta \cos^4 \theta_2 + \sin^4 \theta_2 - \eta \log \eta (\sin^2 2\theta_2)} \quad (6)$$

where $\eta = m_c^2/m_t^2$ (terms of order η^2 have been neglected in N and D) and $\theta_1 = \theta_{\text{Cabbibo}}$, θ_2 , θ_3 and δ are KM angles.

For the large distance contributions we expect that Fig. 3c gives very small CP-phase effects. These are the "charm sea" effects as discussed in ref. 6 and yield the small $|\epsilon'/\epsilon| \approx 1/450$ discussed there. Indeed, if $f \gtrsim \mu^2/4m_c^2$ we can ignore these contributions entirely. Hence, we see that for large f the dominant CP-phase contribution to $\mathcal{M}_{12}^{\pi^0, \eta^0, \dots}$ comes from Fig. 3a and is given in terms of the milliweak amplitude of Gilman and Wise⁴

$$\text{Im}(\mathcal{M}_{12}^{\pi^0, \eta^0, \dots}) = -2fs_2 c_2 s_3 \sin \delta P_2 \text{Re}(\mathcal{M}_{12}^{\pi^0, \eta^0, \dots}) \quad (7)$$

where again P_2 is subject to large calculable QCD corrections but, in the approximation of neglecting these, we have:⁴

$$P_2 = \ln(\eta) / \left(\log\left(\frac{m_c^2}{\mu^2}\right) - \sin^2 \theta_2 \log \eta \right) \quad (8)$$

(we shall parallel the discussion of Gilman and Wise in the following.⁴ Note the sign in eq. (7) is consistent with eq. (9) and (10) in our definition $\mathcal{M}_{12} = \langle K^0 | T | \bar{K}^0 \rangle$).

We also have an additional contribution to $\text{Im } \mathcal{M}_{12}$ from the convention of defining A_0 in the amplitude

$$\langle 2\pi, I=0 | H_{\Delta S=1} | K^0 \rangle = A_0 e^{i\delta_0} \quad (9)$$

to be real. Before this redefinition Gilman and Wise give

$$\begin{aligned} A_0 &= (1 + i f s_2 c_2 s_3 \sin \delta P_2) \text{Re } A_0 \\ &\equiv e^{i\xi} \text{Re } A_0; \quad \xi \approx f s_2 c_2 s_3 \sin \delta P_2 \quad . \end{aligned} \quad (10)$$

Hence, after the redefinition:

$$\begin{aligned} |K_0\rangle &\rightarrow e^{-i\xi} |K^0\rangle \\ |\bar{K}_0\rangle &\rightarrow e^{i\xi} |\bar{K}^0\rangle \end{aligned} \quad (11)$$

we have the additional contribution to $\text{Im } \mathcal{M}_{12}$:

$$(\text{Im } \mathcal{M}_{12})_{\text{redef.}} = 2f s_2 c_2 s_3 \sin \delta P_2 (\text{Re } \mathcal{M}_{12}^{\text{box}} + \text{Re } \mathcal{M}_{12}^{\pi^0, \eta^0}) \quad (12)$$

Taking together the three contributions of eqns. (5, 7, and 12,) we have:

$$\begin{aligned}
 (\text{Im } \mathcal{M}_{12}) &= (2s_2 c_2 s_3 \sin \delta) P_1 \text{Re } \mathcal{M}_{12}^{\text{box}} - (2fs_2 c_2 s_3 \sin \delta) P_2 \text{Re } \mathcal{M}_{12}^{\pi^0, \eta^0} \\
 &+ (2s_2 c_2 s_3 \sin \delta) P_2 (\text{Re } \mathcal{M}_{12}^{\text{box}} + \text{Re } \mathcal{M}_{12}^{\pi^0, \eta^0}) \quad (13)
 \end{aligned}$$

$$= (2s_2 c_2 s_3 \sin \delta) (P_1 + P_2) \text{Re } \mathcal{M}_{12}^{\text{box}} \quad (14)$$

Hence, we observe that the large distance CP phase contribution of Fig. 2a cancels against the phase redefinition, large distance piece of eq. (12). The result of eq. (14) in the notation of ref. 4 is just

$$\text{Im } \mathcal{M}_{12} = (\epsilon_m + 2\xi) \cdot \text{Re } \mathcal{M}_{12}^{\text{box}} \quad (15)$$

The important point of the present discussion is that the rhs of eq. (14) and eq. (15) involve only the short distance contribution to $\text{Re } \mathcal{M}_{12}$. By eq. (4) above this is not directly identifiable with $\Delta m_{K_L K_S}$.

Let us define a parameter z as follows:

$$2\text{Re } \mathcal{M}_{12}^{\pi^0, \eta^0} = -z \Delta m_{K_L K_S} \quad (16)$$

Hence we obtain from eq. (4):

$$\frac{2\text{Im} \mathcal{M}_{12}}{\Delta m_{K_L K_S}} = |(\epsilon_m + 2\xi)(1+z)| \equiv 2\sqrt{2}|\epsilon| \quad . \quad (17)$$

Also, from the definition of ϵ' one has:⁴

$$|\epsilon'| \approx \frac{1}{\sqrt{2}} |\xi\omega| \quad , \quad \omega \equiv \frac{A_2}{A_0} \quad . \quad (18)$$

So our final result is:

$$\left| \frac{\epsilon'}{\epsilon} \right| \approx \left| \omega \left(\frac{2\xi}{2\xi + \epsilon_m} \right) \right| \cdot \left| \frac{1}{1+z} \right| \quad (19)$$

where $|\omega| \approx 1/20$.

If we take the PCAC estimate of eq. (2) seriously we have $z \approx 1.5$ and hence

$$\left| \frac{\epsilon'}{\epsilon} \right| \approx \frac{1}{50} \left| \frac{2\xi}{2\xi + \epsilon_m} \right| \quad , \quad (20)$$

which is of course $1/2.5$ smaller than $|\epsilon'/\epsilon|$ in refs. 4,5.

Now we expect that this result is roughly true only in the penguin dominance, $f \approx 1$ limit, since in that limit is the (penguin)² process Fig. 3a clearly separable from the box diagrams of Fig. 2. Hence in general there will be some dependence upon f in z .

It is amusing that in the limit $\epsilon_m/2\xi \approx 0$ that eq. (20) predicts $|\epsilon'/\epsilon|$ to be equal to the experimental upper limit $\approx 1/50$. This can only occur if $P_1/P_2 \approx 0$ in the (KM) scheme but also may conceivably occur in Higgs induced CP-violation models if the box contributions (which now involve W and Higgs together) are anomalously small due to cancellations. This result is the extreme limit of a milliweak model in which the CP-violating $\Delta S=2$ amplitude occurs only through the large distance $(\Delta S=1)^2$ processes.

Gilman and Wise discuss several possibilities in their analysis with $z=0$. The most dangerous limit is $m_c=1.5$, $m_t=15$ GeV, $\mu=1$ and $\theta_z=15^\circ$ with $f=.75$. The result is $|\epsilon'/\epsilon|_{\text{GW}}=1/13$ which for us is reduced to the value $\approx 1/33$, closer to the upper bound. We expect that even the pure "superweak" result of ref. 6 will be reduced by the large distance corrections to $|\epsilon'/\epsilon|_{\text{superweak}} \leq 1/1000$, though this involves the limit $f \approx 0$, which is expected to be less reliable as discussed above.

After completing this work we realized that similar ideas were discussed previously by Wolfenstein.¹⁵ We differ from ref. 15 primarily in our use of the convention $\text{Im}A_0=0$, and hence in observing the cancellation of eq. (14), and our identification of the (penguin)² diagrams of Fig. 3, which we feel clarifies the separation between large and small distance contributions. Wolfenstein also attaches greater unreliability to estimates of the parameter he defines as $D(=-z)$. We expect that in fact $1 \leq z \leq 2$ as discussed above for

$f \gtrsim \mu^2/m_c^2$. We are therefore confident that a measurement of $|\epsilon'/\epsilon| \gtrsim 1/100$ would be a strong indication that penguins are important provided that possible Higgs (or other) induced CP-violation effects can be eliminated. Higgs becomes decreasingly likely as the mass of allowed residual charged Higgs scalars increases.¹⁶

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FIGURE CAPTIONS

- Fig. 1: Penguin diagram.
- Fig. 2: Box and crossed box diagrams.
- Fig. 3: (a) (Penguin)² diagram; (b) Corrections to pure (Penguin)² process.