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SU(5) WITHOUT SU(5): CONSERVATION LAWS IN UNIFIED MODELS OF QUARKS AND LEPTONS*

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ABSTRACT

The most general S matrix for the 30 first generation quarks and leptons shows many features of SU(5) with no symmetry constraints imposed beyond global conservation of electric charge and weak isospin. All helicity-conserving four-point amplitudes conserve baryon number if there are no transitions from u_R or $\bar{u}_L$ quarks to other states and must violate B conservation if such transitions occur. B-L conservation is violated in four-point-functions only in single-helicity-flip amplitudes.

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The existence of an $SU(2) \times U(1)$ classification[1] for two completely different types of particles which seem to belong together in common multiplets motivates a search for a higher symmetry to unify the two and be the non-Abelian gauge theory of the world.[2] However, this has happened before with the $SU(2) \times U(1)$ of isospin and strangeness and the $SU(3)$ gauge theory of the eightfold way which brought strange and nonstrange particles into unified multiplets. Today the unification of strange and nonstrange particles into flavor $SU(3)$ remains, but the ρ, ω and K^* are no longer an octet of gauge bosons and flavor $SU(3)$ has been revealed to be an accidental symmetry based upon our incomplete knowledge of the number of flavors. To prepare for a repetition of history we look for those properties of quark-lepton unification independent of higher symmetries or more general gauge theories.

We therefore examine the most general S-matrix for the thirty quarks and leptons in the first generation, commonly classified in the 5, 5*, 10 and 10* representations of $SU(5)$, under the assumptions only of conservation of electric charge and weak isospin. The resulting selection rules and conservation laws are easily demonstrated with the aid of the following unorthodox linear combinations of well known quantum numbers

$$\kappa = 3(B - L) - H \tag{1a}$$

$$P = \frac{3(B - L) - H}{4} - \frac{3(Q - I_3)}{5} = \frac{\kappa}{4} - \frac{3}{5}(Q - I_3) \tag{1b}$$

$$\beta = \frac{3(B - L) - H}{4} - (Q - I_3) = P - \frac{2}{5}(Q - I_3) \tag{1c}$$

where B is the baryon number, L the lepton number, H

the helicity, defined as +1 and -1 respectively for right and left handed states, Q the electric charge and I_3 the third component of the weak isospin. The motivation for introducing these quantum numbers is seen in the simple structure of their eigenvalues for the fifteen quark and lepton states of the first generation and their fifteen antiparticles listed in Table I. The pentality quantum number P has the following remarkable properties:

1. Although it is a linear combination of two terms with quarter-integral and tenth-integral values, the values of P for the first generation fermions are only fifth-integral.

2. The total pentality of a multifermion state cannot change by a fifth-integral amount in any transition which conserves Q and I_3. Thus the pentality is an additive quantum number with fifth-integral eigenvalues which is allowed to change only by integral amounts in any transition which conserves Q and I_3. This gives the pentality selection rule

$$\delta P = \delta\beta = 0,\ 1,\ 2\ \ldots\ldots \tag{2a}$$

This immediately implies that

$$\delta\kappa = 4n\ , \tag{2b}$$

where n is an integer. Since δH and $\delta(B-L)$ are both even integers, the selection rule (2b) requires that $\delta H/2$ and $\delta(B-L)/2$ must either be both even integers or both odd integers. Thus transitions with odd and even helicity flips have different selection rules for B-L.

For odd-helicity-flip transitions,

$$\delta H = 2(2n+1) \Rightarrow \delta(B-L) = 2(2n'+1) \tag{2c}$$

where n and n' are integers. Thus B-L cannot be conserved in odd helicity flip transitions. For helicity conserving and even flip transitions

$$\delta H = 4n \Rightarrow \delta(B-L) = 4n' \tag{2d}$$

where n and n' are integers. In four point functions $\delta H = 0$, 2 or 4 and $\delta(B-L) = 0$ or 2, since $\delta(B-L) = 4$ can be achieved only in a transition between two leptons and two antileptons which cannot conserve Q and I_3. Thus B-L is conserved in all helicity-conserving and double-flip four point functions which conserve Q and I_3.

We can now classify all three types of helicity amplitudes occurring in four point functions.

A. Helicity conserving processes: $\delta H = 0$, $\delta P = \delta\kappa = \delta\beta = \delta(B-L) = 0$

These have the form

$$(P_1, P_2) \rightarrow (P_1, P_2) \tag{3a}$$

where P_i denotes any state having the eigenvalue $P = P_i$, together with all other processes obtained from (3a) by crossing.

B. Double-flip processes: $\delta H = \pm 4 = -\delta\kappa$, $\delta P = \pm 1 = \delta\beta$, $\delta(B-L) = 0$.

The only processes allowed by the pentality selection rule (2a) have the form

$$(2/5, 2/5) \rightarrow (-2/5, 1/5) \tag{3b}$$

together with all other processes obtained from (3b) by crossing.

C. <u>Single-flip processes</u>: $\delta H = \pm 2$. The only processes allowed by the pentality selection rule (2a) have $\delta P = \pm 1 = \delta\beta$, $\delta(B-L) = \pm 2$ and the form

$$(1/5, 1/5) \rightarrow (-1/5, -2/5) \tag{3c}$$

together with all other processes obtained from (3c) by crossing.

All these results obtained from the pentality selection rule which assumed only conservation of Q and I_3 can now be seen to be also SU(5) results. The eigenvalues of P are seen in Table I to label representations of SU(5). All the states in a given representation of SU(5) have the same eigenvalue of P. Thus P turns out to be an SU(5) invariant, although P is defined in terms of B and L which are completely outside of SU(5) and there is no group-theoretical reason connecting P with SU(5). The "pentality" quantum number P is seen to have the property analogous to triality for SU(3); it has the eigenvalues $\pm(1/5)$ for the fundamental representations 5 and 5* of SU(5) and the eigenvalues $\pm(2/5)$ for the 10 and 10* representations which are built from two fundamental representations. Note that the fifth-integral quantum numbers arise naturally in this description, without invoking any SU(5), and that the selection rule (2a) that the fifth-integral pentality number P can only change by an integral amount is also derived without SU(5).

The relations (3) are just the four point couplings allowed by SU(5). The $\delta H = 0$ transitions (3a) have the form

$$m \times n \rightarrow m \times n \tag{4a}$$

where m and n denote any representation of SU(5) among the 5, 5*, 10 and 10*. The $\delta H = \pm 4$ and ± 2 transitions (3b) and (3c) correspond respectively to the SU(5) couplings

$$10 \times 10 \rightarrow 10^* \times 5 \tag{4b}$$

$$5 \times 5 \rightarrow 5^* \times 10^* \tag{4c}$$

Our approach here is to note these connections with SU(5) as a guide to our formulation, but to keep everything completely independent of any symmetry assumptions beyond global SU(2) x U(1). Thus all the results obtained for four-point functions involving quarks and leptons will hold in any generalization of the standard model which keeps charge and weak isospin conservation regardless of how many additional Higgses, Schmiggses or Technicrats are introduced. The SU(5)-like results must therefore be present in any such formulation regardless of whether or not these symmetries are explicitly assumed.

The "quasi-baryon number" β has the very interesting property of being exactly equal to the baryon number B for all first generation particles except the u_R and $\bar{u}_L$

$$\beta = B - n(u_R) + n(\bar{u}_L) \quad . \tag{5}$$

Thus in any process where β is conserved, which includes all helicity conserving transitions of the form (3a), baryon number is conserved for all processes in which $n(u_R) - n(\bar{u}_L)$ does not change, and must be violated in processes where this difference does change. In the standard model of $SU(2) \times U(1)$ where u_R and $\bar{u}_L$ couple only via neutral currents which do not change quantum numbers, baryon number is conserved. But baryon number must be violated in any extension which introduces helicity conserving bosons coupling non-trivially to u_R or $\bar{u}_L$.

Almost any model which introduces new particles must admit $\delta H = 0$ four-point functions in which the only external particles are first generation quarks and leptons. Including other generations with identical values of these quantum numbers does not change these conclusions. Unless these new processes treat u_R and $\bar{u}_L$ in the same trivial way as the standard model of $SU(2) \times U(1)$, they must necessarily violate baryon number conservation. <u>This is the basic reason why baryon number nonconservation must arise in any such unification models</u>. It is already required by consistency with charge and weak isospin conservation and the observed quantum numbers of the first generation of quarks and leptons. No further gauge theory is needed. This also explains why attempts to gauge baryon number would encounter inconsistencies.

We now examine the three types of helicity amplitudes given in Eqs. (3) in more detail.

A. <u>Helicity conserving $\delta H = 0$ transitions</u>.

These can be characterized by the selection rules $\delta H = \delta P = \delta \kappa = \delta \beta = \delta(B-L) = 0$. They conserve $B-L$ and β, and therefore conserve baryon number if $n(u_R) - n(\bar{u}_L)$ does not change, but <u>must violate baryon number conservation</u> if it does change. Note that these processes are the only ones of the three types that can be mediated by a Yukawa coupling via a helicity conserving vector boson or by any boson classified in the adjoint representation of $SU(5)$, and that the adjoint representation contains bosons which change $n(u_R) - n(\bar{u}_L)$.

B. Double flip $\delta H = \pm 4$ transitions.

These can be characterized by the selection rules $\delta H = \pm 4 = -\delta\kappa$; $\delta P = \mp 1 = \delta\beta$; $\delta(B-L) = 0$. These conserve B-L but have $\delta\beta = \mp 1$. They violate baryon number conservation unless $n(u_R) - n(\bar{u}_L)$ changes by one unit in the proper direction to match the change in β. These processes have the form (4b) in SU(5) and cannot be mediated by a boson classified in the adjoint representation of SU(5) but could be mediated by a particle such as a scalar Higgs which flips helicity and is classified in a 5 or 5* representation. Equation (4b) shows that in an SU(5) description, such a four point function can only go via intermediate states in the 5, 5*, 45 or 45* representations.

C. Single-flip $\delta H = \pm 2$ transitions.

These can be characterized by the selection rules $\delta H = \pm 2 = \delta(B-L)$; $\delta P = \pm 1 = \delta\beta$, $\delta\kappa = \pm 4$. These violate everything, including B-L and B. However, they correspond to the SU(5) couplings (4c) and can only arise in an SU(5) formalism via an intermediate boson classified in the 10 or 10* representations with derivative couplings. Thus as long as no Higgses or other particles classified in the 10 or 10* are introduced with derivative couplings to quarks or leptons, these processes will not occur and B-L will be conserved.

We now list explicitly all processes which violate baryon conservation and examine the implications for proton decay.

A. $\delta H = 0$ transitions (3a)

These are the only ones which can go via vector exchange. The quasi-baryon number β is conserved, and $\delta B \neq 0$ requires the disappearance of a u_R quark or its equivalent under crossing. Thus the most general $\delta B \neq 0$, $\delta H = 0$ process has the form:

$$u_R + X \rightarrow Y(P = -2/5) + X' \tag{6}$$

where fermions X and X' must have the same eigenvalues of P. The only candidates for Y are the companions of u_R in the $P = -2/5$ multiplet (the 10^* of $SU(5)$) which are the e_R^-, $\bar{u}_R$ and $\bar{d}_R$. The e_R^- is immediately excluded, because the transition $u_R \rightarrow e_R^-$ changes electric charge by $\delta Q = -5/3$, and can only be balanced by the reciprocal transitions $e^- \rightarrow u$ which restores baryon conservation. The transitions $u_R \rightarrow \bar{u}_R$ and $u_R \rightarrow \bar{d}_R$ both involve a change in quasi-baryon number $\delta\beta = +1/3$, and are related to one another by a weak isospin reflection. They must be balanced by a transition $X(\text{quark}) \rightarrow X'(\text{lepton})$ to conserve β. The only quark-lepton transitions which can balance the $\delta Q = -4/3$ of the $u_R \rightarrow \bar{u}_R$ transition are the $d \rightarrow e^+$ transitions. The allowed baryon-number violating processes with $\delta H = 0$ are therefore:

$$u_R + d_R \rightarrow \bar{u}_R + e_R^+ \tag{7a}$$

$$u_R + d_L \rightarrow \bar{u}_R + e_L^+ \tag{7b}$$

together with their weak isospin reflections,

$$u_R + d_R \rightarrow \bar{d}_R + \bar{\nu}_R \tag{7c}$$

$$u_R + u_L \rightarrow \bar{d}_R + e_L^+ \;. \tag{7d}$$

This immediately gives the result that the lepton emitted in nucleon decay must be either positive or an antineutrino, but cannot be negative, nor a left handed neutrino.

The pairs of transitions (7a-7c) and (7b-7d) related by isospin reflection are required to be equal. However, there is no relation between the two pairs at this level, as they involve different $SU(2) \times U(1)$ multiplets. In SU(5) these pairs are related only at the level of a gauge theory by the universal couplings of gauge bosons. At the global SU(5) level with independent Yukawa couplings for different SU(5) multiplets the two pairs involve the couplings of the 10 and 5 of SU(5) respectively to the bosons classified in the 24. These couplings are independent if there is no higher symmetry like SO(10) or gauge condition which relates them.

Note, however, that isospin relations alone are sufficient to predict that the positron decay modes are stronger than the neutrino decay modes, since the neutrino decay (7c) is equal to one of the three positron decay modes (7a) by isospin, and there are two additional positron decays (7b) and (7d).

These results can be expressed as nucleon decays by adding a spectator u or d quark to the equation to give a nucleon on the left hand side and a pion on the right hand side. From (7a) and (7b) we obtain

$$p \rightarrow e^+ + \pi^\circ \tag{8a}$$

$$n \rightarrow e^+ + \pi^- \tag{8b}$$

From (7c) we obtain

$$p \rightarrow \bar{\nu}_R + \pi^+ \tag{8c}$$

$$n \rightarrow \bar{\nu}_R + \pi^\circ \tag{8d}$$

From (7d) we obtain (8a) again.

B. Double-flip $\delta H = 4$ processes (3b).

These are most conveniently listed in the crossed representation with three $P = 2/5$ particles going into one with $P = 1/5$ and looking at each allowed value of the total electric charge. The complete set of $\delta H = 4$ processes are

$$Q = +1 \qquad e^+_L + u_L + \bar{u}_L \to e^+_R \qquad \delta B = \delta L = 0 \tag{9a}$$

$$u_L + u_L + d_L \to e^+_R \qquad \delta B = \delta L = -1 \tag{9b}$$

$$Q = 0 \qquad e^+_L + d_L + \bar{u}_L \to \bar{\nu}_R \qquad \delta B = \delta L = 0 \tag{9c}$$

$$u_L + d_L + d_L \to \bar{\nu}_R \qquad \delta B = \delta L = -1 \tag{9d}$$

$$Q = -\tfrac{1}{3} \qquad d_L + u_L + \bar{u}_L \to d_R \qquad \delta B = \delta L = 0 \tag{9e}$$

$$e^+_L + \bar{u}_L + \bar{u}_L \to d_R \qquad \delta B = \delta L = +1 \tag{9f}$$

These always conserve B-L, but have $\delta\beta = -1$ and violate baryon number conservation unless they involve a single u_R or $\bar{u}_L$ in which case the baryon number always turns out to be conserved. For each charge there is one $\delta B = 0$ and one $\delta B = \delta L = \pm 1$ process. Here also nucleons decay only into e^+ and $\bar{\nu}$.

C. Single-flip $\delta H = \pm 2$ processes (3c). These are also conveniently listed by electric charge in the crossed representation with three $P = 1/5$ particles going into one with $P = -2/5$.

$$Q = -1 \qquad d_R + d_R + d_R \to e^-_R \qquad \delta B = -1, \delta L = +1 \tag{10a}$$

$$Q = -\tfrac{2}{3} \qquad d_R + d_R + \bar{\nu}_R \to \bar{u}_R \qquad \delta B = -1, \delta L = +1 \tag{10b}$$

$$Q = +\frac{1}{3} \qquad e_R^+ + d_R + d_R \to \bar{d}_R \qquad \delta B = -1, \delta L = +1 \tag{10c}$$

$$Q = +\frac{2}{3} \qquad e_R^+ + \bar{\nu}_R + d_R \to u_R \qquad \delta B = 0, \delta L = +2 \tag{10d}$$

These all have $\delta(B-L) = -2$ and $\delta\beta = -1$. All violate lepton conservation. Processes (10d) which conserve baryon number have $\delta L = 2$. The others all have $\delta B = \delta L = -1$ and give nucleon decays into e^- and ν.

We thus have demonstrated the following SU(5) like properties of four-point functions without explicit assumptions of SU(5).

1. All selection rules obtained from full SU(5) symmetry are already required by conservation of electric charge and weak isospin.

2. The four point functions are naturally classified into three types with different helicity structures, namely helicity conserving, double flip and single flip. This classification is in one-to-one correspondence with SU(5) classifications.

3. The violation of the conservation laws of B and B-L can be stated very generally and simply:

A. All helicity conserving and double flip amplitudes conserve B-L and allow nucleon decays only to antileptons. All single-helicity-flip amplitudes have $\delta(B-L) = \pm 2$ and allow nucleon decays only to leptons.

B. Baryon number conservation is violated in helicity conserving amplitudes <u>only in those amplitudes</u> which can be transformed by crossing to the decay of a right-handed up quark u_R into two antiquarks and an antilepton. All other helicity conserving amplitudes conserve B. Thus B is conserved in the standard model of weak interactions where the u_R couples only via neutral currents and cannot change into another state.

C. The role of the u_R is reversed in double-flip transitions. All double-flip transitions which involve the creation or annihilation of a single u_R or $\bar{u}_L$ conserve baryon number. All double-flip transitions which do not involve u_R or $\bar{u}_L$ or which create or annihilate a pair of them violate baryon number conservation.

4. The three types of four-point functions have simple interpretations in terms of the SU(5) and Lorentz quantum numbers of exchanged bosons which could give rise to these couplings if they are coupled to the fermions with Yukawa couplings.

Some of these results have been previously obtained using specific models[3] or symmetries,[4] particularly results on the conservation of B-L. Our results are more general, being model independent and assuming no higher symmetries. We also point out for the first time the close connection between the helicity structure of the amplitude and conservation of B and B-L.

These results are easily extended to treat any n-point function. We first note that a pair of incoming or outgoing particles with equal and opposite pentality have no effect on the selection rules (2) and can be disregarded to give the same selection rules as an n-2-point function. We therefore consider only "irreducible" n-point functions which contain no such pairs either directly or by crossing. There are therefore only two types of irreducible n-point functions which are natural generalizations of the two four-point functions denoted by B and C above.

Type B. These have the general form

$$(2/5)^x \rightarrow (1/5)^{n-x} \tag{11a}$$

and have a change of pentality

$$\delta P = (n - 3x)/5 = \delta\beta\ , \tag{11b}$$

where x is any integer which satisfies the pentality condition

$$(n - 3x)/5 = \text{an integer}\ . \tag{11c}$$

These transitions have changes in helicity and B-L given by

$$\delta H = n \tag{11d}$$

$$\delta(B\text{-}L) = 3\delta P + x\ . \tag{11e}$$

Type C. These have the general form

$$(2/5)^y \rightarrow (-1/5)^{n-y} \tag{12a}$$

and have a change in pentality

$$\delta P = -(n + y)/5 = \delta\beta \tag{12b}$$

where y is any integer satisfying the pentality condition

$$(n + y)/5 = \text{an integer.} \tag{12c}$$

These transitions have changes in helicity and B-L given by

$$\delta H = -n + 2y \tag{12d}$$

$$\delta(B\text{-}L) = 3\delta P + y \tag{12e}$$

Note that if x satisfies the pentality condition (11c), $y = n - x$ automatically satisfies the pentality condition (12c), since $n + y = 2n - x = 2(n - 3x) + 5x$.

These two types of transitions correspond to the SU(5) couplings

$$\text{Type B.} \qquad (10)^{x} \to (5)^{n-x} \tag{13a}$$

$$\text{Type C.} \qquad (10) \to (5^*)^{n-y} \tag{13b}$$

These results are seen to hold for the case of four-point functions, where the type A processes are reducible, the type B processes can be crossed to the form (11a) with $x = 3$ and the type C processes can be crossed to the form (12a) with $y = 1$. Note that $x + y = 4 = n$, and that δP, δH and $\delta(B-L)$ are in agreement with Eqs.(11) and (12).

For six-point functions, we have only one solution to the condition (11c), $x = 2$, which has its companion solution to (12c), $y = 4$. The irreducible six-point functions thus have the form

Type B. $x = 2$, $\delta P = 0 = \delta\beta$, $\delta H = 6$, $\delta(B-L) = 2$,

$$(2/5)^{2} \to (1/5)^{4} \tag{14a}$$

with the SU(5) couplings

$$(10)^{2} \to (5)^{4} \tag{14b}$$

Type C. $y = 4$, $\delta P = -2 = \delta\beta$, $\delta H = +2$, $\delta(B-L) = -2$.

$$(2/5)^{4} \to (-1/5)^{2} \tag{15a}$$

with the SU(5) couplings

$$(10)^4 \rightarrow (5^*)^2 \, . \tag{15b}$$

For eight-point functions there are two solutions to the condition (11c), $x=1$ and $x=6$, with the companion solutions to (12c), $y=7$ and $y=2$. These give the irreducible eight-point functions:

Type B. $x=1$, $\delta P = 1 = \delta\beta$, $\delta H = 8$, $\delta(B-L) = 4$

$$(2/5) \rightarrow (1/5)^7 \tag{16a}$$

$$(10) \rightarrow (5)^7 \tag{16b}$$

and $x=6$, $\delta P = -2 = \delta\beta$, $\delta H = 8$, $\delta(B-L) = 0$

$$(2/5)^6 \rightarrow (1/5)^2 \tag{16c}$$

$$(10)^6 \rightarrow (5)^2 \tag{16d}$$

Type C. $y=7$, $\delta P = -3 = \delta\beta$, $\delta H = 6$, $\delta(B-L) = 2$

$$(2/5)^7 \rightarrow (-1/5) \tag{17a}$$

$$(10)^7 \rightarrow (5^*) \tag{17b}$$

and $y=2$, $\delta P = -2 = \delta\beta$, $\delta H = -4$, $\delta(B-L) = -4$

$$(2/5)^2 \rightarrow (-1/5)^6 \tag{17c}$$

$$(10)^2 \rightarrow (5^*)^6 \tag{17d}$$

One example of the use of these higher n-point functions is in the suggested process of neutron-antineutron mixing or neutron oscillations[5] which would be described by a six-point function. We examine the possibilities by looking at the disappearance of two u-quarks and four d-quarks into the vacuum,

$$(u)^4 + (d)^2 \to |0\rangle \tag{18}$$

This process has $\delta B = -2 = \delta(B-L)$. There are three possible six-point functions which can give these selection rules. The irreducible six-point functions of type B, Eq. (14) and type C, Eq. (15) both have $\delta(B-L) = 2$. The reducible six-point function constructed from a single-helicity-flip four-point function of type C, and an additional "spectator" pair also has $\delta(B-L) = 2$. We consider each individually.

We first note that u-quarks appear in the 10 and 10* representations of SU(5) and d quarks in the 5 and 10. Thus the most general SU(5) coupling which can describe the process (18) is

$$(10^*)^r \times (5)^s \times (10)^{6-r-s} \to |0\rangle \tag{19}$$

where $r \leq 4$ and and $s \leq 2$. Additional constraints on values of r and s are now provided by the selection rules which choose three allowed sets of values. For convenience we use the SU(5) labels for the multiplets. However, all results are independent of SU(5) as above and could be written with the pentality labels.

A. <u>Reducible Six-Point Function with a Conjugate Pair Added to (4c)</u>.

Since the 5* does not appear in Eq. (19) the only conjugate pair which can be added to (4c) is 10-10*. This gives $r = 1$, $s = 3$ in Eq. (19) and the SU(5) coupling

$$10^* \times (5)^3 \times (10)^2 \rightarrow |0\rangle \tag{20a}$$

corresponding to the particular transition

$$u_R + 3d_R + u_L + d_L \rightarrow |0\rangle \tag{20b}$$

This transition clearly has $\delta H = -2 = \delta(B-L)$ and $\delta\beta = 1$, as expected for a single-helicity flip four-point function.

B. Irreducible Six-Point Function of Type B.

This has $r = 2$, $s = 4$ in Eq. (19) and the SU(5) coupling

$$(10^*)^2 \times (5)^4 \rightarrow |0\rangle \tag{21a}$$

corresponding to the particular transition

$$2u_R + 4d_R \rightarrow |0\rangle \tag{21b}$$

This transition clearly has $\delta H = -6$, $\delta(B-L) = -2$, $\delta\beta = 0$, as expected for Type B.

C. Irreducible Six-Point Function of Type C.

This has $r = 0$, $s = 2$ in Eq. (19) and the SU(5) coupling

$$(5)^2 \times (10)^4 \rightarrow |0\rangle \tag{22a}$$

corresponding to the particular transition

$$2d_R + 2d_L + 2u_L \rightarrow |0\rangle \tag{22b}$$

This transition clearly has $\delta H = 2$, $\delta(B-L) = -2 = \delta\beta$, as expected for Type C.

The three transitions (20b), (21b) and (22b) are the only ones allowed by conservation of electric charge and weak isospin for neutron-antineutron mixing. Note that all of these involve helicity flip, and that (21b) is a triple-helicity-flip transition.

All these results were rigorously obtained without any SU(5) symmetry assumptions. Yet one may be suspicious of hidden connections because the particular linear combinations (1) were chosen ad hoc and turned out to have SU(5) properties. We therefore investigate the properties of such linear combinations in more detail and look for possible implicit connections between them and higher symmetries.

In any scheme which unifies quarks and leptons and places them into common multiplets, quantum numbers are needed like generalizations of hypercharge with the same eigenvalues for a large number of states including both quarks and leptons. The quantum number P has only two pairs of equal and opposite eigenvalues ±1/5 and ±2/5 for all sets of 30 states in the first three generations. Among the quantum numbers labeling quantities conserved in SU(2) x U(1), the linear combination of B-L and $Q-I_3$ appearing in (1b) is almost uniquely chosen by the requirement that only two pairs of eigenvalues should appear. Since baryon number and electric charge are both third-integral for quarks and integral for leptons, a linear combination of these two must be found which is not third-integral in order to have a common eigenvalue for quarks and leptons. The combination $Q - I_3$ is used to give the same eigenvalue for both members of an isospin doublet. The most general linear combination $B - x(Q - I_3)$ has the eigenvalues $(1/3) - (1/6)x$ for the left handed quark doublet and $(1/3) - (2/3)x$ and $(1/3) + (1/3)x$ for the right handed u and d quarks respectively. This gives three independent eigenvalues for the quarks alone, without considering antiquarks and leptons. These can be simplified only if two of these three eigenvalues are equal and opposite. The three values of x for which this occurs are $x = 4/5$, 2 and -4. With these values we define

$$\lambda = (B-L) - \tfrac{4}{5}(Q-I_3) = (4P + H)/3 \tag{23a}$$

$$\mu = (B-L) - 2(Q-I_3) = -2I_{3R} \tag{23b}$$

$$\nu = (B-L) + 4(Q-I_3) = 5\tilde{\lambda} \tag{23c}$$

where the additional term $-L$ is needed to keep the same eigenvalues for quarks and leptons, and λ is seen to be simply related to the pentality P. The quantum number μ is also seen to be identical with the right handed isospin recently introduced by Marshak and Mohapatra.[5] The number ν is seen to be $5\tilde{\lambda}$ where $\tilde{\lambda}$ is the eigenvalue of λ for the isospin mirror state, $u \leftrightarrow d$ and $e \leftrightarrow \nu$.

This condition already suggests one of the general features found in unification schemes, the classification of quarks and antiquarks in the same multiplet. The condition that two quark states have equal and opposite eigenvalues of an additive quantum number which is constant in a multiplet gives quarks and antiquarks the same eigenvalue of this qauntum number.

Values of λ, μ and ν are also given in Table I. But λ which is related to the pentality P is the only one with fractional eigenvalues. Thus the fifth-integral eigenvalues are a natural result of the SU(2) x U(1) classification of quarks and leptons and the condition restricting the eigenvalues to only two pairs. The normalization of the operator (11) is chosen to keep the coefficient of B-L equal to unity. Thus all three are conserved if B-L is conserved and conserved modulo 2 if B-L is not conserved. The fractional eigenvalues makes conservation modulo 2 a much more serious constraint for λ than for μ and ν.

The mysterious relation between the fifth integral eigenvalues of the operator λ and SU(5) is clarified by noting that these eigenvalues have a simple interpretation in the SO(10) classification when the right handed neutrino is included which has an eigenvalue of -1. They are just the eigenvalues in the spinorial representation normally used to classify the quarks and leptons of the U(1) generator[4] which appears in the SU(5) x U(1) subgroup of SO(10). If the operator λ is identified with this U(1) generator in SO(10), the classification and all the quantum numbers in Table I are evident. However, B and L are not defined in the usual SO(10) description, and the physical meaning of this U(1) generator is not obvious. There may be a deep underlying significance to the fact that the eigenvalues of λ are the same as those of the U(1) generator for the 16 dimensional spinorial representation in which the quarks and leptons are classified. This does not necessarily require that the operator as defined by Eq.(23a) should also be equivalent to this U(1) generator for other representations in which Higgs or technicolor particles might be classified.

We thus see that there is an underlying SU(5)-like structure present in the quantum numbers of the existing particles. This seems to go beyond the well known result that they just "happen to fit" into two complete irreducible representations of SU(5) and their conjugates. There are also SU(5)-like properties of the four-point S-matrix as well. The tantalizing question still remains whether these properties indicate a basic underlying symmetry or merely an accidental symmetry like flavor SU(3).

The first indication that all was not well with the flavor SU(3) gauge theory was ω-ϕ mixing. Two inequivalent representations of SU(3) were degenerate for reasons completely outside SU(3) and mixed badly to give states far from SU(3) eigenstates. This should not happen in the gauge theory of the world. Such mixing can test any higher symmetry, but cannot yet test SU(5) because there are no pairs of particles like ω and ϕ classified in inequivalent representations of the symmetry group and allowed to mix by the existing conservation laws. For SU(5) two inequivalent representations are needed with the same pentality. The observed quarks and leptons are classified in the 5, 5*, 10 and 10* representations of SU(5) which all have different values of pentality and whose states cannot mix without violating the conservation laws for the known additive quantum numbers of electric charge, weak isospin and color.

The absence of a pair of states like ω-ϕ which can mix makes unambiguous tests of SU(5) very difficult and explains why so many SU(5)-like results are obtainable without SU(5). Since SU(5) is of rank 4, the same as SU(3) x SU(2) x U(1), all the additive conserved quantum numbers in SU(5) are already in SU(3) x SU(2) x U(1). Thus imposing SU(5) invariance can give no new selection rules based on additive quantum numbers. Since pentality is determined by the eigenvalues of the additive quantum numbers, the pentality selection rule is already implied by SU(3) x SU(2) x U(1). Non-trivial selection rules based on the non-Abelian quantum numbers of SU(5) are possible only when representations other than 5, 5*, 10 and 10* are present, so that couplings exist which are forbidden by SU(5) but allowed by pentality, and mixing can occur. As long as no other representations of SU(5) are present, the only kinds of SU(5) predictions which are not already present without SU(5) are relations between processes like (7a) and (7b) which depend upon SU(5) Clebsches. These are not easily tested.

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TABLE I

QUANTUM NUMBERS OF FIRST GENERATION FERMIONS

Fermion	H	Q	I_3	κ	P	B	β	λ	μ	ν
$\bar{d}_R$	1	+1/3	+1/2	-2	-2/5	-1/3	-1/3	-1/5	0	-1
e^-_R	1	-1	0	-4	-2/5	0	0	-1/5	1	-5
u_R	1	+2/3	0	0	-2/5	+1/3	-2/3	-1/5	-1	+3
$\bar{u}_R$	1	-2/3	-1/2	-2	-2/5	-1/3	-1/3	-1/5	0	-1
e^+_L	-1	+1	0	+4	2/5	0	0	+1/5	-1	+5
$\bar{u}_L$	-1	-2/3	0	0	2/5	-1/3	2/3	+1/5	1	-3
u_L	-1	+2/3	+1/2	+2	2/5	+1/3	+1/3	+1/5	0	1
d_L	-1	-1/3	-1/2	+2	2/5	+1/3	+1/3	+1/5	0	1
e^+_R	1	+1	+1/2	+2	1/5	0	0	+3/5	0	3
$\bar{\nu}_R$	1	0	-1/2	+2	1/5	0	0	+3/5	0	3
d_R	1	-1/3	0	0	1/5	+1/3	+1/3	+3/5	1	-1
e^-_L	-1	-1	-1/2	-2	-1/5	0	0	-3/5	0	-3
ν_L	-1	0	+1/2	-2	-1/5	0	0	-3/5	0	-3
$\bar{d}_L$	-1	+1/3	0	0	-1/5	-1/3	-1/3	-3/5	-1	+1

RIGHT HANDED NEUTRINOS

Fermion	H	Q	I_3	κ	P	B	β	λ	μ	ν
$\bar{\nu}_L$	-1	0	0	+4	1	0	1	1	1	1
ν_R	+1	0	0	-4	-1	0	-1	-1	-1	-1