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Gluon Jets from Quantum Chromodynamics

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ABSTRACT

Following the idea of Sterman and Weinberg, we calculate the jet angular radius of a gluon jet and its energy-dependence in the framework of perturbative quantum chromodynamics. The result is free of infrared and mass singularities and does not depend on the gluon fragmentation function. We find that the gluon jet spreads much more (i.e. less jet-like) than a quark jet; this renders the detection of gluon jets very difficult.

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The hope that quantum chromodynamics (QCD) provides a complete description of hadron physics is widely spread. One of the key characteristics of QCD is the existence of gluons. A crucial verification of QCD would thus be the observation of its gluonic structure. Partly motivated by the observation of hadronic jets (which we shall refer to as quark jets, since they are believed to be jets arising from energetic quarks) in e^+e^- annihilation,[1] we expect the gluons to reveal themselves in the form of gluon jets (i.e. hadronic jets arising from energetic gluons). In fact, gluon jets are expected to be produced in many experimental processes such as large p_T scattering, e^+e^- annihilation, lepton-proton deep inelastic scattering and Drell-Yan di-muon experiments.

Recently it has been emphasized that, in QCD, the three-gluon decay mode[2] of the upsilon Υ[3] (or some other heavy quark-antiquark bound states) would provide a very clean place to find and study the gluon jet.[4] From e^+e^- annihilation we learn that a quark jet is observable when the initial quark has an energy $E_q \geq 3$ GeV. Since the mass of Υ is around 9 GeV, the average energy per gluon is 3 GeV. Hence, whether a gluon jet from Υ decay is observable or not depends crucially on its jet-like nature, i.e. the jet angular radius and its energy-dependence as compared to those of the quark jet.

In this paper we shall study these gluon jet properties using the approach suggested by Sterman and Weinberg;[5] they showed that the quark jet structure follows from a perturbative calculation in QCD without assuming a transverse-momentum cut-off. Furthermore, their result does not require any knowledge of the fragmentation function.[6] This means that their approach is particularly useful to the study of gluon jets since how a gluon fragments into hadrons is totally unknown (presumably, fragmentation is a consequence of quark confinement, an issue we shall not discuss here).

Instead of studying a possible three-gluon jet-like structure in the Υ decay directly, we choose to consider a simplified process where a pair of energetic gluons is produced by a color-singlet, gauge-invariant scalar source $\left(F^a_{\mu\nu}\right)^2 = \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu\right)^2$ in the center-of-mass frame. For the two gluon decay mode, this scalar source effectively represents a color-singlet 0^{++} quark-antiquark system in the limit of heavy quark mass. It is our basic assumption (which, we believe, is very plausible) that the jet angular radius and its energy dependence do not depend on the source. Calculations using the pseudoscalar source $F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}$ have also been carried out.[7] The result supports our assumption. This will mean that the essential features of gluon-jet structures do not depend on how the energetic gluon is produced and that our results are applicable to gluon jets produced in all scattering processes.

To study gluon jets we calculate, to order g^2, the partial cross section $\sigma(E, \varepsilon, \delta)$ for the two-gluon jet-like events, where all but a small fraction ε of the total energy E is emitted within some pair of oppositely directed cones of half angle $\delta << 1$, lying within the solid angle $d\Omega$ ($\pi\delta^2 << d\Omega << 1$). For high-energy phenomena in perturbative QCD, there appear degenerate states within the resolution of detector devices. These degenerate states lead to infrared divergences and mass singularities,[8] which, however, are expected to be cancelled for physically sensible cross sections such as $\sigma(E, \varepsilon, \delta)$.

Fig. 1 shows six types of diagrams that contribute to $\sigma(E, \varepsilon, \delta)$ to order g^2. Quark masses are set to zero. Each diagram divided at line (1) (diagrams (i)-(v)) represents a real particle (i.e. gluon, ghost or quark) emission process. The hard Bremsstrahlung region (where the energies of emitted particles $k^0 > \varepsilon E$ and $\ell^0 > \varepsilon E$ are within the angular cone $\pi\delta^2$) gives rise to mass singularities and determines the jet angular radius. (For diagram (iii), quark pairs are produced within the angular cone $\pi\delta^2$). The soft Bremsstrahlung region (where $k^0 < \varepsilon E$ or

$\ell^0 < \varepsilon E$) gives rise to infrared divergences as well as mass singularities. As we shall see below, these infrared divergences and mass singularities are cancelled by those arising from the virtual corrections to the external gluon lines and the source;[9] diagrams (i)-(vi) divided at line (2) represent these virtual corrections.

We perform our calculation in the Feynman gauge and use the dimensional regularization method[10] to control the ultraviolet and infrared divergences and mass singularities. Proper combinatoric factors are taken into account. The real particle emission processes give the partial cross section

$$\sigma(1) = \sigma_o\left(\frac{g^2}{4\pi^2}\right) f(D)\left[C_2(G)\left\{\left(\frac{4}{D^2}-\frac{11}{3D}\right) E^D - (4\ln(2\varepsilon)+\frac{11}{3})\ln(\delta) - \frac{5\pi^2}{6}+\frac{67}{9}\right\} + \frac{4}{3}T(R)N_f\left\{\frac{1}{D}E^D + \ln(\delta) - \frac{23}{12}\right\}\right] \tag{1}$$

as the space-time dimension $n = 4 + D \to 4$, where $f(D) = [(2\sqrt{\pi})^D\,\Gamma(\frac{D}{2}+1)]^{-1}$, and $\sigma_o \equiv (d\sigma/d\Omega)_o$ is the lowest-order differential cross section for the two-gluon production from the source. N_f is the number of quark flavors and $C_2(G)$ and $T(R)$ are color factors for the gluon representation G and the quark representation R respectively[11]: in QCD, $C_2(G) = 3$ and $T(R) = \frac{1}{2}$ for quark triplets. The double pole term $1/D^2$ and the $\ln(2\varepsilon)\ln(\delta)$ term originate solely from diagram (v) where an infrared divergence and a mass singularity show up simultaneously. Finite terms proportional to ε and/or δ have been neglected.

The virtual corrections (diagrams (i)-(vi)) lead to, after renormalization, the partial cross section

$$\sigma(2) = \sigma_o\left(\frac{g^2}{4\pi^2}\right) f(D)\left[C_2(G)\left\{-\frac{4}{D^2}E^D + \frac{11}{3}\left(\frac{1}{D}\right)\mu^D - \frac{67}{18}\right\} + T(R)N_f\left\{-\frac{4}{3}\left(\frac{1}{D}\right)\mu^D + \frac{20}{9}\right\}\right] \quad . \tag{2}$$

We have carried out the renormalizations of the gluon propagator and of the source by subtracting only the ultraviolet divergent terms at an arbitrary off-mass-shell point $p^2 = \mu^2$ for gluon momenta[12] and on the mass shell $p^2 = E^2$ for the outgoing source momentum. As expected, cancellations of infrared divergences and mass singularities take place in the sum of $\sigma(1)$ and $\sigma(2)$. Taking the limit $D = 0$, we obtain

$$\sigma(1) + \sigma(2) = \sigma_o \frac{g^2}{4\pi^2}\left[C_2(G)\left\{ -\frac{11}{3}\ln(E/\mu) - \left(4\ln(2\varepsilon) + \frac{11}{3}\right)\ln(\delta) + \frac{67}{18} - \frac{5\pi^2}{6}\right\} + \frac{4}{3}T(R)N_f\left\{\ln(E/\mu) + \ln(\delta) - \frac{1}{4}\right\}\right] . \quad (3)$$

The coefficient of the $\ln(E/\mu)$ term is simply the anomalous dimension[13] of the source $(F^a_{\mu\nu})^2$. This term reflects the nature of the source and is not related to that of the gluon jet. Let us choose $\mu = E$ so that $g^2/4\pi \equiv \alpha(E)$ is the coupling constant renormalized at $\mu = E$. Then, to order $\alpha(E)$, the partial cross section $\sigma(E, \varepsilon, \delta)$ is given by

$$\sigma(E, \varepsilon, \delta) = \sigma_o\left[1 - \frac{\alpha(E)}{\pi}\left\{4\, C_2(G)\ln(2\varepsilon) + \frac{11}{3}C_2(G) - \frac{4}{3}N_f T(R)\right\}\ln(\delta)\right] , \quad (4)$$

where only the logarithmic terms in eq. (3) are kept. The E dependence occurs only in the definition of $\alpha(E)$. It is amusing to observe that the coefficient of the single logarithmic term is precisely the one-loop β function[14] of QCD.

To facilitate comparison to the quark jet we integrate $\sigma(E, \varepsilon, \delta)$ over the whole solid angle $\Omega = 4\pi$ and define the fraction f of all jet-like events which have a fraction $(1 - \varepsilon)$ of the total energy E inside some pair of opposite cones of half angle δ:

$$f(\text{gluon}) = 1 - \frac{\alpha(E)}{\pi}\left\{4\, C_2(G)\ln(2\varepsilon) + \frac{11}{3}C_2(G) - \frac{4}{3}N_f T(R)\right\}\ln(\delta) . \quad (5)$$

Here f is independent of the gluon production source.[15] The same formula for the quark jet case[5] is

$$f(\text{quark}) = 1 - \frac{\alpha(E)}{\pi}\left\{4\ln(2\varepsilon) + 3\right\} C_2(R)\ln(\delta) \quad , \tag{6}$$

where the color factor $C_2(R) = 4/3$ for a quark triplet. For the same values of f, E and ε, the ratio of the quark-jet angular radius to the gluon-jet angular radius is independent of $\alpha(E)$:

$$\frac{\ln[\delta(\text{quark})]}{\ln[\delta(\text{gluon})]} = \frac{C_2(G)\ln(2\varepsilon) + \frac{11}{12}C_2(G) - \frac{4}{3}N_f T(R)}{C_2(R)\left[\ln(2\varepsilon) + \frac{3}{4}\right]} \simeq \frac{C_2(G)}{C_2(R)} = \frac{9}{4} \quad , \tag{7}$$

where the approximation is qualitatively valid for $\varepsilon < 0.2$ and $N_f = 4$. Hence $\delta(\text{gluon}) \simeq (\delta(\text{quark}))^{4/9}$ where δ is measured in radians; this shows that the gluon jet spreads more than the quark jet. If we assume $\alpha(E)$ to be the running coupling constant in QCD, then the decrease of the gluon-jet radius with increasing energy E is much slower than that of the quark-jet radius. This general qualitative feature is mainly a consequence of the difference in the color factors associated with the representations of the quark and the gluon in QCD. Typically, a quark jet of angular radius 10° (5°) corresponds to a gluon jet of angular radius 27° (19°). Also the threshold energy for the detection of a gluon jet may be as much as an order of magnitude bigger than that (~ 3 GeV) for the detection of a quark jet. This renders the observation of gluon jets very difficult.[16] It is intriguing to consider this as the reason why gluon-jet structures have so far escaped detection.

Although we expect a marked increase in sphericity (or spherocity) when the energy in e^+e^- annihilation approaches that of a heavy-quark-anti-quark bound state, its distinct three-gluon angular distributions would be difficult to measure.

better place to test the gluonic structure in QCD is probably the photon angular and energy distribution in the two-gluon-one-photon decay mode[17] of Υ. This test is even better if there exists another heavy charged 2/3 quark.

We were informed that this problem is also under investigation by M.B. Einhorn and B. Weeks. We thank our colleagues at Fermilab, in particular C.K. Lee, for discussions.

REFERENCES AND FOOTNOTES

[1] G. Hanson, et al., Phys. Rev. Lett. 35, 1609 (1975); G. Hanson, SLAC Report No. SLAC-PUB-1814 (1976).

[2] T. Appelquist and H.D. Politzer, Phys. Rev. Lett. 34, 43 (1975).

[3] S. Herb, et al., Phys. Rev. Lett. 39, 252 (1977); W.R. Innes, et al., ibid. 39, 1240 (1977).

[4] T.A. DeGrand, Y.J. Ng and S.-H.H. Tye, Phys. Rev. D16, 3257 (1977); K. Koller and T.F. Walsh, Phys. Lett. 72B, 227 (1977); H. Fritzsch and K.-H. Streng, Phys. Lett. 74B, 90 (1978); A. De Rujula, J. Ellis, E.G. Floratos and M.K. Gaillard, CERN preprint TH-2455-CERN (1978).

[5] G. Sterman and S. Weinberg, Phys. Rev. Lett. 39, 1436 (1977). We refer the readers to this work for a general discussion on the validity and applicability of their method. Here we follow their notations.

[6] H. Georgi and M. Machacek, Phys. Rev. Lett. 39, 1237 (1977); E. Farhi, ibid. 39, 1587 (1977).

[7] This pseudoscalar source will be suitable for the description of the two-gluon decay from a possible 0^{-+} $q\bar{q}$ bound-state produced in the radiative decay of Υ. However, instanton effects may be important in this case. Neglecting this

complication, the scalar and pseudoscalar sources give, to order g^2, the same gluon jet angular radius and its energy dependence (i.e. eq. (1)). Here we choose to discuss the scalar source since the renormalization of this source has been extensively studied in the literature. See, e.g. H. Kluberg-Stern and J.B. Zuber, Phys. Rev. D12, 467 (1975); C.K. Lee, ibid. D14, 1078 (1976).

[8] The problem of infrared and mass singularities has been discussed exhaustively in the literature. We refer the readers to ref. 5 for an extensive list. In addition, see also J.M. Cornwall and G. Tiktopoulos, Phys. Rev. D13, 3370 (1976); J. Frenkel and J.C. Taylor, Nucl. Phys. B116, 185 (1976); T. Kinoshita and A. Ukawa, Phys. Rev. D16, 332 (1977); A. Sugamoto, Phys. Rev. D16, 1065 (1977); G. Sterman, Phys. Rev. D17, 616 (1978). For specific applications motivated by ref. 5, see e.g. C.L. Basham, L.S. Brown, S.D. Ellis and S.T. Love, Phys. Rev. D17, 2298 (1978); S.Y. Pi, R.L. Jaffe and F.E. Low, MIT preprint CTP-715 (1978); D. Amati, R. Petronzio and G. Veneziano, CERN-TH-2470 (1978).

[9] The scattering of a gluon by an external source $(F^a_{\mu\nu})^2$ has been studied by A.G. Alvarez, Nucl. Phys. B120, 355 (1977). He demonstrated the finiteness of the leading order correction and obtained a formula which is similar to ours. Here we investigate the production of two gluons in the center of mass frame with the inclusion of quarks.

[10] G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972); C.G. Bollini and J.J. Giambiagi, Nuovo Cimento 12B, 120 (1972). See also, W.J. Marciano and A. Sirlin, Nucl. Phys. B88, 86 (1975); R. Gastmans and R. Meuldermans, Nucl. Phys, B63, 277 (1973).

[11] Here we follow the group notation of D.J. Gross and F. Wilczek, Phys. Rev. D8, 3633 (1973).

[12]By means of this off-mass-shell renormalization procedure for gluon momenta the ultraviolet divergences are separated from the infrared and mass singularities. In eq. (2) we have included the contribution arising from the ultraviolet-finite renormalization factor $(Z_3(\mu^2)/Z_3(0))^{1/2}$ for each external gluon line; this factor contains an infrared divergent piece although it is ultraviolet finite.

[13]The gluon part of the anomalous dimension in eq. (3) agrees with that obtained in ref. (7). In our present case, where the external gluons are massless and the outgoing source momentum is time-like, there is no mixing of the anomalous dimension with those of other operators at the one-loop level, as shown in C.K. Lee, ref. (7).

[14]That this coefficient is simply the β function (first evaluated by H.D. Politzer, Phys. Rev. Lett. 26, 1346 (1973), and D. Gross and F. Wilczek, ibid. 26, 1343 (1973)) is observed by A.G. Alvarez, ref. 9. The inclusion of the quark contribution in our case makes this connection more transparent.

[15]In eq. (3), terms independent of logarithms are somewhat ambiguous in the sense that they are related partly to the renormalization of the source and partly to gluon-jet properties. We believe that the former part is removed when one considers the ratio $f = 4\pi\sigma(E,\epsilon,\delta)/\sigma_{tot}(E)$ defined in eq. (5), where $\sigma_{tot}(E)$ is the total cross section computed to order $\alpha(E)$. In eq. (5) we simply use the lowest-order expression for $\sigma_{tot}(E)$, since here we are interested only in the leading contribution proportional to $\ln(\delta)$. As we shall see, our main result depends only on the $\ln(2\epsilon)\ln(\delta)$ term. Hence our conclusion is applicable to all scattering processes if the coefficient of this double logarithmic term is independent of the source. This is a weak form of the assumption we mentioned earlier.

[16]We have implicitly assumed that the transverse momentum of the gluon fragmentation is small compared to that of the Bremsstrahlung effect. Otherwise, gluon jets would be even harder to detect. It has even been speculated that gluon jets may not exist. See e.g. G.L. Kane and Y.-P. Yau, University of Michigan preprint UM-HE-77-44 (1977).

[17]S.J. Brodsky, D.G. Coyne, T.A. DeGrand and R.R. Horgan, Phys. Lett. 73B, 203 (1978); M. Krammer and H. Krasemann, ibid. 73B, 58 (1978).

FIGURE CAPTION

Fig. 1: Leading-order corrections to the two gluon production cross section. Wavy lines are gluons, dotted lines are Faddeev-Popov ghosts and solid lines are massless quarks. The dashed lines numbered 1 and 2 denote the unitary cuts. Mirror reflections of the diagrams shown above and diagrams that do not contribute in our calculation are not shown.

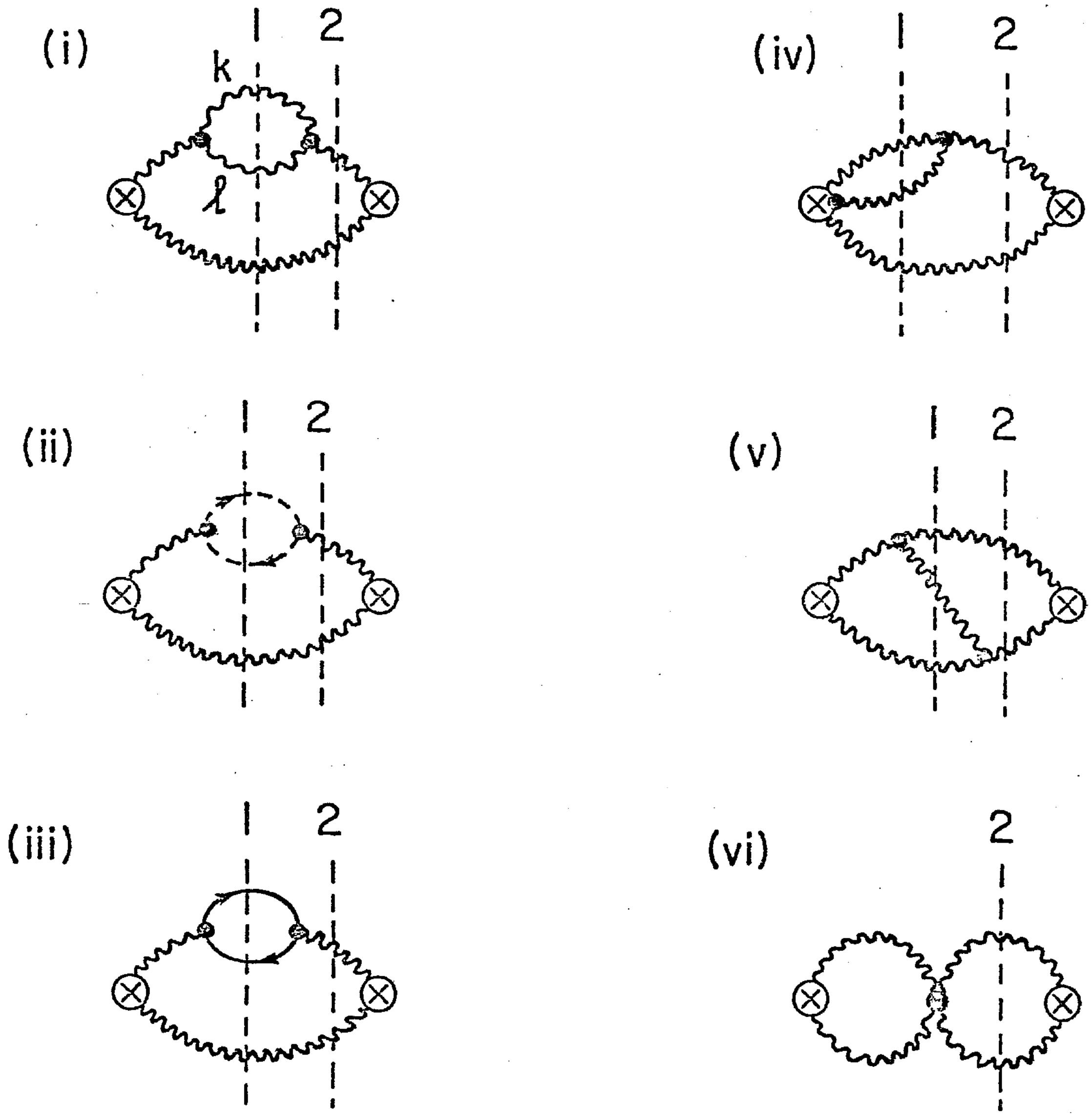

FIG. 1