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Large Order Estimates for Perturbation Theory of a Yang-Mills Field Coupled to a Scalar Field

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ABSTRACT

Using an interation procedure we have found the six possible forms of classical solutions to coupled Yang-Mills and isodoublet scalar fields which are needed to learn the high order behavior of the perturbation series of that field theory. For some of these solutions we have obtained solutions in closed form; for others we numerically estimate their contribution to the asymptotic behavior of the perturbation theory coefficients. It turns out that in the limit of a pure Yang-Mills theory, the most important contribution comes from an instanton-anti-instanton configuration and leads to the conclusion that the perturbation theory is not definable by a Borel summation technique.

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I. INTRODUCTION

Recently a simple method for large order estimates in quantum field theory (QFT) was suggested.[1] It is based on the calculation of the Feynman path integral for Green's functions in large orders of perturbation theory by using the steepest descent method. At the first step of the calculation the classical solution of the field equations must be found. At the next stage we should verify that this solution satisfies the necessary conditions to be a saddle point of the functional integral and that it supplies the maximum possible value for the integrand. Finally the quantum fluctuations near the saddle point should be calculated. This gives the possibility for finding an overall constant in the asymptotic formulae. For renormalizable scalar theories such a program was carried out.[1,2] The Sobolev inequalities[3] were used in Ref. (4) to prove that spherically symmetrical solutions of the classical equations give the maximal possible value of the integrand. For gauge theories the problem of showing the satisfaction of the saddle point conditions looks more complicated. However, in Ref. (5) a method was suggested that allowed this for the case of scalar electrodynamics: namely, to find the forms of the classical solutions and to verify the necessary saddle point conditions for them in a certain region of parameters. In this paper we apply this method to the model of Yang-Mills field interacting with a scalar field. This model is very close to the well known Weinberg-Salam theory of the electromagnetic and weak interactions.[6] We show below that in this case there are six forms of the solutions that can be obtained by using an interaction procedure in the parameter m/k where m and k are perturbative orders in the Yang-Mills coupling constant g^2 and in the scalar self-coupling. In particular, when $m/k \to \infty$, the solution with the maximal value of the integrand has the form of two instantons.[7] A short version of this work is published.[8]

The method of Ref. (1) can be considered as a certain quantative formulation of Dyson's original arguments[9] of the divergence of perturbation series as a result of instability of the ground state for a negative sign of the coupling constant. The applicability of these arguments to the problem of the anharmonic oscillator in quantum mechanics was verified in Ref. (10). Furthermore, it was demonstrated that the asymptotics of coefficients of the perturbative expansion was inimately related to the discontinuity of Green's functions on the cut in the coupling constant plane in the vicinity of $g = 0$. This discontinuity can be calculated at small $g < 0$ by using the semiclassical approximation for the probability of penetration through a potential barrier.[10] The saddle point method for this problem was carried out in Refs. (11-16) and was generalized to the case of QFT in Refs. (15, 16). The Borel summability of the perturbative expansion was discussed in Refs. (1, 17-19). Very interesting problems arise in spinor theories.[20,21]

In the next section we formulate the model--the large perturbative orders of which, we are going to study.

II. MODEL

The action for the SU(2) Yang-Mills theory with a triplet of vector fields $\vec{A}_\mu$ interacting with a doublet of charged scalar fields ϕ can be written in Euclidean space in the form

$$S \equiv \int H\, d^4x = \int d^4x \left\{ \frac{1}{4} \vec{F}^2_{\mu\nu} + \left| \left(\partial_\mu - ig \frac{\vec{\tau}}{2} \cdot A_\mu \right) \phi \right|^2 + \frac{\lambda |\phi|^4}{2} \right\}. \quad (1)$$

Here

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, \epsilon_{abc} A^b_\mu A^c_\nu , \quad (2)$$

is the intensity of the Yang-Mills, τ^a are the Pauli matrices.

We set the mass of the scalar field equal to zero because we are interested only in the short distance behavior of Green's functions. The momentum μ of the normalization point for the invariant charges $g^2(p^2)|_{p^2=\mu^2} = g^2$, $\lambda(p^2)|_{p^2=\mu^2} = \lambda$ is taken to be larger than the normalized mass of the scalar particle: $\mu >> m$.

Green's functions $G(x_1 \dots x_M;\, y_1 \dots y_N;\, z_1 \dots z_N)$ in this theory can be calculated by using the perturbative expansion in the charges g and λ:

$$G(x_1,,, z_N) = \sum_{k,m} (g^2)^m \lambda^k G_{km}(x_1 \dots z_N) , \quad (3)$$

where $G_{km}(x_1 \dots z_N)$ is given in the Lorentz gauge by the Feynman path integral[22]

$$G_{km}(x_1 \ldots x_M;\ y_1 \ldots y_N;\ z_0 \ldots z_N) =$$

$$= Z_0^{-1} \int \prod_{x,\nu,a,z} dA_\nu^a(x)\, d\phi^r(x)\, d\phi^{*r}(x)\, \delta(\partial_\sigma A_\sigma^a)\, \mathrm{Det}(\partial_\sigma \nabla_\sigma(A))$$

$$\cdot \prod_{i=1}^{M} A_{\mu_i}(x_i) \prod_{j=1}^{N} \phi_{r_j}(y_j)\, \phi^*_{r_j}(z_j) \oint \frac{dg^2}{(2\pi)(g^2)^{m+1}} \oint \frac{d\lambda}{(2\pi)\lambda^{k+1}}$$

$$\exp\left[-S(A,\phi,g,\lambda) - \int d^4x\, H'(A,\phi,g,\lambda)\right]. \tag{4}$$

Here H' is a counter term corresponding to the renormalizations of the scalar field mass, of the wave function, and of the charges g and λ. It can be calculated in low orders of perturbation theory. The factor Z_0 is chosen in such a way that G_{00} is equal to the product of the free Green's functions. The integration over g^2 and λ in Eq. (4) is performed along contours closed around zero. The contribution of the disconnected diagrams to Eq. (4) is not essential at large k and m (see Ref. 1).

According to the method of Ref. (1) we should find the saddle point in the integral (4) for each variable A, ϕ, g, λ. This saddle point can be obtained as a solution of the stationarity conditions of the functional

$$\Phi = S + m\,\ell n\, g^2 + K\ell n\,\lambda, \quad \delta\Phi = 0\,. \tag{5}$$

The variations of A and ϕ give the usual classical equations. We should find their solution with a finite action. This means that the solutions must be space-limited. Due to translational and dilatational invariance of the action (1), an arbitrary solution can be expressed in terms of one having its center at $x = 0$ and its scale equal to unity:

$$\phi^{x_0, y}(x) = \frac{1}{y}\tilde{\phi}\left(\frac{x - x_0}{y}\right), \vec{A}_\nu^{x_0, \lambda} = \frac{1}{\lambda}\tilde{\vec{A}}_\nu\left(\frac{x - x_0}{\lambda}\right). \tag{6}$$

The solution $\tilde{\phi}(x)$ or $\tilde{\vec{A}}_\nu(x)$ is not invariant under the shift and scale transformations. Therefore, we can hope to conserve only invariance of the solution under the 10 parameter subgroup of the total 15 parameter symmetry group of the massless action (1). It is convenient to make the invariance obvious by passing to five dimensional coordinate space according to the equation[23]

$$Z_\mu = \frac{2x_\mu}{1 + x^2}, \quad \mu = 1, 2, 3, 4; \; Z_5 = \frac{x^2 - 1}{x^2 + 1}, \; Z_i^2 = 1$$

$$\int d^5 Z \; \delta\left(\sqrt{Z_i^2} - 1\right) \equiv \int dS_5 = \int d^4 x \left(\frac{2}{1+x^2}\right)^4, \tag{7}$$

and to new fields $\vec{A}_i$ and Y

$$\phi(x) = \frac{2}{Hx^2} Y(Z)$$

$$\vec{A}_\nu(x) = \frac{\partial Z_i}{\partial x_\nu} \vec{A}_i(Z), \; \frac{\partial Z_i}{\partial x_\nu} \frac{\partial Z_k}{\partial x_\nu} = \left(\frac{2}{1+x^2}\right)^2 \left(\delta_{ik} - Z_i Z_k\right). \tag{8}$$

Here the five components of A_i satisfy the constraint[23]

$$Z_i \vec{A}_i = 0. \tag{9}$$

Using the previous formulas we can rewrite the action (1) in a form manifestly invariant under the 10 parameter group of rotations of the five-dimensional space:

$$S \equiv \int dS_5 \mathscr{L} = \int dS_5 \left\{ \frac{1}{12} \left[L_{ij} A_k^a + g Z_i \epsilon_{abc} A_j^b A_k^c + \begin{pmatrix} i \to j \\ \nwarrow k \swarrow \end{pmatrix} + \begin{pmatrix} i \to k \\ \nwarrow j \swarrow \end{pmatrix} \right]^2 + \right.$$

$$\left. + \frac{1}{2} \left| \left[L_{ij} - ig \frac{\vec{\tau}}{2} (Z_i \vec{A}_j - Z_j \vec{A}_i) \right] Y \right|^2 + 2 \left| Y \right|^2 + \frac{\lambda}{2} \left| Y \right|^4 \right\} , \tag{10}$$

where

$$L_{ij} = Z_i \partial_j - Z_j \partial_i , \tag{11}$$

is the infinitesimal anti-hermitean generator of rotations in the plane (i, j).

Using the invariance of the functional (10) under gauge transformations, we can choose the following Lorentz-like gauge condition[23]

$$[\partial_i - Z_i (Z \partial)] A_i = 0 . \tag{12}$$

Then the classical equations for the action (10) have the form

$$\left(-\frac{1}{2} L_{ij}^2 + 2\right) A_k^a + g \epsilon_{abc} \left[-4 A_j^b \partial_j A_k^c - 2 \left(\partial_k A_j^b\right) A_j^c \right] + g^2 \left(2 A_j^c A_j^c A_k^a - 2 A_k^c A_j^c A_j^a \right) +$$

$$+ \frac{g}{2} Z_i \left[\left(L_{ik} Y^* \right) i \tau^a Y - Y^* i \tau^a L_{ik} Y \right] + \frac{g^2}{2} A_k^a \left| Y \right|^2 = 0$$

$$\left(-\frac{1}{2} L_{ij}^2 + 2 + \lambda \left| Y \right|^2 + \frac{1}{4} A_i^2 \right) Y - \frac{1}{2} L_{ij} (i\tau^a) Z_i A_j^a Y + \frac{1}{2} Z_i A_j L_{ij} (i\tau^a) Y = 0$$

$$\left(-\frac{1}{2} L_{ij}^2 + 2 + \lambda \left| Y \right|^2 + \frac{1}{4} A_i^2 \right) Y^* + \frac{1}{2} L_{ij} (i\tau^a) Z_i A_j^a Y^* - \frac{1}{2} Z_i A_j L_{ij} (i\tau^a) Y^* = 0 . \tag{13}$$

These equations should be combined with conditions (9), (12):

$$Z_i A_i = 0 \quad (\partial_i - Z_i (Z\partial)) A_i = 0 \ . \tag{14}$$

Variation of the functional (5) with respect to g^2 and λ leads to the relations

$$k = -\frac{\partial S}{\partial \ln g^2} \ , m = -\frac{\partial S}{\partial \ln \lambda} \ , \tag{15}$$

that fix the saddle point value for g^2 and λ in Eq. (4). In the next section we find the form of the solutions of Eq. (13) that satisy the necessary saddle point conditions.

III. THE FORM OF SOLUTIONS OF CLASSICAL EQUATIONS

To solve Eq. (13) we need some Ansatz for the form of their solution. For this purpose we use the same procedure that was applied to scalar electrodynmaics in Ref. (5).

To begin with, let us consider the case

$$m << k \ . \tag{16}$$

Then it is natural to expect that the saddle point in Eq. (4) will be close to that of the purely scalar theory. In the scalar theory the solution fulfilling the necessary saddle point conditions is shown to be a constant on the 5-dimensional sphere.[1,2] It can be obtained easily if we put $A = 0$ in Eq. (13):

$$Y^{(0)} = \sqrt{-\frac{2}{\lambda}} \, U, \ U^* U = 1 \ , \tag{17}$$

where U is a constant spinor. The solution (17) provides the maximum possible value for the integrand in the path integral for the large order coefficients of perturbation theory.[3,4] Other solutions with the same property can be obtained by using the shift and scale transformations (see(6)). We shall fix the position of the solution in the four dimension Eucliedean space and its scale by imposing on it the following constaint:

$$\int dS_5 \, \mathscr{L}(\tilde{A}, \tilde{Y}) Z_i = 0 . \tag{18}$$

In the approximation $A_i^{(0)} = 0$ the solution (17) satisfies this constraint.

Using (15) in region (16) we get the following characteristic values for fields A_i and Y

$$Y \sim \sqrt{k}, \; A_i \sim \sqrt{m} << \sqrt{k} . \tag{19}$$

Therefore we can omit in the first approximation all nonquadratic terms for the field A in action (10) when calculating the integral (4) in region (16). It corresponds to the linearization of the first equation of (13):

$$\left\{ -\frac{1}{2} L_{ij}^2 + 2 + \frac{g^2}{2} \left| Y^0 \right|^2 \right\} \vec{A}_2^{(1)} = 0 . \tag{20}$$

The term of the zeroth order in A vanishes due to the relation $L_{ij} Y^{(0)} = 0$. A nontrivial solution of Eq. (20) does not exist for all values of g. In the general case we have the following solution

$$\vec{A}_r^{(1)}(n) = \sum_{i_1, i_2 \ldots i_n} \vec{\eta}_{r i_1 \ldots i_n} \left\{ Z_{i_1} \ldots Z_{i_n} \right\} , \tag{21}$$

where $\left\{Z_{i_1} \ldots Z_{i_n}\right\}$ is the symmetrical traceless polynomial $P^n(Z_r)$:

$$\delta_{i_r i_{r'}} \left\{Z_{i_1} \ldots Z_{i_n}\right\} = 0 , \tag{22}$$

and $\vec{\eta}_{r\, i_1 \ldots i_n}$ are some arbitrary coefficients.

The eigenvalue $\tilde{g}^{(1)}_{(n)}$ corresponding to functions (21) are

$$\left(g^{(2)}_{(n)}\right)^2 = \frac{-2}{\left|Y^{(0)}\right|^2} (n+1)(n+2). \tag{23}$$

The series (3) in the region $m >> 1$ has a finite radius of convergence in g^2 which is determined by the minimal eigenvalue for $g^{(1)}$ in Eq.(20). This minimal value corresponds to $n = 0$ in Eq. (23) but the corresponding eigenfunction (21) does not satisfy the subsidary conditions (14). Hence we must take the next value for $g^{(1)}$ in Eq. (23) that corresponds to the eigenfunction with $n = 1$ in Eq. (21):

$$\left(g^{(1)}\right)^2 = - \frac{12}{\left|Y^0\right|^2} = 6\,\lambda \sim \frac{1}{k}$$

$$\vec{A}^{(1)}_r = \epsilon\, \vec{\eta}_{rr'}\, Z_{r'} \ . \tag{24}$$

Here we have separated a factor ϵ in order to normalize η in a suitable way (see (32), (40)). The matrices $\vec{\eta}_{rr'}$ should be antisymmetric as is seen from conditions (14):

$$\vec{\eta}^{\,T} = - \vec{\eta} \ . \tag{25}$$

Other constraints on $\vec{\eta}_{rr'}$ are obtained below from the condition that the iteration of Eq. (13) in the small parameter ϵ is possible. We can expand the coupling constant g and the fields $\vec{A}_r$, Y in the series:

$$\vec{A}_r = \epsilon\, \vec{\eta}_{rr'} Z_{r'} + \epsilon^2 \vec{A}_r^{(2)} + \epsilon^3 \vec{A}_r^{(3)} + \ldots$$

$$Y = \sqrt{-\frac{2}{\lambda}}\; U + \epsilon^2 Y^{(2)} + \ldots$$

$$g^2 = +6\lambda + \epsilon \left(g^{(2)}\right)^2 + \ldots \tag{26}$$

and calculate the coefficients in the series from relations (15). For example we have the following equation in the second order in ϵ:

$$\left(-\frac{1}{2} L_{ij}^2 - 8\right) A_k^{(2)} = -\frac{\left(g^{(2)}\right)^2}{2} \left(-\frac{2}{\lambda}\right) \eta^a_{kk'} Z_{k'} - 6\, \eta^b_{kj} \eta^c_{jk'} Z_{k'}\, \epsilon_{abc} \;. \tag{27}$$

The left hand side of Eq. (27) does not contain the first harmonics due to our choice of $\tilde{g}^{(1)}$ (24). It means that Equation (27) has a solution for $A_k^{(2)}$ only if η satisfies the constraint

$$\left[\eta^b, \eta^c\right] = C_1\, \epsilon_{abc}\, \eta^a , \tag{28}$$

where C_1 is an arbitrary constant.

In such a case we can find $g^{(2)}$ to make the right hand side of Eq. (27) equal to zero

$$\left(g^{(2)}\right)^2 = 6\lambda C_1 . \tag{29}$$

Analogous reasoning in the third order in ϵ gives two more constraints:

$$\eta^a \eta^a \eta^b = C_2 \eta^b , \tag{30}$$

$$\mathrm{Sp}(\eta^a \eta^b)\eta^b = C_3 \eta^a , \tag{31}$$

where C_2 and C_3 are arbitrary constants. It can be verified that the fulfilling of the constraints (25), (28), (30) and (31) is sufficiently for conducting the above iteration procedure to the arbitrary order in ϵ. Below we shall find all solutions of these constraints.

To begin with, let us consider the case when $C_1 \neq 0$ in Eq. (28). Then we can set C_1 equal to unity thus eliminating the ambiguity in extracting the factor ϵ in Eq. (24):

$$[\eta^b \eta^c] = \epsilon_{abc} \eta^a . \tag{32}$$

So, we must find three pure imaginary antisymmetric matrices $i\eta^a$ that have the commutation relations of the generators of the SU_2 group. In a general case such matrices provide a representation of the SU_2 algebra in some subspace of the five dimentional coordinate space. Due to the constraint (30) this representation is irreducible or it consists of irreducible representations with the same weights. We have only three possibilities for such matrices. Namely they can correspond to representations with isospins $T = 2, 1$ and $(1/2 \times 1/2)$ and can be written in the form (see Table 1):

$$\text{I} \qquad \eta^a_{ij} = \gamma^i_{cd}\,\gamma^j_{c'd'}\,(-\epsilon_{acc'}\,\delta_{dd'} - \epsilon_{acd'}\,\delta_{dc'}) \;, \tag{33}$$

$$\text{II} \qquad \eta^a_{ij} = \gamma^i_c\,\gamma^j_{c'}\,(-\epsilon_{acc'})\,;\; a, c', c = 1, 2, 3 \;, \tag{34}$$

$$\text{III} \qquad \eta^a_{ij} = \gamma^i_\mu\,\gamma^i_\nu\,\eta^a_{\mu\nu}\;;\; \mu, \nu = 1, 2, 3, 4 \;, \tag{35}$$

where the matrices γ^i_{cd}, γ^i_c, γ^i_μ satisfy the conditions

$$\begin{aligned}
&\gamma^i_{aa} = 0,\; \gamma^i_{ab} = \gamma^i_{ba},\; \mathrm{Sp}(\gamma^i\gamma^k) = \delta_{ik}, \\
&(\gamma^i)_{ab}\,(\gamma^i)_{cd} = \frac{1}{2}(\delta_{ac}\,\delta_{bd} + \delta_{ad}\,\delta_{bc}) - \frac{1}{3}\,\delta_{ab}\,\delta_{cd} \\
&\gamma^i_c\gamma^i_{c'} = \delta_{cc'},\; \gamma^i_c\gamma^j_c = (P)_{ij},\quad P^2 = P \\
&\gamma^i_\mu\gamma^i_{\mu'} = \delta_{\mu\mu'},\; \gamma^i_\mu\gamma^j_\mu = (P)_{ij},\quad P^2 = P \;,
\end{aligned} \tag{36}$$

and can be written in the following particular form after an appropriate orthogonal transformation in the five dimensional space (see Table 1)

$$\hat{Z} \equiv Z_i\;\gamma^i_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} \left[Z_4 - \frac{1}{\sqrt{3}}\,Z_5\right] & Z_1 & Z_2 \\ Z_1 & \left[-Z_4 - \frac{1}{\sqrt{4}}\,Z_5\right] & Z_3 \\ Z_2 & Z_3 & +\frac{2}{\sqrt{3}}\,Z_5 \end{pmatrix}_{ab} \;,$$

$$\gamma^i_c = \delta_{ic},\; \gamma^i_\mu = \delta_{i\mu} \,. \tag{37}$$

The matrices $\eta^a_{\mu\nu}$ in Eq. (35) differ from the 't Hooft matrices only by a factor (see Table 1)

$$\eta^3 = \frac{1}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & +1 & 0 \end{pmatrix}, \quad \eta^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \eta^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \tag{38}$$

Now we consider the other possibility $C_1 = 0$ in Eq. (31). Then it can be shown that by an appropriate ortogonal transformation

$$\eta^a_{ij} = \gamma^i_{i'} \gamma^j_{j'} \eta^a_{i'j'} \ , \quad \gamma^i_{i'} \gamma^j_{i'} = \delta_{ij} \ , \tag{39}$$

These three matrices can be transformed to the following form

$$\eta^a = \ell^a_1 \eta_1 + \ell^a_2 \eta_2, \quad \eta_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \eta_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{40}$$

and ℓ^a_1, ℓ^a_2 are some vectors in isotopic space.

If we substitute expression (40) in Eq. (33), we get three possibilities (see Table 1):

$$\text{IV} \qquad (\ell^a_1)^2 = (\ell^a_2)^2 = 1, \ \ell^a_1 \ell^a_2 = 0 \tag{41}$$

$$\text{V} \qquad \ell^a_1 = \ell^a_2, \ (\ell^a_1)^2 = 1 \tag{42}$$

$$\text{VI} \qquad (\ell^a_1)^2 = 1, \ \ell^a_2 = 0 \ . \tag{43}$$

The case $(\ell_2^a)^2 = 1$, $\ell_1^a = 0$ can be reduced to Eq. (43) by an appropriate orthogonal transformation (39). In Eqs. (41)-(43) we choose a certain normalization of matrices η thus eliminating the ambiguity in separating the factor ϵ in Eq. (24).

Now we can pass on to finding the form of the solution of Eq. (13). We remember that the iteration of Eq. (13) in terms of ϵ does not meet any difficulty provided the matrices $\vec{\eta}$ in Eq. (27) satisfy the conditions (28), (31) and (33). The solution is expressed as a series in $\epsilon\vec{\eta}$. It can depend on the following invariants

$$(\eta^a\eta^a)_{ij} Z_i Z_j , \quad (\eta^a\eta^a\eta^b\eta^b)_{ij} Z_i Z_j , \left\{(\eta^a\eta^b)_{i_1 j_1} Z_{i_1} Z_{j_1}\right\}^2 \ldots , \tag{43}$$

and so on. The use of constraints (28), (30), (31) gives us the possibility to reduce the number of independent invariants.

To begin with, let us consider case I (33). Then an arbitrary invariant function can be expressed in terms of the following invariant structures (see (37))

$$S_n = \mathrm{Sp}\,(\hat{Z}^n) . \tag{44}$$

Furthermore, independent invariants can be obtained from the expansion of the characteristic polynomial

$$\begin{aligned} P^3(\lambda) \;\equiv\; \mathrm{Det}\,(\hat{Z} - \lambda I) &= -\lambda^3 + \lambda^2\, \mathrm{Sp}\,\hat{Z} - \lambda\, \frac{1}{2}\left(\mathrm{Sp}(\hat{Z}^2) - (\mathrm{Sp}\hat{Z})^2\right) + \\ &+ \frac{1}{6}\,(\mathrm{Sp}\hat{Z})^3 - \frac{1}{2}\,(\mathrm{Sp}\hat{Z})\;\mathrm{Sp}(\hat{Z}^2) + \frac{1}{3}\,\mathrm{Sp}(\hat{Z}^3) . \end{aligned} \tag{45}$$

Due to Eq. (36) we get

$$\mathrm{Sp}\, Z = 0, \; \mathrm{Sp}(Z^2) = Z_i Z_i = 1 , \tag{46}$$

and therefore in this case we have only one independent invariant

$$S = \frac{1}{3} \; \mathrm{Sp}(Z)^3 = \mathrm{Det}\; Z . \tag{47}$$

If we introduce the matrix

$$h_{ab} = \eta^a_{ik} \eta^b_{i\ell} Z_k Z_\ell = 2(2 - 3\hat{Z}^2)_{ab} . \tag{48}$$

We can express S in terms of matrices η_{ik} by using the formula

$$\mathrm{Sp}(h^3) = 6(11 - 108S^2) . \tag{49}$$

Further, there is the following useful relation:

$$\hat{Z}^3 = -\frac{1}{2}\hat{Z} + S \cdot I , \tag{50}$$

because the eigenvalues of $\hat{Z}$ due to Eq. (45) satisfy the equation

$$-\lambda^3 - \frac{1}{2}\lambda + S = 0 . \tag{51}$$

Now we want to find the possible structures on which the vector potential A^a_i can depend. Generally these structures may be of the form

$$(\hat{Z}^n \gamma_i \hat{Z}^m)_{bc} \epsilon_{a'bc} (\hat{Z}^k)_{aa'} , \tag{52}$$

with arbitrary n, m, k. But if we take into account relation (50) and the identity

$$\epsilon_{a'bc}\,\delta_{aa''} = \epsilon_{abc}\,\delta_{a'a''} - \epsilon_{abc}\,\delta_{ca''} - \epsilon_{aa'c}\,\delta_{ba''}\,, \tag{53}$$

the number of independent structures can be reduced to three. Thus we can look for a solution in the form

$$A_i^a = \epsilon_{abc}\left[a_1(s)\widehat{Z}\,\gamma_i + a_2(s)\widehat{Z}^2\,\gamma_i + a_3(s)\widehat{Z}^2\,\gamma_i\widehat{Z}\right]_{cb}$$

$$Y = U\,Y(s)\,,\ U^*U = 1\,, \tag{54}$$

where $a_i(s)$ and $Y(s)$ are functions of S (47). The expression (54) can be written in terms of matrices η^a_{ij} if we use the relations (see (48)):

$$\eta^a_{ik}Z_k = -2\epsilon_{abc}(\widehat{Z}\gamma_i)_{cb}\,,\ h_{ab}\eta^b_{ik}Z_k = -2\epsilon_{abc}(\widehat{Z}\gamma_i - 6\widehat{Z}^2\gamma_i\widehat{Z})_{cb}\,,$$

$$(h^2)_{ab}\,\eta^b_{ik}Z_k = -2\epsilon_{abc}(\widehat{Z}\gamma_i - 30\,\widehat{Z}^2\gamma_i\widehat{Z} - 18\,\widehat{Z}^2\gamma)_{cb}\ . \tag{49a}$$

Ansatz (54) does not contradict the system of Eq. (13). The equations for $a_i(s)$, $Y(s)$ can be easily found if we express the action (10) in terms of these functions. Using the formula

$$\int d^5Z\,\delta\,(\sqrt{Z_i^2} - 1)\,\delta\,(\mathrm{Det}\,\widehat{Z} - s) = 4\sqrt{6}\,\pi^2\,\theta\left(\frac{1}{54} - s^2\right)\,, \tag{55}$$

we get the following expression for the action (10) in terms of the functions $a_i(s)$ and $Y(s)$:

$$S = \sqrt{6}\ \pi^2 \int_{-1/3\sqrt{6}}^{1/3\sqrt{6}} ds \Big\{ W\Big(\frac{1}{2}\, a'^2 + \frac{1}{12}\, Wa'^2_2 + \frac{1}{36}\, Wa'^2_3 - 2a'a_2\Big) + 18a^2 + 9\, Wa_2^2 + \frac{10}{3}\, Wa_3^2$$

$$+ g\Big[3a^3 - 3SWa_2^3 + \frac{1}{9}\, Wa_3^3 + \frac{1}{2}\, Wa_2^2\,(3a - a_3)\Big] + g^2\Big[\frac{9}{8}\, a^4 + \frac{1}{32}\, W^2 a_2^4 + \frac{1}{216}\, W^2 a_3^4 +$$

$$+ \frac{3}{8}\, Wa^2 a_2^2 + \frac{1}{2}\, Wa^2 a_3 (a - 9Sa_2) - \frac{1}{4}\, W^2 a_2^2\, a_3\,(a - Sa_2)\Big] +$$

$$+ \frac{2}{3}\, W(Y')^2 + 8\, Y^2 + 2\lambda\, Y^4 + g^2\, Y^2 \Big(\frac{3}{2}\, a^2 + \frac{1}{4}\, Wa_2^2 + \frac{1}{12}\, Wa_3^2\Big)\Big\}, \qquad (56)$$

where

$$W = 1 - 545^2;\ a = a_1 + 3Sa_2 - \frac{1}{3}\, a_3\,. \qquad (57)$$

The differential equations for a_i and Y are easily obtained from the condition of stationarity of the action (56)

$$\delta S = 0\ . \qquad (58)$$

Let us consider now cases II, III, V and VI (see (33)-(35) and (40)-(43)). Here we can construct only one independent invariant:

$$S = \eta_{ij_1}\, \eta_{ij_2}\, Z_{j_1}\, Z_{j_2}\ , \qquad (59)$$

and only one possible structure for $A_i(S)$. Therefore the solution of Eq. (13) can be found in the form

$$A_i^a = \eta_{ik}^a\, Z_k\, a(S),\ Y = U\, Y(s),\ U^+ U = 1\ . \qquad (60)$$

The simplest way to obtain the equations for a(S) and Y(S) is to express (10) in terms of a(S) and Y(S) is to express (10) in terms of a(S) and Y(S):

$$S = \frac{2\pi^{5/2}}{\Gamma\left(\frac{\eta_\perp}{2}\right)\Gamma\left(\frac{5-\eta_\perp}{2}\right)} \int_0^1 dS\, S^{\eta_\perp/2-1}(1-S)^{(3-\eta_\perp)/2}\Big\{\left[T(T+3-\eta_\perp)+1\right]$$

$$\left[2S^2(1-S)(a')^2+3Sa^2+\frac{g^2}{4}Sa^2Y^2\right]+T(T+1)\left(-gSa^3+\frac{T}{4}g^2S^2a^4\right)+$$

$$+4S(1-S)(Y')^2+2Y^2+\frac{\lambda}{2}Y^4\Big\}, \qquad (61)$$

and then to use the stationarity condition (58).

In Eq. (61) T is the total isospin of the representation realized by matrices η and $\eta_\perp$ is the dimension of the subspace in which it acts (see Table 1).

At last we consider the case IV (41). In this case we can construct two independent invariants

$$S_1 = -(\eta_1^2)_{ij} Z_i Z_j, \quad S_2 = -(\eta_2^2)_{ij} Z_i Z_j, \qquad (62)$$

and two possible structures for A_i^a:

$$A_i^a = \ell_1^a(\eta_1)_{ik} Z_k a_1(S_1, S_2) + \ell_2^a(\eta_2)_{ik} Z_k a_2(S_1, S_2)$$

$$Y = U\,Y(S_1, S_2), \quad U^+U = 1\,. \qquad (63)$$

The solution in the form (63) satisfies Eq. (13) and the differential equations for $a_i(S_1, S_2) Y(S_1, S_2)$ can be obtained from the stationarity condition (58) if we express action (10) in terms of these functions:

$$S = 8\pi^2 \int_0^1 dS_1 \int_0^{1-S_1} dS_2 \frac{1}{\sqrt{1-S_1-S_2}} \left\{ S_1(1-S_1) \left[S_1 \left(\frac{\partial a_1}{\partial S_1} \right)^2 + S_2 \left(\frac{\partial a_2}{\partial S_1} \right)^2 + 2 \left(\frac{\partial Y}{\partial S_1} \right)^2 \right] \right.$$

$$+ S_2(1-S_2) \left[S_1 \left(\frac{\partial a_1}{\partial S_2} \right)^2 + S_2 \left(\frac{\partial a_2}{\partial S_2} \right)^2 + 2 \left(\frac{\partial Y}{\partial S_2} \right)^2 \right] - 2 S_1 S_2 \left[S_1 \frac{\partial a_1}{\partial S_1} \frac{\partial a_1}{\partial S_2} + \right.$$

$$\left. + S_2 \frac{\partial a_2}{\partial S_1} \frac{\partial a_2}{\partial S_2} + 2 \frac{\partial Y}{\partial S_1} \frac{\partial Y}{\partial S_2} \right] + \frac{3}{2} \left(S_1 a_1^2 + S_2 a_2^2 \right) + \frac{g^2}{4} S_1 S_2 a_1^2 a_2^2 + Y^2 + \frac{\lambda}{4} Y^4 + \frac{g^2}{8} \times$$

$$\left. \times Y^2 \left(S_1 a_1^2 + S_2 a_2^2 \right) \right\} . \qquad (64)$$

We have found in this section all possible forms of solutions which satisfy constraints (14) and can be obtained by iteration Eq. (13) in the parameter $\epsilon \sim \sqrt{m/k}$ (see (24)). In the next section we shall find exact solutions for some cases and estimate the action for others.

IV. GREEN'S FUNCTIONS IN HIGH ORDERS OF PERTURBATION THEORY

We consider at first the cases II, III, V and VI. The action (61) can be rewritten in the form

$$S = \frac{2\pi^{5/2}\left(T(T+3-n_\perp)+1\right)}{g^2\cdot\Gamma\left(\frac{n_\perp}{2}\right)\Gamma\left(\frac{5-n_\perp}{2}\right)}\int_0^\infty d\xi\,(\mathrm{sh}\,\xi)^{4-n_\perp}\Bigg\{\dot{A}^2+2(n_\perp-2)A^2+\dot{\phi}^2+(n_\perp-3)\phi^2+$$

$$+\frac{T(T+1)}{T(T+3-n_\perp)+1}\left(-2A^3+\frac{T}{2}A^4\right)+\frac{1}{4}\left[T(T+3-n_\perp)+1\right](A^2\phi^2+\chi\,\phi^4)\Bigg\}, \qquad (65)$$

if we introduce new variables ξ and new functions A, ϕ by the following definition

$$S=\frac{1}{\mathrm{Ch}^2\xi}, \; a=\frac{1}{gS}\,A(\xi);\; Y=\frac{1}{g}\left[\frac{T(T+3-n_\perp)+1}{2S}\right]^{1/2}\phi(\xi),\; \chi=\frac{\lambda}{g^2}\,. \qquad (66)$$

In the case III ($T = 1/2$, $n_\perp = 4$) we obtain the following Euler equations

$$\begin{cases}\left(-\dfrac{d^2}{d\xi^2}+4+\dfrac{3}{16}\phi^2\right)A+\left(-3A^2+\dfrac{1}{2}A^3\right)=0\\[2ex] \left(-\dfrac{d^2}{d\xi^2}+1+\dfrac{3}{16}A^2\right)\phi+\dfrac{3}{8}\chi\,\phi^3=0\,.\end{cases} \qquad (67)$$

It can be easily verified that there is an exact solution of this system of equations

$$A = \frac{4\,\mathrm{Sh}\,2\xi_0}{\mathrm{Ch}\,2\xi + \mathrm{Ch}\,2\xi_0}\;,\;\phi = \pm i\,\frac{8\,\ell^{-\xi_0}}{\sqrt{\mathrm{Ch}\,2\xi + \mathrm{Ch}\,2\xi_0}}\;, \tag{68}$$

where the parameter ξ_0 is related to χ by the formula

$$e^{\xi_0} = (12\chi - 1)^{1/4}\;. \tag{69}$$

The solution (68) can be obtained by using the iteration procedure in the parameter ϵ (see (17) and (24))($\epsilon \sim \chi - 1/6$).

In terms of four-dimentional variables and fields this solution takes the form

$$A^a_\mu(x) = \frac{4\eta^a_{\mu\nu}x_\nu}{g}\;\frac{\rho^4 - 1}{(x^2+\rho^2)(1+\rho^2x^2)}\;,\;\phi = \frac{u}{g}\;\frac{4(\pm i)\sqrt{3}}{\sqrt{(x^2+\rho^2)(1+\rho^2x^2)}}$$

$$\rho = (12\chi - 1)^{1/4}\;. \tag{70}$$

We see from this equation that our exact solution for the vector field is proportional to the product of two instantons

$$A^a_\mu = \frac{4\eta^a_{\mu\nu}x_\nu}{g}\;\frac{1}{x^2+\lambda^2}\;, \tag{71}$$

with two different scales $\lambda_1 = \rho$ and $\lambda_2 = \rho^{-1}$. The action (65) can be calculated for the solution (68)

$$S = \frac{16\pi^2}{g^2}\;,\;S_0 = -2 + \frac{3(\mathrm{Sh}\,4\xi_0 - 4\xi_0)}{2\,\mathrm{Sh}^2\,2\xi_0}\;. \tag{72}$$

According to the method of Ref. (1) Green's functions in large orders of the perturbation theory are given by the following expression (see (4)):

$$G_{km} \sim \exp\left(-\tilde{\mathcal{F}}(k,m)\right) , \tag{73}$$

where $\tilde{\mathcal{F}}(k,m)$ is the value of functional (5) at the solutions of its stationarity equations (13) and

$$k = -g^2 \frac{\partial S}{\partial g^2} , \quad m = -\lambda \frac{\partial S}{\partial \lambda} . \tag{74}$$

For our exact solution (68) we obtain by using (72) the equation

$$\left\{ \begin{array}{l} f(2\xi_0) \equiv e^{2\xi_0} \, \mathrm{th}2\xi_0 \, \dfrac{\xi_0 + \frac{1}{3}\,\mathrm{Sh}^2 2\xi_0 - \frac{1}{2}\,\mathrm{Sh}\,2\xi_0\,\mathrm{Ch}\,2\xi_0}{2\xi_0\,\mathrm{Ch}\,2\xi_0 - \mathrm{Sh}\,2\xi_0} = \dfrac{k+m}{k} \\ S = k+m . \end{array} \right. \tag{75}$$

It turns out that the solution of this equation $\xi_0 = \xi_0(k+m/k)$ is complex for real values of $k+m/k$. In Fig. 1 the curve in the complex plane $2\xi_0$ is plotted on which $\mathrm{Im}\, f(2\xi_0)$ equals zero. When $k+m/k$ grows from unity to infinity the solution of Eq. (75) moves along this curve. There are two different solutions of the equation. For the first one, the point in the $2\xi_0$ plane moves from $2\xi_0 = 0$ at $m=0$ to $2\xi_0 = \pi/2\, i$ at $m=\infty$. For the second one the point moves from the point $2\xi_0 \simeq 1.3 + 2.8\, i$ to $2\xi_0 = \infty + i\pi/2$. Therefore for one classical solution (68) we have two saddle points in the integral (4).

Taking into account the solutions with complex conjugate values for $2\xi_0$ we can write Eq. (73) in the form

$$G_{k,m} = A \cdot \left(\frac{k+m}{16\pi^2 e}\right)^{k+m} R_e \left[C\left(\frac{m}{k+m}\right)\right]^{k+m} , \qquad (76)$$

where for our case C(m/k+m) due to (5) and (75) has the form (see (72))

$$C\left(\frac{m}{k+m}\right) = S_0^{-1}(2\xi_0)\left(\frac{e^{4\xi_0}+1}{12}\right)^{-\frac{k}{k+m}} , \qquad (77)$$

and ξ_0 should be determined from Eq. (75) as a function of m/k+m. We computed the modulus and phases of C(m/k+m) for the two possible branches 1 and 2 (see Table 2). At $m/k \to \infty$ the value of ξ_0 for the second branch tends to infinity. From Eq. (68) we see that ϕ vanishes in the same limit and the relative scale of two instantons goes to infinity. The scalar field becomes inessential and we end up with the pure Yang-Mills theory. In particular the action in the limit is twice that of a single instanton (71). An appropriate solution of such form was used by Bogomolny and Fateev[7] in their work on large order estimates in the pure Yang-Mills theory. Note, that the expression (68) is not the only solution of Eq. (67). For sufficiently large χ there are other real solutions. In the limit $\chi >> 1$ the vector potential for these solutions almost coincides with the one for exact solution (68) and the scalar field tends to zero but instead of a plateau at $|\xi| << \xi_0$ as in (68) the scalar field here has one or several bumps of a finite size. The action on these solutions at $\chi \to \infty$ almost equals the one for solution (68) and is smaller on the values of the order of $1/\chi$. However it can be verified numerically that the region where other solutions of Eq. (67) give the larger value for $C(m/k+m)$ in Eq. (76) is negligibly small $(|m/k+m-1| << 1)$.

We can expand the action (72) near its value for the pure scalar theory:

$$S = \frac{16\pi^2}{g^2}\left[1 - 6 \sum_{h=0}^{\infty} y \frac{h(-1)^h}{(h+1)(h+2)}\right], \; y = 12\left(\chi - \frac{1}{6}\right) . \tag{78}$$

Using Eq. (75) we obtain the following equations for the calculation of y and g

$$m = S + (2+y)\frac{\partial S}{\partial y} = \frac{16\pi^2}{g^2}\, 6 \sum_{h=2}^{\infty} \frac{y^h(-1)^h(h-1)}{(h+2)(h+3)}$$

$$k+m = S = \frac{16\pi^2}{g^2}\left[-2 - 6 \sum_{h=1}^{\infty} y^h \frac{(-1)^h}{(h+1)(h+2)}\right] . \tag{79}$$

We can find the solution of Eq. (79) in a series in $\sqrt{m/k+m}$:

$$y = \pm i \sqrt{\frac{20}{3}}\sqrt{\frac{m}{k+m}} - \frac{25}{9}\frac{m}{k+m} + \ldots \tag{80}$$

which gives us the possibility of calculating $C(m/k+m)$ in Eq. (77) for small $m/k+m$:

$$\ell n\ C\left(\frac{m}{k+m}\right) = -\ell n\ S_0(y) - \frac{k}{k+m}\ \ell n\,\frac{y+2}{12} = i\,\pi + \ell n\,3 - \frac{m}{k+m}\,\ell n\,6 - \frac{5}{18}\left(\frac{m}{k+m}\right)^2 \pm$$

$$\pm\ i\,\frac{1}{3}\ \sqrt{\frac{20}{3}}\ \left(\frac{m}{k+m}\right)^{3/2} + \ldots \tag{81}$$

The appearance of $i\pi$ in Eq. (81) results in sign-alternating coefficients G_{km} (76) at sufficiently small values of m/k.

Now we pass to the discussion of the solutions of the Euler-Lagrange equations for all other cases (see (56), (61) and (64)). Using the above mentioned procedure of iteration in the small parameter $\epsilon \sim (\chi - 1/6)$ we can find these solutions and the action on these solutions (cf. (78)). Furthermore, this parameter can be found from the saddle point Eq. (74) as a series in $\sqrt{m/k+m}$ (cf. (80)) and we can obtain $C(m/k+m)$ in Eq. (76) in terms of the series at small $m/k+m$ (cf. (81)):

$$\ell n\,\left|C_i\left(\frac{m}{k+m}\right)\right| = \ell n\,3 - \frac{m}{k+m}\ \ell n\,6 - B_i\left(\frac{m}{k+m}\right)^2 + \ldots\ , \tag{82}$$

where the values for B_i are given in Table I for all six cases. We see from the Table the maximum contribution to the large order asymptotics comes for small $m/k+m$ from the classical solution with $T = 2$ (54). For arbitrary values of $m/k+m$ we have estimated the actions (56), (61) and (64) by using appropriate trial functions. The simplest trial functions are the lowest spherical harmonics (a_1 = const, Y = const, $a_2 = a_3 = 0$ for Eq. (51); a_i = const, Y = const for cases (60) and (63)). This estimate is in rather good agreement with the value of the action in the case when we have the exact formula (72). It turns out that just as in case III we have in cases I, II and IV two solutions when we use Eq. (74) in order to determine the saddle points for g^2 and λ. The values of $\ell n|C(m/k+m)|$ for the first solution can be calculated from Eq. (82) with high precision in the whole region $0<m/k+m<1$. The values of $\ell n|C(m/k+m)|$ for the second solution in cases I, II and IV are significantly smaller than those in cases III. In Table I we give the results of numerical estimates of $C(m/k+m)$ for these two solutions in the limit $m/k \rightarrow 0$. For the second solution we obtain in this limit Y equal to zero for all four cases and therefore there are four different forms of solutions of the pure-Yang Mills equations. For the case II ($T=1$, $n_\perp = 3$) (see (65)) that solution can be found in an exact way

$$A = \frac{2\sqrt{2}\,\mathrm{Sh}\,2\xi_0}{\mathrm{Ch}\,2\xi + \mathrm{Ch}\,2\xi_0}\,;\ \mathrm{th}\,\xi_0 = \sqrt{\frac{3}{2}}\,e^{-i\,\pi/6}\,, \tag{83}$$

and the action for this solution is

$$\frac{g^2}{16\pi^2}S = C^{-1}\left(\frac{m}{k+m}\right)\Bigg|_{m\to\infty} = \frac{4}{3}\,e^{-i\,\pi/3}\left(i\,\frac{\sqrt{3}}{2} - \frac{\mathrm{arctg}(\mathrm{Sh}\,\xi_0)}{\mathrm{Sh}\,\xi_0}\right) \simeq 2.275\,e^{-i67.3^\circ}, \tag{84}$$

which agrees with the estimate obtained by using the trial functions above.

Thus, we have found six different forms of solution of the classical equations for the SU(2) Yang-Mills theory with a scalar field by using our iteration procedure and have estimated the contribution of each of these solutions to the asymptotic behaviour of coefficients of the perturbation series. More exact asymptotic formulas for $G_{k,m}$ can be obtained only if we calculate in Eq. (4) the integral over small fluctuations near these saddle points. It is a rather difficult problem because we have found only one exact solution. In the case of the other five solutions we can obtain only in the form of a series over the parameter $(\chi - 1/6)$.

In Ref. (24) it was argued that in the Yang-Mills theory for scattering amplitudes on the mass shell there are factorial contributions from certain Feynman diagrams resulting from renormalization effects. It would be desirable to find the total contribution of all such diagrams.

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TABLE I

	T	$n_\perp$	η^{a}_{ik}	B	$C(m/k\to\infty)$			
					① $\phi \neq 0$		② $\phi = 0$	
					$\ell n\lvert C\rvert$	arg C	$\ell n\lvert C$	arg C
I	2	5	$-2\epsilon_{abc}\gamma_i^{bd}\gamma_k^{cd}$	$\frac{1}{9}$	-1.097	150°	-1.040	125°
II	1	3	$-\epsilon_{ika}$	$\frac{10}{63}$	-1.436	125°	-0.822	67.3°
III	$\frac{1}{2}\oplus\frac{1}{2}^{*}$	4	$\frac{1}{2}(\epsilon_{4ika}+\delta_{4i}\delta_{ka}-\delta_{4k}\delta_{ia})$	$\frac{5}{18}$	-1.633	113°	0	0
IV		4	$\ell^a_1\eta^{ik}_1+\ell^a_2\eta^{ik}_2$	$\frac{5}{42}$	-1,083	180°	-3.045	180°
V		4	$\ell^a_1(\eta^{ik}_1+\eta^{ik}_2)$	$\frac{5}{21}$	-1.371	180°	-	-
VI		2	$\ell^a_1\eta^{ik}_1$	$\frac{5}{28}$	-1.289	180°	-	-

Variables for the various cases considered in the text.

TABLE II

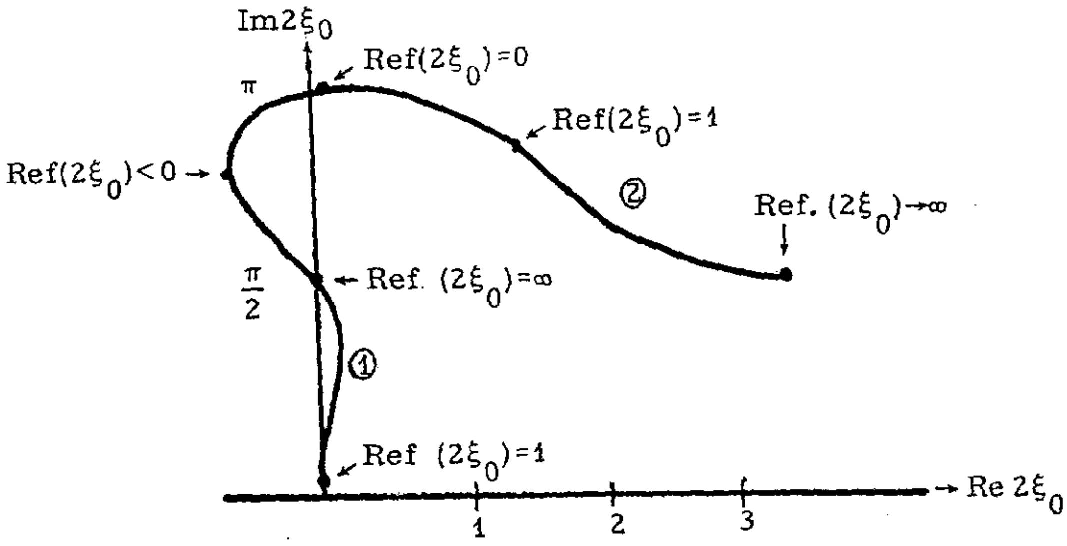
$\frac{m}{k+m}$	0	0.1974	0.3289	0.4751	0.6315	0.8095	0.9150	1	
$\ln\lvert C\rvert$	1.099	1.962	1.863	1.714	1.511	1.227	0.037	-1 633	1
arg C	180°	175.4°	169.7°	161.2°	149.5°	133.3°	122.4°	113.0°	

$\frac{m}{k+m}$	0	0.2685	0.6549	0.8339	0.9243	1	
$\ln\lvert C\rvert$	-1.436	-1.340	-0.973	-0.623	-0.356	0	2
arg C	276.3°	191.5°	82.2°	37.5°	16.4°	0	

Tabulated values for $C(m/k+m)$ in Eq. (77) for two different solutions of Eq. (75).

FIGURE CAPTION

Fig. 1: The curve on which $\mathrm{Im}\, f(2\xi_0) = 0$ in Eq. (75).



Im2ξ0
π
Ref(2ξ0)=0
Ref(2ξ0)=1
Ref(2ξ0)<0
②
Ref. (2ξ0)→∞
π/2
Ref. (2ξ0)=∞
①
Ref (2ξ0)=1
1
2
3
Re 2ξ0

Fig. 1