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CONDENSATION OF $(G^a_{\mu\nu})^2$ IN QUANTUM CHROMODYNAMICS†

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This talk is based on the results obtained in collaboration with R. Fukuda.[1]

In the following, I wish to discuss the property of the QCD vacuum by focusing on the expectation value of the color-singlet composite operator $(G^a_{\mu\nu})^2$, the square of the gluon field strength tensor. Just as the $\bar{q}q$ operator plays the primary role in studying the "quark content" of the vacuum, especially its chiral property, $(G^a_{\mu\nu})^2$, measuring the "gluon content" of the vacuum, gives us a valuable bit of information on its complex structure. Theoretically, if one assumes importantce of certain field configurations, such as dilute gas of instantons,[2] one finds $\langle 0|(\alpha_s/\pi)(G^a_{\mu\nu})^2|0\rangle$* to be non-zero and positive. On the other hand, through the QCD sum rules[3] this quantity is directly related to the properties of physical resonances and has been determined experimentally to be $\sim 0.012\ \mathrm{GeV}^4$.

Now a natural and an important question is whether we can deduce from QCD that $(G^a_{\mu\nu})^2$ condenses quite generally without assuming dominance of certain configurations. We have addressed ourselves to this question and have the answer in the affirmative.[1] In the true non-perturbative vacuum of QCD, $(G^a_{\mu\nu})^2$ condenses with positive (magnetic) sign.

The standard method to study condensation of an operator, call it $\hat{\phi}$, is to construct the effective potential for it by first computing the generating functional in the presence of a source term $J\hat{\phi}$ and then Legendre-transforming the result. This procedure, although straight forwardly implementable in such theories as Gross-Neneu model[4] or O(N)-symmetric $\lambda\phi^4$ model[5] in the large N limit, is next to impossible to carry out for QCD at present. This calls for a trick. A trick is provided by the trace anomaly equation[6,7] in the presence of the constant source J coupled to the operator $\hat{\phi}\equiv\int d^4x\ 1/4(G^a_{\mu\nu})^2$. Let us recall that in QCD with massless quarks, the trace anomaly equation, in the vacuum state, takes the form $\langle 0|\theta^\mu_\mu|0\rangle=(2\beta(g)/g)\langle 0|\ 1/4(G^a_{\mu\nu})^2|0\rangle$. Because of the Poincaré invariance of the vacuum, the left hand side is nothing but 4 times the energy density $\varepsilon=\langle 0|\theta_{00}|0\rangle$. Thus if we can derive a similar equation in the presence of J, we will obtain $\varepsilon(J)$, from which the effective potential can be constructed.

Our ability to derive such an equation rests on the fact that the operator $\hat{\phi}$ appears already in the action. In the generating functional Z=expiW, the addition of the above source term amounts to a simple change $\int d^4x\ 1/4(G^a_{\mu\nu 0})^2\rightarrow(1+J_0)\int d^4x\ 1/4(G^a_{\mu\nu 0})^2$ (where 0 signifies bare quantities). In the axial gauge specified by the constraint $\delta(\eta_\mu A^\mu_0)$, this $(1+J_0)$ factor in turn may be eliminated by

*It is to be emphasized that, throughout, $\langle 0|(G^a_{\mu\nu})^2|0\rangle$ means the difference between $(G^a_{\mu\nu})^2$ in the true vacuum and in the perturbative vacuum.

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the rescaling $g_{J0}^2 \equiv g_0^2/(1+J_0)$, $A_{J0}^{\mu} \equiv (1+J_0)^{1/2} A_0^{\mu}$, and we are left with the generating functional identical in form to the one without the source. This we know how to renormalize and we can derive the anomaly equation from it. There is however one complication, which can be readily seen by expanding $W(J)$ in powers of J; $W(J_0)=W(0)+J_0\delta W/\delta J_0+\frac{1}{2}J_0^2\delta^2 W/\delta J_0^2+\ldots$. As $(-\delta/\delta J_0)$ effects an insertion of a "hard" operator, we need to remove the extra infinities caused by the multiple insertions of ϕ_0. This turns out to lead to mixing of infinite number of operators $\{\phi_0^n\}_{n=1,2,\ldots,\infty}$ under renormalization. Without giving the details let me just assert that this is handled by renormalizing J by a Z-factor which itself depends on the source: $J_0=JZ_J[J]=J(Z_J^{(0)}+JZ_J^{(1)}+J^2Z_J^{(2)}+\ldots)$. Here the renormalized source J is defined such that $(-\delta/\delta J)^n W(J)|_{J=0}$ gives the renormalized n-fold insertion of $\hat{\phi}$. This evidently is quite analougous to the case of the usual coupling constant renormalization and is of no mysterious nature.

Now that we have done away with technical complications, we can follow the analysis of Ref. 7 and get the desired anomaly equation. For the vacuum state it reads

$$\frac{1}{4}\langle 0|\theta^{\mu}_{\mu J}|0\rangle = \varepsilon(J) = (\beta(g_J)/2g_J)(1+J)\phi \quad , \tag{1}$$

where $g_J^2=g^2/(1+J)$ and $\phi\equiv(-\delta/\delta J)W(J)$. This is an exact result, including, in particular, long wave length excitations. With $\varepsilon(J)$ at hand, we can perform the Legendre transform and obtain the effective potential

$$V(\phi)=(\beta(g_J)/2g_J)(1+J)\phi-J\phi \ . \tag{2}$$

It seem at first that we still do not know how to compute $\varepsilon(J)$ or $V(\phi)$. The important observation is that ϕ is the independent variable so that we must substitute $J=-\partial V/\partial\phi$ everywhere J appears in the above equation. We then, realize that it is a non-linear differential equation for $V(\phi)$, which may be solved at least for small g. With $\beta(g_J)=-b_0g_J^3-b_1g_J^5-\ldots$, we get the solutions sketched in Fig. 1. (For $\phi<0$, the energy will only go up.) Referring the details to the original paper, we see from this figure that we have a stable non-perturbative vacuum for which $\phi>0$. (Stationary value for the upper curve, e.g., is $g^2\phi=\mu^4\exp(-c-1-2/b_0g^2)$, c is a constant, which clearly shows the non-perturbative nature of the vacuum.) This is the result announced in the beginning. It is worth emphasizing that the negative character of β function is crucial for this result.

A brief remark would be helpful here: a similar potential obtained in the past by various people[8] should not be confused with the one we have discussed. Their potential is not for the composite singlet operator $(G^a_{\mu\nu})^2$, but for A^a_μ, the octet vector potential. Vacuum expectation value of the composite operator is what is relevant for QCD sum rules.

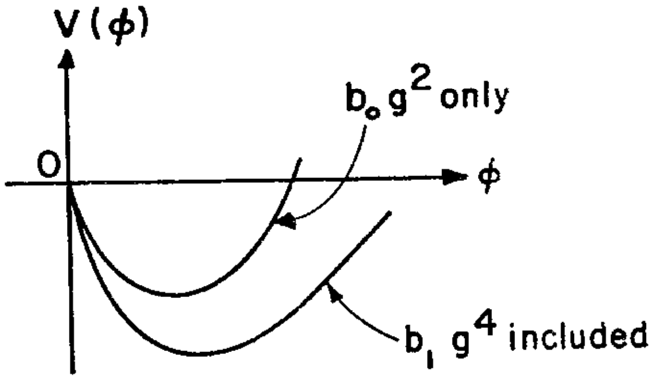

Fig.1. Effective potential corresponding to two different truncations of β function

In the remainder, I would like to describe how the condensation of $(G^a_{\mu\nu})^2$ is related to confinement. For the confinement problem a useful object is the vacuum average of the Wilson loop operator

$\Psi(C)\equiv\langle\Psi(C)\rangle\equiv\langle 1/N\,\mathrm{Tr}P \exp ig\oint_C A_\mu dx^\mu\rangle$, where C denotes a closed loop.

Recently it has been shown[9] that this object can be made finite by the usual renormalization procedure once one isolates the self-energy of the test particle. Then, in the axial gauge the renormalized $\Psi(C)$ satisfies the renormalization group equation, $(\mu\partial/\partial\mu+\beta(g)\partial/\partial g)\Psi(C)=0$. Let us for simplicity take the loop to be a circle of radius r. Noting that $g^2\partial/\partial g^2$ effects an insertion of $\hat{\phi}$, after Wick rotation, the above equation can be readily be transformed into

$$(\partial/\partial\sigma+M^2(\sigma))\Psi(C)=0 \ , \tag{3}$$

where σ is the area of the loop and $M^2(\sigma)=(\beta(g)/g\sigma)\Delta\Phi$, $\Delta\Phi\equiv\int d^4x(\phi_c(x)-\phi)$. Here $\phi_c(x)(\phi)$ is the vacuum expectation value of $\phi(x)$ in the presence (absence) of the loop, i.e. $\phi_c(x)=\langle\phi(x)\Psi(C)\rangle/\langle\Psi(C)\rangle$. This equation tells us that the area dependence of the Wilson loop is governed by the integrated difference of condensation with and without the loop. If the local difference tends to zero rapidly away from the loop, $\Delta\Phi$ would be proportional to σ (i.e. $M^2(\sigma)\sim$ constant) and one obtains so called the area law. The fact that the difference enters is of great importance for the correct sign. The Wilson loop being a source of chromo-electric flux, $\phi_c(x)$ near the loop is less magnetic than φ i.e., $\Delta\Phi<0$. This together with the negative character of β function gives the desired sign for the area dependence.

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