

Coherent Production on Nuclei Does Not Measure Total Cross Sections for Unstable Particles

HANNU I. MIETTINEN

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

JON PUMPLIN

Department of Physics, Michigan State University, East Lansing, Michigan 48824

ABSTRACT

We calculate diffractive production in nucleon-nucleus collisions using a parton model. We fit the result to the standard optical model formula, and obtain estimates of 12-30 mb for the cross section of the diffractively produced system on a nucleon. The actual value of that cross section in the model is 65 mb. Estimates for the total cross section of an unstable hadron which are based on the A -dependence of coherent production should therefore not be trusted.



It was suggested more than a decade ago that coherent production experiments on nuclei may yield information on the cross sections of diffractively produced systems on nucleons.¹ Because these systems are unstable, their cross sections cannot be measured directly. The optical model, which has become standard for analyzing coherent production data, is illustrated by Fig. 1. A beam hadron propagates in a nucleus, and interacts to form the excited state D. The probability that D will escape from the nucleus depends upon its cross section on nucleons, $\sigma_{\text{tot}}(\text{DN})$, and on the amount of nuclear matter it must traverse $\propto A^{1/3}$. The explicit formula, in large A approximation, is²

$$\frac{d\sigma(aA \rightarrow DA)}{dt} = \left(\frac{d\sigma(aN \rightarrow DN)}{dt} \right)_{t=0} |F|^2 ;$$

$$F = \frac{4\pi}{\sigma_1 - \sigma_2} \int_0^\infty J_0(B\sqrt{t}) \left(e^{-\sigma_1 \rho(B)/2} - e^{-\sigma_2 \rho(B)/2} \right) B dB . \quad (1)$$

Here, $\sigma_1 = \sigma_{\text{tot}}(aN)$, $\sigma_2 = \sigma_{\text{tot}}(\text{DN})$, $\rho(B)$ is the nuclear density at impact parameter B, and t is the four-momentum transfer squared.

Analyses of diffractive production using π , K, and p beams have consistently led to estimates for unstable state--nucleon total cross sections which are smaller than one might naively expect.³ For example, in $pA \rightarrow p\pi^+\pi^-A$, the cross section varies from 15 to 35 mb, depending on the $p\pi^+\pi^-$ mass.⁴

A crucial assumption in the optical model approach is that the time required for excitation to take place is short compared to the time required for propagation through the nucleus. Stated differently, the model assumes that the new-born diffractive state is immediately ready to interact, with its full hadronic cross section. Modern understanding of the space-time development of high-energy collisions opposes this assumption. Rather than being instantaneous, hadronic

processes take place over a long time, because interactions among fast constituents are slowed down by a Lorentz time dilatation factor $\gamma_{p_{lab}}$.⁵ This view is supported by the experimental observation that multi-hadron production on nuclei displays little or no "cascading."⁶

If the space-time basis of the optical model is open to criticism, the estimate the model provides for the total cross section of an unstable particle should also be questioned. To study this problem, we have devised a "theoretical experiment," which goes as follows. We calculate diffractive production on nuclei in a theoretical model, and fit the resulting cross sections with the optical model to obtain an estimate of $\sigma_{tot}(DN)$. We then calculate the true value of $\sigma_{tot}(DN)$ and compare it with the estimate. Our result is that the true value and the estimate differ by a factor of 2-5, and hence the optical procedure cannot be relied upon.

To make our "theoretical experiment" meaningful, the model employed in it must describe diffractive production on single nucleons reasonably well. It should also possess the theoretically-favored long time-scale structure. For these reasons, we use a simple parton model, which has recently been found to describe diffractive production on protons.⁷ The model has the standard space-time structure of constituent models, with a total interaction time $\propto p_{lab}$. Its main assumptions are:

(i) A fast moving hadron is a superposition of states $|\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N\rangle$ which contain various numbers, types and configurations of structureless partons. The rapidity distribution of the partons is approximately flat, extending from the rapidity of the beam particle down to rapidity zero.

(ii) Interactions between partons are of short range in rapidity. Hence soft hadronic collisions are initiated by interactions between the wee partons of the beam and target.

(iii) Diffraction scattering arises as the shadow of non-diffractive multi-particle production. Since the various components of the colliding hadrons' wave function interact with different strengths, shadow scattering leads to diffraction dissociation as well as to elastic scattering.⁸

(iv) The parton states are the eigenstates of diffraction. The partons interact independently with the target, so that the total interactions probability is

$$T(\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N; \vec{B}) = 1 - \prod_{i=1}^N [1 - \tau(\vec{b}_i + \vec{B}, y_i)] \quad (2)$$

where \vec{B} is the impact parameter of the incident hadron and τ is the interaction probability for a single parton.

The total and beam dissociation cross sections are given by

$$\begin{aligned} d\sigma_{\text{tot}}/d^2\vec{B} &= 2\langle T \rangle \\ d\sigma_{\text{diff}}/d^2\vec{B} &= \langle T^2 \rangle - \langle T \rangle \end{aligned} \quad (3)$$

where the averages are taken over the probability distribution of the beam partons. We neglect—as usual—any contributions from real parts, which are expected to be small at high energy. For quantitative calculations, we use explicit forms for the parton distributions and the interaction probability, which are similar to those of reference 7, except that we make the $y > 0$ parton distribution, and the $y < 0$ interaction probability, constant in order to avoid the somewhat artificial definition of wee partons in that reference. We also changed the proton radius parameter slightly, to fit pp scattering in the $p_{\text{lab}} = 200\text{-}300$ GeV/c region.⁹

We now turn to the analysis of diffractive production on nuclei. We assume that the parton states are the eigenstates for diffraction on nuclei. When a parton configuration $|\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N\rangle$ of the beam particle scatters on a nucleus with A nucleons at impact parameters $\vec{b}_1, \dots, \vec{b}_A$, the interaction probability is, analogous to Eq. (2)

$$T_A(\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N; \vec{b}_1', \dots, \vec{b}_A') = 1 - \prod_{i=1}^N \prod_{j=1}^A [1 - \tau(\vec{b}_i - \vec{b}_j' + \vec{B}, y_i)] \quad (4)$$

where \vec{B} is the impact parameter of the incident hadron and \vec{b}_i, \vec{b}_j' are measured from the centers of beam hadron and target nucleus. The total and beam-dissociation cross sections are given by Eq. (3), with the expectation values interpreted to include an average over the ground-state distribution of nucleons in the target nucleus. This average can be calculated explicitly if one neglects nuclear correlation effects. We calculated the averages over beam parton distributions numerically, using a Monte Carlo technique.

To describe the nuclear density, we use the standard Woods-Saxon formula, with a skin thickness of 0.52 fm. We chose the radii to fit neutron-nucleus total cross section data.¹⁰ The values obtained, $R = 2.21, 3.22, 4.56, 5.59, 6.91$ fm for $A = 12, 27, 64, 112, 207$; are entirely reasonable.

Results of our parton model for the coherent beam dissociation cross section are shown in Fig. 2, together with the predictions of the optical model. In calculating these curves, we avoided the large- A approximations which lead to Eq. (1); but results using that equation are almost identical. One sees that a cross section of ≈ 12 mb would be inferred according to the optical picture from an experiment described by our model.

The optical model curves in Fig. 2 contain an arbitrary normalization factor. This is common practice in the analysis of actual experiments. It finds some justification in the fact that $(d\sigma/dt)_0$ may not be accurately known on hydrogen, and it may anyway contain a non-pomeron exchange background which is "filtered out" of the coherent nuclear data. If we use the absolute normalization of the optical model, we obtain a rather poor "best fit" with $\sigma_2 \approx 30$ mb.

The calculation of the actual $\sigma_{\text{tot}}(\text{DN})$ in our model is as follows. The diffractive state which emerges from a proton-nucleon collision at impact parameter \vec{B} can be separated into an elastic and an inelastic part:

$$T(\vec{B})|p\rangle = \langle T(\vec{B}) \rangle |p\rangle + |D\rangle \quad . \quad (5)$$

The inelastic part $|D\rangle$ is orthogonal to the single proton: $\langle p|D\rangle = 0$. The cross section for producing it is $\langle D|D\rangle = \langle (T(\vec{B}) - \langle T(\vec{B}) \rangle)^2 \rangle = \langle T(\vec{B})^2 \rangle - \langle T(\vec{B}) \rangle^2$, in agreement with Eq. (3). The total cross section for $|D\rangle$ on a second nucleon at impact parameter \vec{B}' is $2\langle D|T(\vec{B}')|D\rangle/\langle D|D\rangle$. Integrating over \vec{B}' and averaging over the production impact parameter \vec{B} yields the D-nucleon total cross section¹¹

$$\sigma_{\text{tot}}(\text{DN}) = \frac{2 \int d^2\vec{B} d^2\vec{B}' \langle (T(\vec{B}) - \langle T(\vec{B}) \rangle)^2 T(\vec{B}') \rangle}{\int d^2\vec{B} \langle (T(\vec{B}) - \langle T(\vec{B}) \rangle)^2 \rangle} \quad . \quad (6)$$

Averaging over the parton distributions and evaluating the integrals numerically, we obtain $\sigma_{\text{tot}}(\text{DN}) = 65$ mb for the true DN cross section in our model.

The qualitative basis of our result is simple and independent of the details of the model. The existence of nearly transparent states which pass through a large nucleus with little absorption implies sizeable diffractive production at all impact parameters. The integrated diffractive cross section at large A therefore rises

almost as fast as the nuclear area $\propto A^{2/3}$. The optical formula interprets this rapid rise in terms of a small σ_2 . Meanwhile, the true $\sigma_{\text{tot}}(\text{DN})$ is large, because a major contribution to the dispersion $\langle T^2 \rangle - \langle T \rangle^2$, which is responsible for diffractive production, comes from impact-parameter fluctuations of the beam partons. The diffractive states are therefore rather diffuse in impact parameters. Their large effective radius leads to a large cross section for Dp collisions.

In calculating the diffractive cross section, we treat the parton states as eigenstates of diffraction in nuclear matter. This is only approximately correct, because the slow partons do not undergo significant time dilatation, so their distribution can evolve significantly during passage through the nucleus. This will reduce the A-dependence of σ_{diff} somewhat and hence increase the value of σ_2 obtained from the optical model fit. We have attempted to estimate this "evolution" effect and our results—though model dependent—suggest that it cannot be large enough to overthrow our basic conclusions.

Several authors have attempted to understand the A-dependence of diffractive production using coupled-channel generalizations of the optical model.¹² Our model could also be translated to that form. We think it would be pointless to do so however, because the long time-scale associated with diffractive processes makes it unnecessarily complicated to describe passage through the nucleus in terms of asymptotic states. Our approach is thus closer to that of References 13, 14. Doubts about the validity of the optical model procedure have also been raised recently by Caneschi and Schwimmer.¹⁵

We have shown that the traditional optical model for estimating total cross sections of unstable hadrons fails badly in a reasonable "theoretical experiment." We conclude that it cannot be applied reliably to real experiments. We know of no reliable method for extracting unstable particle cross sections. The A-dependence

of diffractive production on nuclei remains an important subject for experiment, however, because it provides information about the distribution of cross sections for the eigenstates of diffraction.

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FIGURE CAPTIONS

- Fig. 1: Traditional optical model for coherent production on nuclei. In the first interaction, the beam particle a is excited into a state D . The second interaction measures the D -nucleon total cross section.
- Fig. 2: Beam-dissociation cross section in nucleon-nucleus collisions for various target nuclei. Solid points (●) are calculated according to the parton model described in the text. Curves correspond to the cross sections given by the Kölbig-Margolis model² with $\sigma_2 = \sigma_{\text{tot}}(\text{DN})$ as parameter. We have omitted t_{min} factors to simplify the analysis. This does not affect the estimate for σ_2 , because t_{min} would change both the points and the curves in the same way; but it means that this figure should not be compared directly with experiment.

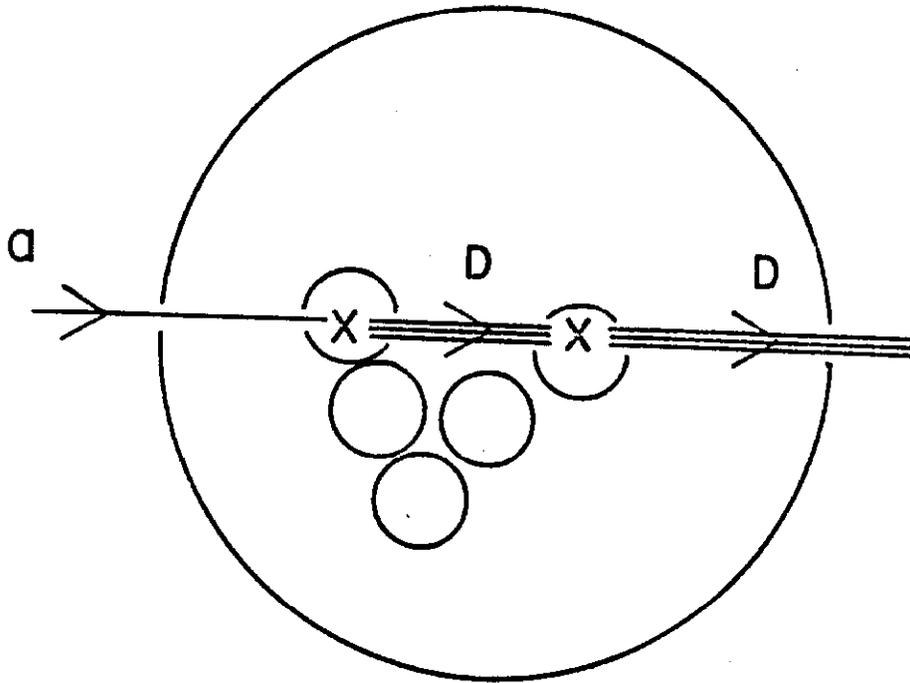


Fig. 1

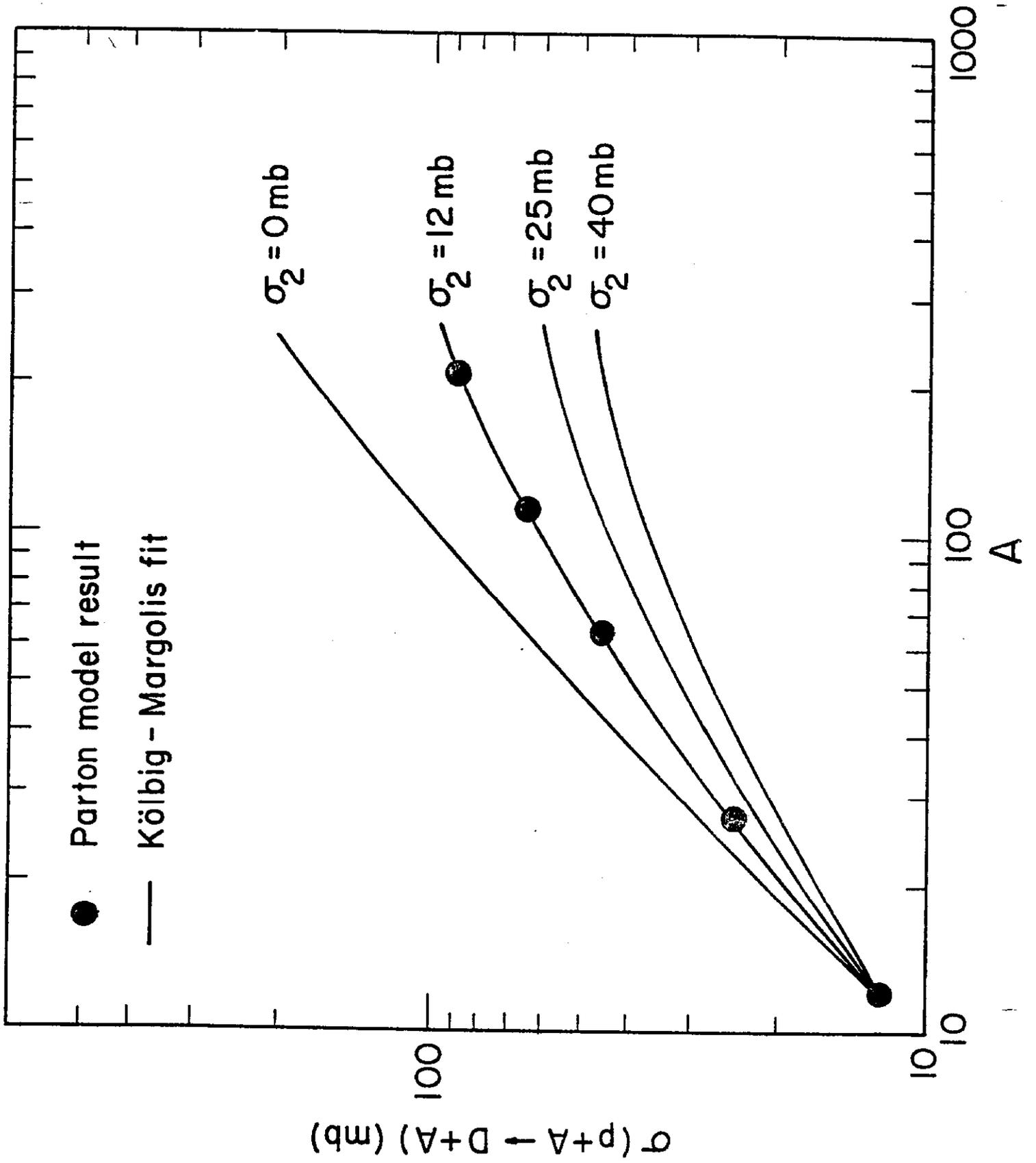


Fig. 2