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WHY MOST FLAVOR DEPENDENCE PREDICTIONS FOR NONLEPTONIC CHARM DECAYS ARE WRONG

FLAVOR SYMMETRY AND FINAL STATE INTERACTIONS IN NONLEPTONIC DECAYS OF CHARMED HADRONS*

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Nonleptonic weak decays of strange hadrons are complicated by the interplay of weak and strong interactions. Models based either on symmetry properties or on the selection of certain types of diagrams are both open to criticism. The symmetries used are all broken in strong interactions, and the selection of some diagrams and neglect of others is never seriously justified. Furthermore, the number of related decays of strange hadrons is small, so that experimental data are insufficient for significant tests of phenomenological models with a few free parameters.

The discovery of charmed particles with many open channels for nonleptonic decays has provided a new impetus for a theoretical understanding of these processes.[1,2] The GIM current provides a well defined weak hamiltonian, which can justifiably be used to first order. The QCD approach to strong interactions gives flavor-independent couplings and flavor symmetry broken only by quark masses. In a model with n generations of quarks and 2n

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flavors, a flavor symmetry group SU(2n) can be defined which is broken only by H_{weak} and the quark masses. Here again, the same two approaches of symmetry and dynamics have been used. But both types of treatment tend to consider only the symmetry properties or dominant diagrams of the weak interaction, including some subtle effects, while overlooking rather obvious effects of strong interactions.[3,4]

A simple example of how strong interactions can completely change flavor dependence predictions is given by the $K^-\pi^+$ and $\bar{K}^\circ\pi^\circ$ decays of the D°. Some treatments suggest that the $\bar{K}^\circ\pi^\circ$ decay mode is strongly suppressed[3,4] relative to $K^-\pi^+$. However, both the $\bar{K}^\circ\pi^\circ$ and $K^-\pi^+$ states are linear combinations of isospin eigenstates with $I = 1/2$ and $I = 3/2$. To see effects of strong interactions, the decay amplitudes should be expressed in terms of these isospin amplitudes. Suppression of the $\bar{K}^\circ\pi^\circ$ mode implies that the two amplitudes nearly cancel in the $\bar{K}^\circ\pi^\circ$ mode and add constructively in the $K^-\pi^+$ mode. This cancelation is changed by final state interactions which shift the relative phases.[5,6]

This phenomenon is seen quantitatively in a simple model[5] which neglects enhancement factors and couplings to inelastic channels and assumes that all final state interactions can be parametrized by phase shift factors, $e^{i\delta_1}$ and $e^{i\delta_3}$. The D° decay amplitudes are

$$A(D^\circ \rightarrow K^-\pi^+) = \sqrt{(1/3)}\ A_3 e^{i\delta_3} - \sqrt{(2/3)}\ A_1 e^{i\delta_1} \tag{1a}$$

$$A(D^\circ \rightarrow \bar{K}^\circ\pi^\circ) = \sqrt{(2/3)}\ A_3 e^{i\delta_3} + \sqrt{(1/3)}\ A_1 e^{i\delta_1} \tag{1b}$$

where A_1 and A_3 denote the $I = 1/2$ and $I = 3/2$ amplitudes when the final state interactions are neglected. The effect of final state interactions on models predicting the suppression of the neutral state (1b) is tested by assuming a complete suppression in the absence of final state interactions. Then

$$A_1 = -\sqrt{2}\ A_3 \tag{2a}$$

and

$$\Gamma(D^\circ \to \bar{K}^\circ \pi^\circ) = \frac{8\Gamma(D^\circ \to K^- \pi^+)}{\{9 \cot^2[(\delta_3 - \delta_1)/2] + 1\}} \quad . \tag{2b}$$

The neutral decay is seen to be suppressed only if $\delta_3 \approx \delta_1$. But the $I = 3/2$ channel is exotic and has no resonances; the $I = 1/2$ channel is not exotic and has many K^* resonances. The D mass is sufficiently close to the resonance region so that the two $K\pi$ phase shifts should be affected very differently by nearby resonances. This is shown dramatically in a recent partial wave analysis of elastic $K\pi$ scattering.[7,8] The $I = 1/2$ s-wave shows a resonance with a mass of 1.5 GeV and a width of 200-300 MeV, giving an s-wave phase at 1.85 GeV varying between 100° and 160° for different solutions. The $I = 3/2$ s-wave shows no resonances and a smooth phase variation well described by an effective range fit with a value around −25° to −30° at 1.85 GeV. For $\delta_3 - \delta_1 = -180°$ the suppression is completely reversed, $\Gamma(D^\circ \to \bar{K}^\circ \pi^\circ) = 8\Gamma(D^\circ \to K^- \pi^+)$, and other predictions[3,4] are drastically modified.

Even above the resonance region there is a considerable energy range in the Regge region where phases of exotic and non-exotic amplitudes are known to be very different. Only at high energies where the Pomeron completely dominates the scattering amplitudes and gives an almost pure imaginary phase can the difference between δ_3 and δ_1 be neglected. Thus the experimental fact that the $\bar{K}^\circ \pi^\circ$ and $K^- \pi^+$ decays are of the same order of magnitude is simply explained by hadronic final state interactions, and any attempts to explain the data only by weak interactions or gluon exchange diagrams without considering hadronic final state interactions and isospin factors are open to serious criticism. Note that the difference between exotic and nonexotic channels is defined by hadron flavor exchange processes and these essential physical features cannot be omitted from any realistic treatment.

A complete description of the nonleptonic decays must take into account such final state interactions by a full dynamical

In the $SU(3)_{uds}$ treatments, for example, the charmed mesons transform like a 3 and H_{weak} has three pieces which transform respectively like a 3^*, a 6 and a 15. Thus there will be independent amplitudes corresponding to all of the representations appearing in the products

$$3 \times 3^* = 1 + 8$$

$$3 \times 6 = 8 + 10$$

$$3 \times 15 = 8 + 10^* + 27 \; .$$

Each of these seven representations can define an independent amplitude for a particular type of final state, unless some are excluded by the allowed final state couplings. In some cases there may even be more amplitudes, as in the case of two nonequivalent octets like vector-pseudoscalar final states, where both D and F type couplings are allowed and independent, and there are ten independent amplitudes.

For the particular case of decays into two octet pseudoscalar mesons, Bose statistics of a 0^+ state excludes the antisymmetric 10 and 10^* representations and uniquely chooses the D coupling for the octet so that only five independent amplitudes remain. But five is still an unwieldy number for extracting the physics of symmetry breaking when the predictions are violated by experiment.

The product $i \times h$ simplifies when the initial state transforms like a singlet under the symmetry group and the relevant terms in H_{weak} are classified in only one irreducible representation. In this case the final state transforms like these relevant terms in H_{weak} and there is only a single amplitude.

As a simple example of this approach, we begin with the isospin group, for which symmetry breaking effects can be completely neglected in the strong interactions. The Cabibbo favored component of the charm-changing part of H_{weak} transforms like the charged components of an isovector. The F^+ meson is a singlet under isospin. It immediately follows that only isovector final states calculation, including enhancement factors and couplings to inelastic channels as well as phase shifts.[6] Since such calculations are not possible at present, two alternative approaches can be used. One is to use phenomenological models together with hadron scattering data and constraints from analyticity and unitarity to estimate the final state interactions.[9] Another is to use subgroups of the full flavor symmetry group SU(2n) which are approximate symmetries of strong interactions, thus automatically taking into account all final state interactions which are invariant under this approximate symmetry.

One popular procedure has been to neglect the mass differences among the light (u,d,s) quarks and to assume that the old $SU(3)_{uds}$ symmetry is a good symmetry broken only by H_{weak}, while rejecting all higher symmetries as being badly broken by masses.[1,2,10,11] In this way, a number of independent SU(3) amplitudes are defined, which are taken as free parameters in fitting the data. The disadvantage of the $SU(3)_{uds}$ approach is that a large number of different amplitudes contribute to any given process, and it is therefore difficult to interpret the underlying reason for any disagreement in fits to data. SU(3) breaking is not easily incorporated into these treatments.

An alternative approach is to use other subgroups of the maximum flavor symmetry, chosen to give simple predictions. The effects of symmetry breaking can then be considered for each individual case, and the simplicity of the predictions makes the underlying physics more transparent. In general, symmetry does not lead to simple predictions because there are too many different independent invariant amplitudes. If the initial state transforms under the symmetry like a member of an i-dimensional representation of the group and H_{weak} transforms like a member of an h-dimensional representation, then the number of independent amplitudes is equal to the number of irreducible representations appearing in the product $i \times h$ which are also allowed for the particular final states considered.

are allowed for Cabibbo-favored F^+ decays and we obtain the well known selection rule,

$$\Gamma(F^+ \to \pi^+\pi^\circ) = 0 \ , \qquad (3)$$

since the spin zero $\pi^+\pi^\circ$ state has isospin 2. This selection rule follows only from the isospin transformation properties of H_{weak} and the isospin invariance for the strong interactions and should be unaffected by SU(3) symmetry breaking, strong final-state interactions or an increase in the number of quark flavors. It is therefore no surprise that this selection rule holds in an SU(3) treatment even when the number of flavors is increased from four to six.[2]

The D^+ and D° mesons constitute an isospin doublet. With an isovector H_{weak}, two values of isospin are allowed for the final state, $I = 1/2$ and $3/2$. Thus isospin gives no simple predictions for D decays; the best obtainable is a triangular inequality relating $D^\circ \to K^-\pi^+$, $\sqrt{2}(D^\circ \to \bar{K}^\circ\pi^\circ)$ and $D^+ \to \bar{K}^\circ\pi^+$. Again it is no surprise that this inequality holds in SU(3) treatments independent of the number of quark flavors.[2] The contrast between this complicated inequality and the selection rule (3) shows the advantage of an initial singlet state.

We next consider U spin, under which the D° meson transforms like a singlet. The importance of U spin for charm-changing nonleptonic decays has been pointed out by Donoghue and Wolfenstein.[12] U spin is particularly useful in the four quark model, in which the charm-changing part of H_{weak} transforms like a pure U spin vector, and the final state in D° decays is pure U vector. The U spin analog of the selection rule (3) is

$$\Gamma(D^\circ \to K^\circ\bar{K}^\circ) = 0 \ . \qquad (4a)$$

Here again the Bose statistics within a multiplet forbids the antisymmetric U spin vector state for a spatially symmetric two-boson system. However, the selection rule (2a) is not as solid as the isospin selection rule (3) for two reasons.

1. U spin symmetry breaking cannot be neglected to the same degree as isospin.

2. H_{weak} transforms like a pure U vector only in the four quark model. With more than four quarks, a U spin scalar component also appears.

The effects of U spin symmetry breaking are similar to those already discussed for an analogous electromagnetic process.[13] We first note that U spin also predicts the following well known equality between charged pion and charged kaon decays of the D°,

$$\Gamma(D^\circ \to K^+K^-) = \Gamma(D^\circ \to \pi^+\pi^-) \; . \tag{4b}$$

The analogous electromagnetic U spin predictions are

$$\sigma(e^+e^- \to \gamma \to K^\circ\bar{K}^\circ) = 0 \tag{5a}$$

$$\sigma(e^+e^- \to \quad K^+K^-) = \quad (e^+e^- \to \pi^+\pi^-) \; . \tag{5b}$$

The two symmetry-breaking mechanisms discussed in connection with the electromagnetic predictions (5) are also directly applicable to the charmed meson decay predictions (4).

1. Violation by SU(3) breaking of equalities or cancellations between pairs of diagrams. The $K^\circ\bar{K}^\circ$ state contains two quark-antiquark pairs, one $s\bar{s}$ and one $d\bar{d}$. In the dominant diagrams contributing to both forbidden reactions (4a) and (5a) one pair is created in a hard electroweak vertex and the other in a soft strong vertex. There are two diagrams in which the roles of the $s\bar{s}$ and $d\bar{d}$ pairs is reversed. In the U spin or SU(3) limit, these two diagrams exactly cancel. The symmetry is broken by the s-d mass difference, which can destroy this cancellation. It is reasonable to assume that the hard electroweak vertices are point-like and are unaffected by the s-d mass difference. But if it is easier to create nonstrange quark pairs out of the vacuum than strange pairs in strong processes, then the diagram in which the $s\bar{s}$ pair is created strongly will not cancel the other diagram

and the selection rule will fail. Whether this U spin breaking is significant at this mass is still an open question, with arguments presented on both sides.[10,13,14]

The equalities (4b) and (5b) do not depend upon such cancellations but upon the equality of contributions from pairs of diagrams in which a $d\bar{d}$ and $s\bar{s}$ pair is created by the hard vertex, and the additional $u\bar{u}$ pair is created in the same way, in both cases either hard or soft. If these diagrams provide the major contribution, the equalities (4b) and (5b) would be less sensitive to symmetry breaking than the selection rules (4a) and (5a). In the electromagnetic case, there is also a dominant diagram in which the $u\bar{u}$ pair is created by the photon and the $d\bar{d}$ and $s\bar{s}$ pairs are created strongly. However, this kind of diagram is absent in the dominant contribution to the D° decays (4b) since the $u\bar{u}$ state is forbidden by U spin for a pure U spin vector state, and additional U spin breaking is required to obtain the diagram in the first place, as in the case of models with more than four quarks discussed below. Once such diagrams are introduced, the mass breaking must also be considered; but the mass breaking along cannot introduce a violation by this mechanism.

2. <u>SU(3) breaking in resonance mass spectra</u>. The predictions (5) are clearly violated at the ϕ mass, where the forbidden reaction (5a) is equal to the allowed production of charged kaon pairs, and there are no charged pion pairs. In the SU(3) symmetry limit, the amplitude for the reaction (5a) via the ϕ would be canceled by contributions from the ρ and ω, and these would also restore a charged pion amplitude satisfying the equality (5b). But because the vector nonet is not degenerate, the relations (5) are strongly violated.

A similar situation clearly obtains for the charmed meson decay predictions (4). If there are any scalar meson resonances near the D° mass which are not in degenerate nonets, the predictions can be strongly violated. Experimental information on s-wave $\pi\pi$ and $K\bar{K}$ scattering amplitudes at the D mass is necessary in order to either take these effects into account properly or to prove that they are negligible. <u>Without such information it is very difficult to trust any calculation which attempts to explain the observed discrepancy between experiment and the prediction (4b)</u>.

The same approach used in Eqs.(1) can be applied to estimate the correction of the selection rule (4a) for final state interactions in the $K\bar{K}$ system. Assuming isospin invariance we define phase shifts δ_0 and δ_1 for the final states of isospin zero and one respectively. Then the amplitudes for the $K\bar{K}$ decays can be written

$$A(D^\circ \to K^+K^-) = (1/2)(A_0 e^{i\delta_0} + A_1 e^{i\delta_1}) \tag{6a}$$

$$A(D^\circ \to K^\circ\bar{K}^\circ) = (1/2)(A_0 e^{i\delta_0} - A_1 e^{i\delta_1}) \tag{6b}$$

where A_0 and A_1 are the amplitudes for the isoscalar and isovector final states when the final state interactions are neglected. The selection rule (4a) implies that $A_0 = A_1$ in the U spin vector approximation. When this is substituted into Eqs.(6a) and (6b) we obtain the correction to the selection rule (4a) due to final state interactions as

$$\Gamma(D^\circ \to K^\circ\bar{K}^\circ) = \Gamma(D \to K^+K^-)\tan^2\left[(\delta_0 - \delta_1)/2\right] \; . \tag{6c}$$

In the SU(3) symmetry limit the isoscalar and isovector phase shifts are equal and the selection rule (4a) is recovered from (6c). However, in view of known symmetry breaking in the structure of isoscalar and isovector $K\bar{K}$ resonances one would not expect that δ_0 and δ_1 would be equal so close to the resonance region.

A scalar resonance denoted by $\varepsilon(1300)$ with a width of 200-400 MeV has been reported under the f meson. If this resonance and the K resonance at 1420 mentioned above are members of an SU(3) nonet, a similar scalar state coupled only to kaons can be expected under the f'(1516). If this resonance has a large width, its tail could still be appreciable at the D mass and affect the decay to the K^+K^- final state with no effect on $\pi^+\pi^-$. A relatively small resonant amplitude interfering constructively with non-resonant background could explain effects of the order of the experimental discrepancies reported for the relation (5b). Until effects of this kind are properly investigated, any attempts to fit the data by introducing new weak interaction contributions are unconvincing.

Predictions from U-spin properties of H_{weak} which are less sensitive to symmetry breaking may be obtained by using the invariance of strong interactions under charge conjugation. The U spin Weyl reflection which interchanges s and d flavors induces the following transformations:

$$K^\circ \leftrightarrow \bar{K}^\circ \tag{7a}$$

$$K^+\pi^- \leftrightarrow K^-\pi^+ \tag{7b}$$

Thus a final state which contains only K° and $\bar{K}^\circ$ mesons together with $K^+\pi^-$ and $K^-\pi^+$ pairs goes into its charge conjugate state under the U spin reflection. Since the D° goes into itself under any U spin transformation, the transformation (7) relates any D° decay into these particles to a D° decay to its charge conjugate state. For example, the assumption that H_{weak} transforms like a pure U spin vector which leads to the relations (4) also gives the relation:

$$\Gamma(D^\circ \to K^\circ K^-\pi^+) = \Gamma(D^\circ \to \bar{K}^\circ K^+\pi^-) \ . \tag{8}$$

Like the predictions (4), this prediction (8) no longer holds if there are more than four quarks, or if there are additional diagrams which introduce a U spin scalar component into the effective H_{weak}. However, the kind of symmetry breaking discussed in connection with Eq.(6) and in particular the effects of resonances should not affect the relation (8). Thus a comparison of the experimental tests of the two predictions (6) and (8) should give an indication of whether the violation of (6) presently observed comes from an additional $U = 0$ component in H_{weak} or from U-spin violating final state interactions. [However, subtle U-spin-symmetry breaking effects can still be present. The contribution of the $K^{*+}K^-$ state to $K^\circ K^-\pi^+$ is not balanced by the U-spin-reflection contribution of $\rho^+\pi^-$ to $\bar{K}^\circ K^+\pi^-$ because the $\bar{K}^\circ K^+$ channel is closed for ρ^+ decay.]

Additional predictions are obtainable from U spin reflections which relate Cabibbo allowed transitions to doubly unfavored transitions. These follow from the property of the terms in H_{weak} which generate these transitions as being related by U spin reflection. Consider the $\Delta C = 1$ part of H_{weak} in the notation of Quigg

$$\begin{aligned}\mathscr{H}(\Delta C = 1) = \{\bar{c}s,\bar{d}u\}V_{11}V_{22} + \{\bar{c}s,\bar{s}u\}V_{12}V_{22} \\ + \{\bar{c}d,\bar{d}u\}V_{11}V_{21} + \{\bar{c}d,\bar{s}u\}V_{12}V_{21}\end{aligned} \tag{9}$$

The first and fourth terms of (9) are seen to be two components of the same U spin vector which go into one another under the U-spin Weyl reflection, except for the difference in the coefficients. These terms describe Cabibbo favored and doubly unfavored transitions respectively. We thus obtain the prediction that any pair of favored and doubly unfavored transitions which go into one another under the U spin reflection satisfy the relation,

$$\frac{\Gamma(D^\circ \to \tilde{f})}{\Gamma(D^\circ \to f)} = \frac{\Gamma(D^+ \to \tilde{f}')}{\Gamma(F^+ \to f'')} = \frac{\Gamma(F^+ \to \tilde{f}'')}{\Gamma(D^+ \to f'')} = \left[\frac{V_{12}V_{21}}{V_{11}V_{22}}\right]^2 \tag{10}$$

where f, f' or f'' denotes any Cabibbo favored final state for the decay considered, and $\tilde{f}$ denotes the doubly unfavored state obtained from f by a U spin reflection.

For the case where the final states contain only the mesons K° and $\bar{K}^\circ$ and the meson pairs $K^+\pi^-$ and $K^-\pi^+$, relations are obtained between final states which are charge conjugates and where effects of final state interactions can be expected to be much smaller. For the case of final states of two pseudoscalar mesons, the π° and η also appear in simple U-spin equalities because the contribution from the $U = 1$ mixture of π° and η vanishes as a result of a selection rule related to the selection rule (4a) by a U spin rotation. Thus we obtain the relations,

$$\frac{\Gamma(D^\circ \to K^+\pi^-)}{\Gamma(D^\circ \to K^-\pi^+)} = \frac{\Gamma(D^\circ \to K^\circ\pi^\circ)}{\Gamma(D^\circ \to \bar{K}^\circ\pi)} = \frac{\Gamma(D^\circ \to K^\circ\eta)}{\Gamma(D^\circ \to \bar{K}^\circ\eta)} = \left[\frac{V_{12}V_{21}}{V_{11}V_{22}}\right]^2 \qquad (11)$$

These predictions would be unaffected by a treatment of final state interactions like Eq.(1a) if the phase shifts are invariant under charge conjugation.

Note that for the particular case of the four quark model, the quantity appearing on the right hand sides of Eqs.(10) and (11), $V_{12}V_{21}/V_{11}V_{22} = \tan^2\theta_C$, where θ_C is the Cabibbo angle. However, the results hold equally well for models with more than four quarks with the parameters in the Hamiltonian (9) taking on the values in the particular model. It is only the second and third terms in the Hamiltonian which describe the Cabibbo singly unfavored transitions which change their U spin transformation properties and introduce a $U = 0$ component when more than four flavors are present. This is discussed in detail below. Unfortunately, relations like (10) and (11) which involve doubly unfavored transitions are more difficult to test experimentally because of the low rates for these transitions.

Doubly unfavored transitions can be enhanced in decay modes with neutral kaons, since the states actually detected are not K° and $\bar{K}^\circ$ but K_L and K_S. These coherent mixtures of K° and $\bar{K}^\circ$ offer the possibility of measuring interference between the favored and doubly unfavored amplitudes,[16] thereby obtaining a signal which is linear in the small amplitude rather than quadratic. However, the determination of the contribution of this interference term requires an additional measurement to normalize the direct term. This is possible in the case of the relations (11) only if both the K_L and

K_S decay modes are measured. The relation (11) then predicts that the K_L and K_S modes are no longer equal, but have a difference proportional to the square root of the right hand side of (11). If there is a nontrivial relative phase between $V_{12}V_{21}$ and $V_{11}V_{22}$ due to CP violation, the decays $D^\circ \to K_S M^\circ$ and $\bar{D}^\circ \to K_S M^\circ$ would no longer be equal, where M° is any neutral pseudoscalar meson. This might be a possible test for CP violation.

We now generalize this approach to the use of larger flavor symmetry groups. Predictions from such higher symmetries are quite likely to be violated. However, if the predictions are simple, it may be possible to analyze the symmetry breaking mechanisms, in a manner similar to the above examples. In this way it might be possible to restore communications between treatments which use only symmetries and treatments which use only dynamical diagrams and which normally ignore one another. To obtain simple predictions, we look for subgroups of the general flavor symmetry group SU(2n) to find those in which initial states are classified as singlets.

A natural generalization of the U spin group $SU(2)_{ds}$ which acts only on d and s quarks is the group which acts on all "down-type" quarks with charge -1/3, and which we denote by $SU(n)_d$. This is just U spin for a four quark model and is $SU(3)_{dsb}$ for a six quark model. We can also define the analogous group $SU(n)_u$ which acts only on "up-type" quarks with charge +2/3. States which contain no "down-type" quarks are singlets in $SU(n)_d$ and simple predictions are obtained for their decays using $SU(n)_d$, as we have already seen from U spin which is the special case of $n = 2$. The same is true for $SU(n)_u$ and decays of states which contain no "up-type" quarks and are singlets in $SU(n)_u$. But charmed mesons contain at least one "up-type" quark and cannot be singlets in $SU(n)_u$; thus only $SU(n)_d$ singlets may be found. For mesons containing b quarks the opposite is true and $SU(n)_u$ may be useful in nonleptonic decays of such states. We do not consider this possibility further here and restrict our treatment to decays of charmed mesons.

We now investigate the transformation properties of H_{weak} under $SU(n)_d$. We first note the chain of subgroups of SU(2n),

$$SU(2n) \supset SU(n)_u \times SU(n)_d \supset SU(n)_C$$

$$SU(2n) \supset SU(n)_u \times SU(n)_d \supset SU(n)_M$$

where $SU(n)_C$ and $SU(n)_M$ are groups obtained by combining the "up" and "down" groups into a single "horizontal symmetry" in two ways which differ by a generalized Cabibbo rotation. $SU(n)_C$ is the group which transforms the n quark doublets into one another, where each quark doublet is defined by the weak current. $SU(n)_M$ is the analogous group in which the quark doublets are defined by eigenstates of the mass matrix. The flavor-changing parts of the generalized GIM current for n generations each contain a single "up" and a single "down" type quark operator. They thus transform like the fundamental representations of $SU(n)_u$ and $SU(n)_d$ respectively; e.g. like the 3 and 3* in a six quark model. The current-current interactions in H_{weak} therefore transforms like a linear combination of singlet and adjoint representations. However, the interaction hamiltonian must transform like a singlet of $SU(n)_C$ to give the GIM cancellations. Thus the representations of $SU(n)_u$ and $SU(n)_d$ in $SU(n)_u \times SU(n)_d$ must be the same, either both singlet or both adjoint. The charm-changing part of H_{weak} cannot be a singlet under $SU(n)_u$, and must transform like conjugate members of the adjoint representation (octet for a six quark model) for $SU(n)_u$ and $SU(n)_d$.

We now note that the neutral charmed mesons as well as charmed baryons containing only c and u quarks must be classified in the singlet representation of $SU(n)_d$, since they have no down-type quarks. Thus decays of these states into states constructed from the same $SU(n)_d$ multiplets are described by a single amplitude, one which transforms like the adjoint representation of $SU(n)_d$.

This can be seen explicitly by noting that the $\Delta C = 1$ part of the Hamiltonian (9) transforms like a member of the adjoint representation of $SU(n)_d$, as the $\bar{c}$ and u quarks are singlets in $SU(n)_d$. For the decay of a neutral charmed meson, or a charmed baryon containing only c and u quarks, the final state transforms under $SU(n)_d$ like (9) if there is no additional flavor symmetry breaking. Thus the final state transforms like the quark-antiquark pair state,

$$|\Psi_f\rangle = V_{11}V_{22}|s\bar{d}\rangle + V_{12}V_{22}|s\bar{s}\rangle + V_{11}V_{21}|d\bar{d}\rangle + V_{12}V_{21}|d\bar{s}\rangle \,. \quad (12)$$

Note that in the conventional GIM four-quark model, where $V_{11} = V_{22} = \cos\theta_C$ and $V_{12} = -V_{21} = \sin\theta_C$ and θ_C is the Cabibbo angle, the coefficients of the $s\bar{s}$ and $d\bar{d}$ terms in (12) are equal and opposite. Thus Ψ_f is a pure U spin vector, and this leads to a number of well known relations between decays. For more than four quarks the coefficients of $|s\bar{s}\rangle$ and $|d\bar{d}\rangle$ are no longer simply related, and there is also a U-spin scalar component in Ψ_f.

This U spin property can also be seen by noting that the state (12) is the projection into the sd subspace of a state which transforms like the adjoint representation of $SU(n)_d$. For the four quark model the sd subspace is the entire space of $SU(2)_d$ and the requirement that a state which transforms like a member of the adjoint representation is orthogonal to the singlet makes the coefficients of $|s\bar{s}\rangle$ and $|d\bar{d}\rangle$ equal and opposite. When the space becomes larger, the orthogonality no longer relates these coefficients, since orthogonality with the singlet can always be fixed by adjusting the coefficient of the $|b\bar{b}\rangle$ term in the wave function which has been omitted from (12) as irrelevant for the decays under consideration.

We can now see the interplay of different types of flavor symmetry breaking. For the case where no flavor symmetry breaking is assumed other than that in H_{weak}, Eq.(9) defines a definite linear combination of U-spin scalar and vector components, with coefficients which are completely determined by the elements V_{ij} of the quark mixing matrix. Thus the breaking of the simple U-spin equalities of the four-quark model$\big($e.g. $D^\circ \to K^+K^- = D^\circ \to \pi^+\pi^-$, Eq.(4b)$\big)$

is completely fixed by these parameters. If flavor symmetry breaking is assumed but $SU(3)_{uds}$ is still assumed to be unbroken, as in the treatments of Refs.1-2, then U spin is conserved, but the U spin scalar and U spin vector components of the final state are no longer related and are described by two independent amplitudes. This can be seen in Quigg's Table I, where there are five independent reduced matrix elements in the SU(3) analysis of charmed meson decays. However, only two linear combinations appear in the decays of D°'s into charged pions and kaons, namely the combinations $2T+E-S$ and $3T+2G+F-E$, corresponding to the U spin vector and scalar components of the final state.

We now consider the possible effects of mass breaking on $SU(n)_d$ flavor symmetry. Since our symmetry only involves negatively charged quarks, there are two distinct mass differences which break the symmetry. For simplicity we consider the six-quark model. The generalization to more generations is trivial. The relevant mass differences are:

1. The d-s mass difference. This also breaks old-fashioned SU(3) and is neglected in the conventional SU(3) treatments.

2. The mass difference between the b quark and the light quarks.

Note that in the tree approximation, no b quarks appear in charmed meson decays, since they are not present in either the initial nor final states. Thus the breaking of $SU(3)_d$ by the high mass of the b quark is irrelevant in the tree approximation. The d-s mass difference can appear in the tree approximation, in diagrams where $d\bar{d}$ or $s\bar{s}$ pairs are produced from colored gluons. Flavor symmetry requires that such pairs be produced with equal amplitudes, but the mass difference can suppress strange quark production. However, no such pair production occurs in the tree diagrams for two-body D° decays into charged kaons and pions. Thus flavor symmetry breaking can be inserted here only when diagrams involving loops are important.

We now apply this formulation to the particular case of D° decays into charged pions and kaons. This case is particularly simple since the charged pions and kaons are in U spin doublets and in the fundamental representations of $SU(n)_d$. There is thus only a single amplitude for the four final states. The results can be read off immediately from Eq.(12), since the two meson final state is obtained from the wave function (12) by simply adding a $u\bar{u}$ pair which is a singlet under $SU(n)_d$. The final state in the charged pion-kaon pair sector is then

$$\Psi_f = V_{11}V_{22}|K^-\pi^+\rangle + V_{12}V_{22}|K^-K^+\rangle + V_{11}V_{21}|\pi^-\pi^+\rangle + V_{12}V_{21}|\pi^-K^+\rangle \tag{13a}$$

$$= V_{11}V_{22}|K^-\pi^+\rangle + (\Sigma+\Delta)|K^-K^+\rangle - (\Sigma-\Delta)|\pi^-\pi^+\rangle + V_{12}V_{21}|\pi^-K^+\rangle \tag{13b}$$

where Σ and Δ are the linear combinations of $V_{11}V_{21}$ and $V_{12}V_{22}$ defined in Ref.1 which project out the U spin vector and scalar parts of the wave function. Note that Δ vanishes in the four quark model where (12) is a pure U spin vector and that Δ is required to be small in the general case in order to fit the experimental information available on the quark mixing matrix. Estimates of the ratio $|\Delta/\Sigma| \leq 1/15$ have been given in the literature.[1] With this value it is impossible to fit the observed branching ratios with the wave function (13).

Note that the wave function (13) corresponds to the following relation between Quigg's amplitudes for the $U=1$ and $U=0$ components of the final state

$$2T + E - S = (1/2)(3T + 2G + F - E) \ . \tag{14}$$

The observed branching ratio, which gives a much higher K^+K^- decay relative to $\pi^+\pi^-$ than indicated by the wave function (13) can be fit in Quigg's formulation by using values of the amplitudes which do not satisfy the condition (14). This corresponds to enhancing the $U=0$ amplitude on the right hand side of (13) relative to the $U=1$ amplitude on the left. From our formulation we see that this violation of (14) cannot take place in the tree approximation, and we are led to consider diagrams involving loops. There are two kinds of loops:

1. A quark-boson loop, in which a quark emits a W and then absorbs it, changing flavor in the process. This is often called the penguin diagram. In the symmetry limit the $c \to u$ transition is forbidden; the three diagrams $c \to d \to u$, $c \to s \to u$ and $c \to b \to u$ exactly cancel. Mass differences can destroy this cancellation and give a contribution which breaks the symmetry. Whether such a diagram can quantitatively explain the data is beyond the scope of the symmetry treatment and is left for specific model builders. Note, however, that in this case the $d\bar{d}$ or $s\bar{s}$ pair in the final state must be created by a gluon from the vacuum. This pair will then be in a $U=0$ state if U-spin breaking is neglected. The contribution to the final state from this loop diagram can be expressed in Eq.(13) as a modification of the parameter Δ from the value given by the quark mixing matrix. If this contribution has the proper phase to increase Δ and the proper magnitude, it could enhance the K^+K^- decay. However, it does not seem reasonable to calculate this diagram without considering U-spin breaking, because there are strong indications that it is easier for a gluon to create a non-strange quark pair from the vacuum than a strange quark pair. This U spin breaking could be estimated by using data from other decays into strange and nonstrange channels.

2. Quark loops. To explain the difference between K^+K^- and $\pi^+\pi^-$ decays, loop diagrams are needed in which $s\bar{s}$ and $d\bar{d}$ pairs are annihilated and created. However, these can simply be called final state interactions in the $\pi\pi$ and $K\bar{K}$ systems. This immediately leads to the question of the behavior of the $\pi\pi$ and $K\bar{K}$ s-wave scattering amplitudes in the vicinity of the D° mass, which has been discussed in detail above in connection

with the simple U spin prediction (4b). Without additional information on the strong amplitudes in this region, models of weak interaction cannot be tested convincingly in these decays.

We now consider the application of $SU(n)_d$ to other charmed hadron decays looking for singlet states. All doubly charged baryons B^{++} must contain three quarks with charge 2/3 and none of charge -1/3 and are therefore all singlets in $SU(n)_d$. All states of such a baryon and two charged mesons, $B^{++}M_1M_2$ transform under $SU(n)_d$ like the meson component M_1M_2, since the baryon B^{++} is invariant. Thus the requirement that the final state must transform under $SU(n)_d$ like the state (12) immediately leads to the result:

$$\frac{\Gamma(B_i^{++} \to B_f^{++} M_1 M_2)}{\Gamma(B_i^{++} \to B_f^{++} M_1' M_2')} = \frac{\Gamma(D^\circ \to M_1 M_2)}{\Gamma(D^\circ \to M_1' M_2')} , \tag{15}$$

where B_i^{++} and B_f^{++} are doubly charged baryons which can be initial and final states in a charm-changing decay; e.g. (B_i^{++}, B_f^{++}) can be (cuu, Δ^{++}) or (ccu, cuu), and (M_1, M_1') and (M_2, M_2') are pairs of charged meson states in the same U spin multiplet. These meson states can also be meson resonances, and M_i and M_i' can be the same state.

From the observation that (p, Σ^+) transform under U spin and $SU(n)_d$ like (π^-, K^-), we obtain

$$\frac{\Gamma(D^\circ \to M_1 K^-)}{\Gamma(cuu \to M_1 \Sigma^+)} = \frac{\Gamma(D^\circ \to M_2 K^-)}{\Gamma(cuu \to M_2 \Sigma^+)} = \frac{\Gamma(D^\circ \to M_3 \pi^-)}{\Gamma(cuu \to M_3 p)} \tag{16}$$

where M_1, M_2 and M_3 are any positive meson states, including meson resonances, which are all in the same U-spin multiplet. Unfortunately the doubly charged (cuu) charmed baryon baryon seems to have a mass above the threshold for the decay into $\Lambda_c \pi^+$, so that weak decay branching ratios are very small.

We thus see a one-to-one correspondence between D° decays into charged mesons and the decays of doubly charged baryons into states containing two particles in the final state which are members of U spin doublets and possibly additional U spin singlet particles. This follows from the Wigner-Eckart theorem for $SU(n)_d$, which states that the matrix element for the transition is given by a reduced matrix element multiplied by a Clebsch-Gordan coefficient for $SU(n)_d$. This does not apply to final states containing neutral mesons because these are classified in the adjoint representation of $SU(n)_d$; e.g. the octet in a six-quark model. Because there are two couplings (commonly called D and F) for this case, there are two allowed $SU(n)_d$ amplitudes for this case and no simple predictions are obtained. In the D° decays, simple predictions are still obtained because Bose statistics for a 0^+ state of two identical octets requires the D-type coupling. For the baryon decays there are no such constraints, unless two mesons in a final state like those of Eq.(14) are in a definite partial wave which requires D-type coupling. This complication does not arise with charged mesons, because they all contain one u-type quark or antiquark and one d-type, and are therefore classified always in the fundamental representation of $SU(n)_d$. The coupling of the fundamental representation and its conjugate to the adjoint representation is always unique.

Further relations between charmed meson decays are obtainable directly from Quigg's tables and the additional relation (14). In each case the symmetry breaking must be examined individually on some dynamical basis, analogous to the discussion above for Eqs.(4-6). If the $SU(n)_d$ symmetry breaking mechanism conserves U spin, Quigg's results are valid with no additional constraints like (14) on the amplitudes. The relation (14) between the $U=0$ and $U=1$ amplitudes in the final state no longer holds, but $SU(3)_{uds}$ symmetry is still good since isospin is always assumed to be good, and U spin combined with isospin is equivalent to $SU(3)_{uds}$. Alternatively one can keep the relation (14) and correct for the strong interaction breaking by explicit models; e.g. introducing phase shifts for the $U=0$ and $U=1$ channels, analogous to the isospin phase shifts used in Eq.(6). It may be possible to obtain bounds on the effects of symmetry breaking by imposing general constraints like analyticity and unitarity on these phase shifts or by reasonable extrapolation of experimentally measured phase shifts at lower energies.

In each case, however, the question arises of whether the $SU(n)_d$ symmetry breaking which conserves $SU(3)_{uds}$ is sufficiently stronger than the breaking of $SU(3)_{uds}$ itself to justify taking one into account and neglecting the other. For the case where the final state is dominated by a non-degenerate nonet of resonances, the breaking of $SU(3)_{uds}$ cannot be neglected. But there may be cases where such mixing of U spin eigenstates is not important and $SU(3)_{uds}$ conserving mechanisms which enhance the small $U=0$ component in the wave function (13) could be crucial.

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