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A PHENOMENOLOGICAL MODEL FOR PSEUDOSCALAR MESON MIXING†

Isaac Cohen*
Weizmann Institute of Science
Rehovot, Israel

and

Harry J. Lipkin**
Weizmann Institute of Science
Rehovot, Israel

and

High Energy Physics Division
Argonne National Laboratory
Argonne, Illinois 60439

and

Fermi National Accelerator Laboratory
Batavia, Illinois 60510

ABSTRACT

Conventional models of η-η' mixing are extended to include radial excitations in the mass matrix. The number of free parameters is minimized by using the Quigg-Rosner logarithmic potential to obtain properties of radially excited states. The remaining parameters are fixed by fitting observed masses. The wave functions obtained by diagonalizing the mass matrix give an η similar to the conventional description, 99% ground state and mainly octet but a very different η' with a 50% admixture of radial excitations, and the $s\bar{s}$ configuration dominant in the ground state component. This η' wave function modifies the bad quark model predictions for peripheral production by introducting form factors which lead to new predictions in agreement with experiment. Additional isoscalar pseudo-scalar states are predicted around 1280 and 1500 MeV. Applications to radiative vector meson decays and charmonium are also discussed.

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*On leave of absence from Departamento de Fisica, Facultad de Ciencias, Universidad Central de Venezuela, Caracas, Venezuela.

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1. Introduction

Theoretical and experimental investigations of the $\eta - \eta'$ systems[1-6] indicate that the conventional mixing description is inadequate and that additional components are present in the wave functions of the physical particles. The admixture of radially excited quark-antiquark wave functions into the lowest state has been suggested,[1-3] since the "annihilation" interaction which mixes strange and nonstrange pairs and causes the deviation from ideal mixing should also mix radial excitations.

In this paper we attempt to estimate the effect of the radial mixing on the cross sections for peripheral production of pseudoscalar mesons. Mixing introduces a form factor which suppresses the production process, because the overlap of the initial and final meson wave functions is no longer unity. It has been pointed out that such a suppression of η' production in charge exchange and strangeness exchange processes is in the proper direction to eliminate the discrepancies found in quark model and SU(3) predictions, which all assume form factors of unity, but quantitative estimates of the effect have not been given. The principal difficulty is to find a credible model which does not have too many free parameters and can use other experimental data to define the amount of radial mixing.

We consider a phenomenological model which extends the standard mixing models of Isgur[7] and DGG[8] to include radially excited states. Only quark-antiquark configurations are considered, and more exotic components such as glue balls[6] or four quark configurations[9] are neglected. Simple phenomenological interactions are used with a minimum of free parameters in order to enable significant predictions. The key ingredient in reducing the number of parameters is the Quigg-Rosner logarithmic potential[10] which provides input information for the radially excited states, namely the excitation energies of these states and the dependence of the matrix elements of the annihilation interaction on the radial excitation quantum numbers. We do not take the log potential seriously nor justify the use of the nonrelativistic quark model for light quarks. We simply assume that the light quark mesons are described to some approximation by quark-antiquark pair wave functions, that radial excitations exist, and that global properties of these excitations like the excitation energies and matrix elements of the annihilation interaction can be estimated to a reasonable approximation by the log model, except for an overall strength parameter which is determined by fitting data. All free parameters are fixed by fitting experimental meson masses which are greater in number than the number of parameters. Very good fits are obtained, but we do not emphasize the mass fits at this stage, since they may be due to general symmetry properties of the model rather than to details. Care must be taken before using the excellence of these fits to justify the details of the model.

The mass fits are used to determine the wave functions and the amounts of radial mixing. We then re-examine predictions for meson production and decay processes based on the old quark model and find that radial mixing removes the disagreements with experiment of the old model. The following interesting new predictions are also obtained:

1. Higher pseudoscalar states. An excited pseudoscalar meson should exist at a mass of about 1280 MeV with properties similar to the η and η'. We denote this by "D". It should be produced in peripheral πN and KN reactions with cross sections comparable to the η and η', and should also be seen in radiative Ψ decays $\Psi \to "D"\gamma$ comparable to $\Psi \to \eta'\gamma$. It is tempting but not necessary to identify this pseudoscalar with the D(1280), since it should have a strong $\eta\pi\pi$ decay mode with an appreciable fraction being $\delta\pi$. An additional state is predicted at about 1500 MeV. Recent experimental evidence for such states has been reported[11].

2. Including $c\bar{c}$ states in the formalism leads to mixing of a small amount of $c\bar{c}$ into the η and η' wave functions which are in qualitative agreement with the strengths of the $\psi \to \eta\gamma$ and $\Psi \to \eta'\gamma$ decays.

3. Form factors should also be present in radiative vector meson decays $V \to P\gamma$. These should be particularly important for decays like $\eta' \to \rho\gamma$.

4. Radial mixing is also produced by the spin-spin (hyperfine) interaction, assumed to be very short range like the Fermi-Breit interaction. The experimental fact that the ρ-π splitting is of the same order as orbital splittings indicates that simple first order perturbation theory is inadequate for the treatment of spin splittings and that higher order effects can be important. Explicit diagonalization of the mass matrix in the models considered shows that the first order contribution is only about 2/3 of the exact spin splitting. Thus using the experimental ρ-π splitting in a first order treatment to fix the magnitude

of the hyperfine interaction overestimates the interaction and leads to errors in the predictions of other hyperfine splittings; e.g. in the $\Psi - \eta_c$ splitting.

5. Fixing the parameters in the $c\bar{c}$ sector to fit the Ψ mass leads to a predicted mass for the η_c considerably above the usually accepted value and possible even above the Ψ. The exact value of the mass depends upon extrapolations from the light quark region to charmonium and may not be reliable. But the presence of an additional effect neglected in conventional charmonium spectroscopy which must push up the η_c mass is clearly indicated[12].

We use two models to check the radial mixing, one in which a squared mass matrix is diagonalized and one in which a linear mass matrix is diagonalized. That qualitatively similar mixings are obtained in these two different models suggests that the basic physics of the mixing is correctly described and nearly model independent. Our approach is to use existing models with existing parameters wherever possible and to introduce the minimum modifications necessary to include radial mixing; i.e. including radially excited states in the mass matrix and using a simple prescription for determining matrix elements for these states.

2. The Extended Isgur Model

The first model considered is a simple extension of Isgur's model,[7] in which the matrix elements of the mass squared operator are given by the expression

$$\langle q_{a'} \bar{q}_{b'} n' | M^2 | q_a q_b n \rangle = \delta_{aa'} \delta_{bb'} \delta_{nn'} \left[(\bar{m}_a + m_b + E_n)^2 + S \right] + \delta_{ab} \delta_{a'b'} A/nn' \tag{1}$$

where $|q_a\bar{q}_b n\rangle$ denotes a state of a quark of flavor a, and an antiquark of flavor b in four states of radial excitation specified by the quantum number $n = 1,2,3$ and 4, m_a and m_b are the masses of quarks of flavors a and b, E_n is the excitation energy of the radially excited state n, and S and A are interaction strength parameters to be determined by fits to experimental masses. The original Isgur model has the same squared mass operator but considers only the states with $n = 1$; i.e. only the ground state radial configurations. The quark mass parameters m_a and m_b and the strength parameters S and A for the "scattering" and "annihilation" interactions are already present in Isgur's model. Our modifications are:

1. Introducing the radial excitation energies E_n. These parameters are fixed by setting $E_1 = 0$ to make the ground state matrix element identical to Isgur's, setting $E_2 = 0.59$ GeV given by the $\Psi - \Psi'$ splitting and using the values from the log potential model[10] for the higher states, $E_3 = 0.91$, $E_4 = 1.13$ GeV.

2. Introducing an n-dependence for the annihilation interaction matrix element as proportional to $1/nn'$. This is obtained by the argument that the annihilation interaction is short range and depends upon the value of the wave function at the origin. For a logarithmic potential, $\Psi_n(0)\Psi_{n'}(0)$ is proportional to $1/\sqrt{nn'}$. This is squared for the squared mass operator.

For the values of the parameters we follow Isgur in setting $m_u = m_d = 337$ MeV obtained from the proton magnetic moment, and set $m_s - m_d = 140$ MeV from the equal spacing of the baryon decuplet.

The value of the parameter S is then fixed to give the mass of the pion as $S = -0.43\ \mathrm{GeV}^2$. This then gives a good value of 0.48 GeV for the kaon mass as noted by Isgur. The value of the parameter A is then fixed by fitting the mass of the η' to the value $A = 0.47$. This differs from Isgur's value of 0.27 because of the effects of radial mixing. The annihilation interaction affects the mass of the η' in two ways in the present treatment, the matrix elements within the $n = 1$ sector raise the mass, as in the conventional treatment, but the mixing produced by the off-diagonal elements lower the mass. Thus the elements in the $n = 1$ sector must give a larger mass shift than the observed mass to fit the data, and the parameter A is larger than that in the Isgur treatment.

With these parameters, we obtain a mass of 0.53 for the η, which is in very good agreement with the experimental mass of 0.54, with no further adjustment of parameters. The "D" appears at a mass of 1260 MeV and the next state is at 1420 MeV.

The physical implications of the good fit to the η mass are quite clear. In the limit where the parameter A is very large, flavor SU(3) becomes a good symmetry, since the symmetry breaking term $m_s - m_u$ is small in comparison with A. In this limit the term in (1) proportional to A contributes only to the masses of SU(3) singlet states. The SU(3) octet states have no radial mixing and masses given by the expectation value of the first term. They therefore satisfy the Gell-Mann-Okubo mass formula. Since the η mass is known to be given reasonably well by the GMO formula, increasing the magnitude of A must always give a reasonably good fit to the η mass, even though its value is set by fitting something else.

3. Results - Mixing and Form Factors

The qualitative features of the radial mixing are also apparent. Since the annihilation term affects only the SU(3) singlet states and must raise them considerably to fit the high mass of the η', the η remains reasonably close to the pure ground state octet configuration. The η', on the other hand, has a large singlet component and can have appreciable radial mixing. These are exactly the properties desired to obtain agreement with the predictions for peripheral charge exchange and strangeness exchange production processes.

The quantitative results for the mixing are obtained directly from the diagonalization of the squared mass matrix (1) with the values of the parameters given above. These are displayed in Tables 1 and 2. The η is seen to be 93% octet and 6% singlet in the ground state configurations with only a 1% admixture of radial excitations. The η' is 45% singlet and 6% octet and loses 49% of the wave function to radial excitations. Note, however, that in the strange-nonstrange basis the η is 56% nonstrange and 42% strange, which is very different from the 1/3 nonstrange and 2/3 strange for a pure octet, while the η' is dominantly strange with 35% strange and 16% nonstrange rather than 2/3 nonstrange and 1/3 strange as for a pure singlet. These large departures from the SU(3) wave functions arise from small mixtures of 6% because of coherence effects. A 6% admixture in probability is 0.25 in amplitude. Note that the η has nearly equal strange and nonstrange components as noted by Isgur, while the η' is dominantly strange and therefore produced much less in nonstrange charge exchange processes than in any description where it is entirely in the ground state configuration and is orthogonal to the nearly equally mixed η. Note that 99% of the ground state octet configuration is accounted for in the $\eta - \eta'$ system but only 51% of the ground state singlet.

These results can now be used to calculate form factors which suppress meson transitions. We consider peripheral production processes at low momentum transfer and radiative M1 $V \to P\gamma$ decays, where the transition amplitude is proportional to an overlap integral between the initial and final meson states. This overlap integral is commonly taken to be unity in conventional quark model calculations. Radial mixing reduces this overlap. The radial form factors for these transitions are listed in Table 2. The quark model amplitudes should be multiplied by these form factors to obtain the transition amplitude for the physical mixed states.

In the extended Isgur model all the initial states for the transitions listed in Table 2 have no radial mixing, and the form factor is proportional to the appropriate ground state component of the final state. For reactions initiated by kaons, the quark model gives two independent amplitudes for η and η' production, corresponding to the production of the strange ($s\bar{s}$) and nonstrange $[(u\bar{u} + d\bar{d})/\sqrt{2}]$ components of these mesons. Since the radial mixings of the strange and nonstrange components are different, the amplitude for the production of the physical meson state is obtained by coherently adding the terms for each component, which are products of the quark model amplitude and the appropriate form factor. The form factors for the transitions to the non-strange and strange components of the final meson are denoted by the letters n and s respecively, and their magnitude and phase are given. All other form factors in Table 2 are given as their squares, since there is only a single contributing amplitude to the transition and the corresponding cross section or width is proportional to the square of the form factor with no coherent summations.

Table 2 also lists form factors for the Linear Mass Model discussed below. In this model there is also radial mixing in the pion and kaon, but the results for transitions between pseudoscalar meson states are essentially unchanged.

4. Results - Peripheral Production

We now consider the comparison with experiment of the new predictions for peripheral production processes obtained by use of the form factors in Table 2. For strangeness exchange processes, the introduction of form factors modifies the conventional sum rule[13] to:

$$\bar{\sigma}(K^-p \to \Lambda\eta) + 2\bar{\sigma}(K^-p \to \Lambda\eta') = 0.9\bar{\sigma}(K^-p \to \Lambda\pi^\circ) + 1.1\bar{\sigma}(\pi^-p \to \Lambda K^\circ), \qquad (2)$$

where $\bar{\sigma}$ denotes the cross section corrected for phase space factors to give the square of the invariant amplitude, and the form factors used are those for the extended Isgur model. The principal difference between this sum rule (2) and of the conventional quark model sum rule where all the coefficients are unity is in the factor of 2 multiplying the $\Lambda\eta'$ cross section.

This is precisely what is needed to correct the disagreement found by Aguilar-Benitez et al [4] for the conventional sum rule. Table 3 shows their data and the comparison with the conventional and the modified sum rules. Comparison is also shown for the sum rule obtained from the linear mass model, which has coefficients of 1.7, 0.9 and 1.1 instead of 2, 0.9 and 1.1.

The more recent data of Marzano et al, [5] which also indicated a disagreement with the old sum rule also agree with the modified sum rule. Their values of 22.9 ± 1.2μb, 47.6 ± 2.8 μb and 66 ± 4.1 μb for the η, η' and π° production cross sections in the forward hemisphere at 4.2 GeV/c gave a prediction of zero for $\sigma(\pi^- p \to \Lambda K^\circ)$ in strong disagreement with experiment. The modified sum rule(2) predicts 52 ± 7 μb which is in reasonable agreement with the value 54 ± 2 quoted for the total cross section for this reaction at 4.5 GeV/c in a different experiment.

For charge exchange processes, Okubo [14] has pointed out that conventional mixing predicts the ratio of η' to η production to be a universal constant for all processes which have no active strange quarks.

$$\frac{\sigma(A+B\to\eta'+X)}{\sigma(A+B\to\eta\ \ +X)} = K = \frac{|\langle\eta'|\eta_{ns}\rangle|^2}{|\langle\eta\ |\eta_{ns}\rangle|^2} \tag{3}$$

where A, B and S do not contain strange quarks and η_{ns} denotes the particular linear combination of η and η' which contains only nonstrange quarks, the analog of the physical ω. Okubo finds a value of $K = 0.5 \pm 0.25$ by analysis of a large number of processes. But recent experiments[2,18] give

$$K = (2.0 \pm 0.68)/(1.27 \pm 0.39) \tag{4}$$

for the ratio of η' to η production in the backward direction in $K^- p \to \Lambda\eta$ or $\Lambda\eta'$. If this is a baryon exchange process, the coupling of the η and η' to nonstrange baryons should also go via the η_{ns} component and the processes should satisfy Okubo's universality relation (3). The fact that $K > 1$ for this case whereas $K < 1$ for all meson exchange processes investigated by Okubo might suggest a difference between meson exchange and baryon exchange, if the discrepancy of less than two standard deviations proves to be statistically significant.

In our model, Eq. (2) holds only at small momentum transfers where the transition amplitude depends upon a form factor which is just the overlap integral listed in Table 2. It should not hold for baryon exchange where there is no overlap integral and the radially excited components in the wave function can also contribute. We obtain a value of $K = 0.3$ for the Okubo parameter (3), in reasonable agreement with the experimental value. We can also define an analogous parameter for the ratio of "D" production to η production.

$$K^{\text{"D"}} = \frac{(\ A + B \to \text{"D"} + X)}{(\ A + B \to \ \eta \ + X)} \tag{5}$$

From the form factors in Table 2, we obtain $K^{\text{"D"}} = 0.2$. Thus this first excited pseudoscalar should be produced in charge exchange with a cross section comparable to that of the η'.

There is also an SU(3) charge exchange sum rule[16], which was originally derived under the assumption that the η is an unmixed pure octet state. This has been found to be in good agreement with experiment. However, the standard mixing prescription would imply that the η' must be a nearly pure singlet if the η is pure octet, and this gives a value of $K = 2$ for Okubo's parameter, in strong disagreement with experiment. The mixing given in Table 1 clearly has the right qualitative features to resolve this paradox, since the η is indeed nearly pure octet, while the value of K is reduced by the mixing of radially excited configurations into the η'. The charge exchange sum rule is modified by the form factors to

$$\sigma(\pi^- p \to \pi^\circ n) + 1.8\sigma(\pi^- p \to \eta n) = \sigma(K^+ n \to K^\circ p) + \sigma(K^- p \to K^\circ n). \tag{6}$$

This differs from the successful SU(3) sum rule by having the factor 1.8 multiplying the η production cross section instead of 3. Although the difference between factors of 1.8 and 3 seems to be large at first sight, it has a small effect on the sum rule where the left hand side is dominated by the much larger $\sigma(\pi^- p \to \pi^\circ n)$. Better data are needed in order to test the difference between the two sum rules.

5. Results - Masses

At this point our model has been shown to resolve the difficulties of the standard description of peripheral production processes. We now ask whether there is any additional new information or predictions which are obtainable. We have already seen that the first radially excited pseudoscalar ("D") must have appreciable mixing of the ground state singlet configuration and therefore be produced together with the η' with comparable cross sections. We now extend our mass matrix to treat the vector mesons introducing the following additional assumptions.

1. A is assumed to be zero for the vector states, since the deviations from ideal mixing are known to be small. The matrix is therefore diagonal and its diagonalization is trivial.

2. The parameter S for the vector mesons is assumed to be (-1/3) times the value for the pseudoscalars, $S_v = -(1/3)S_p$. This rather drastic assumption assumes that the entire "gluon exchange" interaction appears as a spin dependent force proportional to $\vec{\sigma}_1 \cdot \vec{\sigma}_2$. This is completely ad hoc, but gives a remarkable fit to the experimental masses. The values for the masses of the ω, ρ, K^* and ϕ are (770,770,900,1030) MeV while the experimental values are (780,770,890,1020). This can be turned around to state that if S_v is taken to be a free parameter to fit the data, its value turns out to be very close to the value $-(1/3)S_p$ predicted for a pure spin-dependent force.

This raises the interesting question of why meson masses are fit so well by a model which has only the quark masses and a spin-dependent two-body force and no spin-independent two-body force. One answer is that the mass parameters used in the model are not masses of free quarks but effective masses which already include the effect of the spin-independent potential. Note that the value of the quark mass was determined from the proton magnetic moment. The mass parameter appearing in the magnetic moment of a bound Dirac particle is not the mass of a free particle but depends also on the binding energy in a manner which varies with the Lorentz structure of the potential[17]. A Lorentz four-vector potential, like a coulomb potential, does not change the magnetic moment, and the magnetic moment of an electron bound by electrostatic forces (e.g. in a Van De Graaf accelerator) is the same as that of a free electron. A Lorentz scalar potential, however, adds to the mass term in the Dirac equation, and the magnetic moment is determined by the sum of the two; i.e. by the total energy of the system or its mass including the binding energy. Our fit to the meson masses might suggest that the spin-independent confining potential (e.g. the log) is a Lorentz scalar and therefore the quark mass determined from magnetic moments includes the binding energy. However, the spin-dependent force which comes from one gluon exchange is clearly vector and does not appear in the magnetic moment.

6. A Model With Linear Masses

In any case, the suggestion that hadron masses are adequately described by quark masses taken from magnetic moments and only a spin-dependent two-body interaction is worthy of further investigation. We attempt this by considering a logical extension of the DGG[8] model which uses linear masses and a "color-magnetic" spin-dependent force but does not consider mixing of radial excitations. The extension of this model is slightly more complicated than that of the squared-mass Isgur model because of the necessity to consider the spin-dependent force more carefully, in particular its flavor dependence. In the Isgur model the flavor dependence of the spin-dependent interaction was neglected, because spin effects are flavor independent when squared masses are used. This corresponds to the well known but mysterious SU(6) result that $M_{\rho}^2 - M_{\pi}^2 = M_{K^*}^2 - M_K^2 = M_{D^*}^2 - M_D^2$. This has allowed us to disregard flavor dependence of the interaction S, following Isgur.

When linear masses are used with the DGG prescription of attributing the spin forces to color magnetic hyperfine interactions, the interaction strength depends upon the color magnetic moments of the quarks which are inversely proportional to their masses. A good fit to the linear masses is again obtained, with spin forces that are now flavor dependent. But the success of the flavor-independent spin force with squared masses then becomes a mystery or an accident. We do not attempt to resolve this puzzle here, but simply change the ground rules in accordance with conventional procedure and use flavor independence with squared masses and flavor dependence with linear masses. The use of these two different approaches to calculate mixing form factors should then check the model dependence of the mixing formalism and values of form factors.

One additional complication arises in taking the spin force seriously as a hyperfine interaction. This short range force also must have appreciable matrix elements connecting the ground state with radial excitations. The fact that the

ρ-π mass splitting is of the same order as radial excitation energies indicates that treating the interaction responsible for the ρ-π splitting as a perturbation and neglecting its effect on the wave functions is questionable. We therefore introduce radial mixing by the spin force as well as by the annihilation interaction. To keep down the number of free parameters, we note that the Fermi-Breit hyperfine interaction also depends upon the value of the wave function at the origin. We therefore take the mass dependence of the spin force and the annihilation interaction to be the same, and similarly for the dependence on the radial quantum number n. Our mass matrix now has the form

$$\begin{aligned}\langle q_{a'}\bar{q}_{b'}n'|M|q_a\bar{q}_b n\rangle = {} & \delta_{aa'}\delta_{bb'}\delta_{nn'}(m_a + m_b + E_n) \\ & + \delta_{ab}\delta_{a'b'}(A/m_a m_{a'}\sqrt{nn'}) \\ & + \delta_{aa'}\delta_{bb'}(B\vec{\sigma}_a \cdot \vec{\sigma}_b/m_a m_b\sqrt{nn'})\end{aligned} \tag{7}$$

where m_a, m_b and E_n have exactly the same values as in the Isgur model, (1), and the parameters A and B replace the parameters A and S and are adjusted to fit experimental masses as before.

The value of the parameter B is fixed by fitting the ρ-π mass difference to give $(3/4)B/m_u^2 = 0.366$. This allows us to calculate immediately the masses of the π, ρ, K and K^* to obtain the values $(\pi,\rho,K,K^*) = (150,770,470,890)$ MeV in comparison with the experimental values (140,770,500,800). This good fit is rather surprising, especially the good value for the pion mass, which is not easily obtained in other models. Although the quantitative success may be accidental, there is one qualitative feature which may have interesting physical implications; namely the necessity to go beyond first order perturbation theory in treating mass splittings.

In any model where the spin splittings come from an interaction proportional to $\vec{\sigma}_1 \cdot \vec{\sigma}_2$, the vector and pseudoscalar interactions satisfy the relation $S_v = -(1/3)S_p$ discussed above. This implies that if the spin splittings can be taken as a perturbation and treated only to first order, the linear combination $(m_\pi + 3m_\rho)/4 =$ 610 MeV is the unperturbed mass with the spin interactions turned off. But if the unperturbed mass is taken to be that of two quarks, determined from the proton magnetic moment, the unperturbed mass is 670 MeV. Thus fitting the masses with unperturbed masses of 670 MeV and a spin splitting which satisfies $S_v = (-1/3)S_v$ must give too high a value for the pion mass. This may also be true in the bag model where the relevant quark masses in this argument are not the a priori quark masses but an [18] "effective quark mass parameter" which enters into the total energy and into the magnetic moment in a manner similar to our phenomenological model.

The results from the diagonalization of the matrix (7) show that it is necessary to go beyond first order perturbation theory to get a good value for the pion mass. In particular, we see that with the parameters chosen to fit the masses, the first order perturbation result for the pion mass would be 320 MeV. The downward shift of the pion mass from 670 MeV is only two thirds of the total shift obtained by exact diagonalization. Note that the same physics can also be introduced into the bag model to obtain a better result. A short range spin-dependent interaction not only lowers the energy of the pion by its first order effect, it also changes the pion wave function, bringing more of it closer to the origin and lowers the energy by an additional amount.

The value of the parameter A is fixed again by fitting the η' mass. This then gives a good value of 530 MeV for the η mass, compared with 550 for the experimental mass and predicts a mass of 1310 MeV for the "D" and 1490 MeV for the next state.

For the isoscalar vector mesons with A set equal to zero masses of 780 and 1010 MeV are obtained for the ω and ϕ in good agreement with the experimental values of 780 and 1020.

The form factors for peripheral meson production and $V \to P\gamma$ decays are listed in Table 2 and are qualitatively the same as those from the extended Isgur model. The factor multiplying the $\Lambda\eta'$ cross section in the strangeness exchange sum rule (2) is 1.7 instead of 2 as in the Isgur model, but there is still good agreement with experiment. The Okubo parameter $K = 0.28$ instead of 0.3 and the factor appearing in the charge exchange sum rule (6) is not changed. The factor $K^{\text{"D"}}$ comes out 0.16 instead of 0.2. Thus all the results for peripheral processes are not sensitive to the change of model.

7. Application to Charmed Mesons and Charmonium

It is possible to include the charmed mesons in the mass matrix (7) and use the experimental value of the ψ mass to obtain a value for the mass of the charmed quark. This comes out to be $m_c = 1550$ MeV. We then obtain values of 1870 and 2010 for the D and D^* masses, as compared with 1800 and 2010. This agreement is reasonable, but not surprising, since the main ingredient in these predictions is the mass of the charmed quark determined by fitting the Ψ mass. However, the surprising result that the η_c mass is 3100 MeV essentially degenerate with the Ψ is also obtained. This results from two effects:

1. The hyperfine interaction determined from the ρ-π splitting is smaller than conventionally assumed, because it is amplified by the higher order effects as discussed above to produce the observed splitting. The higher order effects are negligible in the charmonium system because the interaction is reduced by the quark mass factors.

2. The annihilation interaction A which is normally neglected and which is also considerably larger than the value obtained by fitting low-lying meson masses without considering radial excitations as discussed above. Both these effects must be present in any model which uses experimental masses of low-lying states to determine parameters relevant to charmonium. Thus while the exact values obtained here may not be accurate, the essential point that the standard hyperfine splitting calculation gives a mass for the η_c which is <u>too low</u> may have a general validity.

It is interesting to use the charmonium results to estimate the radiative decays of the Ψ to $\eta\gamma$ and $\eta'\gamma$. Assuming that these decays go via the mixing of $c\bar{c}$ into the η and

η' wave functions and using the admixtures given by diagonalizing the mass matrix (7) we obtain the result $\Gamma(\Psi \to \eta'\gamma)/\Gamma(\Psi \to \eta\gamma) = 1.9$, which is in reasonable agreement with the experimental value of 1.8 ± 0.8. This also predicts an appreciable value for the radiative decay to "D"; namely 2.9 times the $\eta\gamma$ decay, neglecting phase space factors. Thus it might be of interest to look for this "D" in radiative Ψ decays.

Table 1. Meson Masses and Wave Functions in the Extended Isgur Model

Meson	Mass		Fraction of Ground State Wave Function				
	Theory	Experiment	SU(3) basis Octet	SU(3) basis Singlet	Strange-nonstrange Basics Nonstrange	Strange-nonstrange Basics Strange	Total Ground State
η	530	550	93%	6%	56%	42%	99%
η'	950	960	6%	45%	16%	35%	51%
Total $\eta + \eta'$			99%	51%	72%	77%	
"D"	1260	?		22%	11%	11%	22%

Table 2. Meson Form Factors

Transition	Squares of Form Factors		Form Factors	
	Linear Model	Isgur Model	Linear Model	Isgur Model
$\pi \to \eta$	.55	.56		
$\pi \to \eta'$	.16	.16		
$K \to \eta_n$	.57	.56	+.75	+.75
$K \to \eta_s$	.42	.42	-.65	-.65
$K \to \eta'_n$	.20	.16	+.45	+.4
$K \to \eta'_s$	.39	.35	+.62	+.59
$\pi \to K$	.99	1		
$\rho \to \pi$	.72	1		
$\omega \to \pi$	.72	1		
$\rho \to \eta$	.52	.56		
$\rho \to \eta'$	.46	.16		
$\omega \to \eta$	.52	.56		
$\omega \to \eta'$	.46	.16		
$\phi \to \eta'$	.44	.35		
$\phi \to \eta$	.37	.42		
$K^* \to K$	.82	1		

Table 3. Tests of Strangeness Exchange Sum Rules

	$\bar{\sigma}(K^-p \to \Lambda\eta)$	$+ A(K^-p \to \Lambda\eta')$	$= B\bar{\sigma}(K^-p \to \Lambda\pi^\circ)$	$+ C\bar{\sigma}(\pi^-p \to \Lambda K^\circ)$,
Values of $\bar{\sigma}(\mu b/GeV^2)$ Ref. 4	236 ± 55	469 ± 73	576 ± 52	545 ± 28
Old Sum Rule	LHS = 705 ± 91		RHS = 1121 ± 59	
New Sum Rule (Isgur Model) A=2, B=0.9, C=1.1	LHS = 1174		RHS = 1117	
New Sum Rule (Linear Model) A=1.7, B=0.9, C=1.1	LHS = 1033		RHS = 1129	
Values of $\sigma(\mu b)$ Ref. 5	22.9 ± 1.2	47.6 ± 2.8	66 ± 4.1	[54 ± 2]
Old Sum Rule A=B=C=1	LHS = 70.5		RHS = [120]	
New Sum Rule (Isgur Model) A=2, B=0.9, C=1.1	LHS = 118		RHS = [119]	

Values in brackets are from different experiment at slightly different energy. No phase space corrections are included in using the results of Ref.5, but these should not introduce any qualitative difference.

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