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Deep Inelastic Muon Scattering in DUMAND

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ABSTRACT

The cross section for deep inelastic muon scattering ($\mu + N \rightarrow \mu + X$) has been estimated for energies in the TeV region using structure functions measured at laboratory energies but taking into account the scaling violations predicted by the asymtotically free field theory QCD. Using optimistic assumptions for the flux of muons and the DUMAND array acceptance the counting rate was found to be exceedingly small. The results are very sensitive to the minimum muon scattering angle that can be measured. Unless $\theta_{min} \lesssim 10$ mr can be acheived, which seems highly unlikely, we must conclude that this is not an experiment for DUMAND to undertake.

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I. Differential Cross Section

To investigate the feasibility of measuring deep inelastic muon scattering in the DUMAND array we first must estimate the cross section in the TeV energy range. The scattering process we consider is

$$\mu + N \rightarrow \mu' + X$$

as is shown in Figure 1.

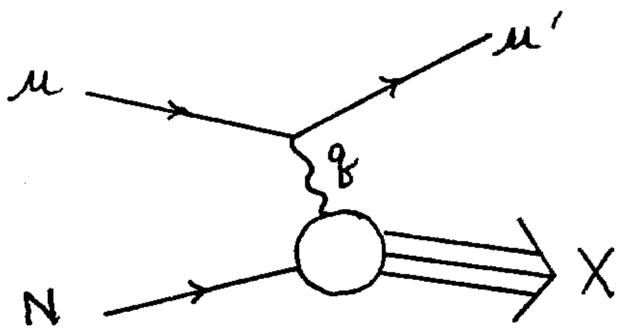

Fig. 1. Deep inelastic muon scattering.

Letting E be the incident muon energy, E' be the scattered muon energy and θ be their scattering angle the usual kinematic variables are the following:[1]

$$q^2 = 4EE' \sin^2(\theta/2) \simeq EE' \theta^2$$

(We will be concerned only with small θ)

$$\nu = E - E'$$

$$x = q^2/2M\nu$$

and

$$y = (E - E')/E = 1 - E'/E.$$

The differential cross section can then be written in the form

$$\frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2}{ME} \left(\frac{2 - 2y + y^2}{y^2 x}\right) F_1 = \frac{2\pi\alpha^2}{ME} \left(\frac{2 - 2y + y^2}{y^2 x^2}\right) F_2 \qquad (1)$$

where we have neglected σ_L/σ_T, which should be a good approximation at high energies, and have assumed the Callan-Gross relation $F_2 = 2x F_1$ for the structure functions.

II. DUMAND Acceptance

In the DUMAND array muon energies below some minimum value E_o cannot be measured. For fixed E this places an upper limit on y since

$$E_o < E' = E(1 - y) \tag{2}$$

implies that

$$y < y_{max} = 1 - E_o/E \tag{3}$$

In addition, muon scattering angles smaller than some θ_o will not be observable and for fixed E this leads to a lower limit on y which depends on x. If $\theta > \theta_o$ then

$$\theta_o^2 < \theta^2 \simeq \frac{q^2}{EE'} = \frac{2M\,x\,y}{E(1 - y)} \tag{4}$$

and

$$y > y_{min}(x) = \left(1 + \frac{2xM}{\theta_o^2 E}\right)^{-1} \tag{5}$$

Clearly, $y_{min}(x)$ is extremely sensitive to θ_o. In Figure 2 we display in the x-y plane the constraints imposed by the requirements that $E' > E_o$ and $\theta > \theta_o$ for fixed E.

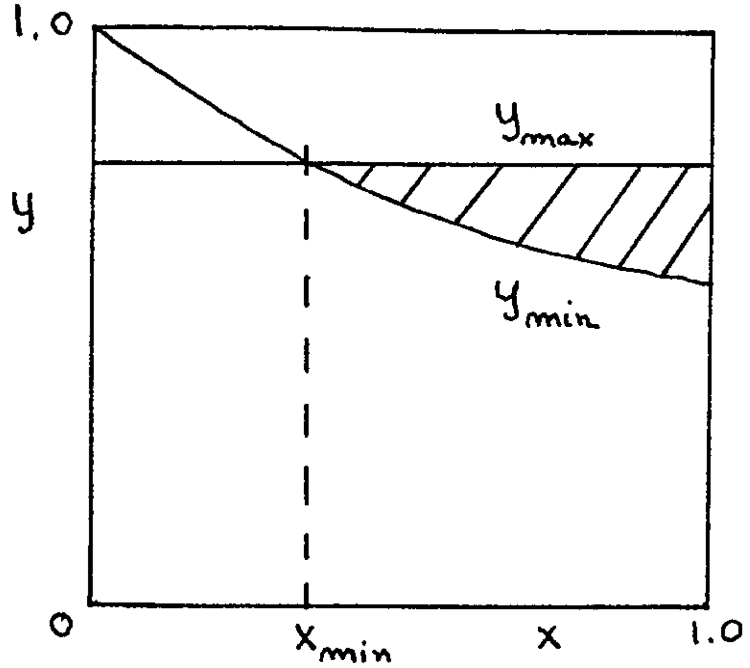

Fig. 2 The restrictions imposed on x and y for fixed E by the requirements $E' > E_o$ and $\theta > \theta_o$. The shaded area is the kinematically allowed region.

From Figure 2 it is clear that there is a minimum value of x below which there will be no acceptance. It is obtained by setting $y_{min}(x) = y_{max}$, i.e.,

$$1 - \frac{E_o}{E} = \frac{1}{1 + \frac{2x_{min}M}{\theta_o^2 E}} \tag{6}$$

or

$$x_{min} = \frac{\theta_o^2 E_o}{2M} \left(1 - \frac{E_o}{E}\right)^{-1}. \tag{7}$$

In order that $x_{min} < 1$, we must have

$$\frac{\theta_o^2 E_o}{2M} < 1 - \frac{E_o}{E} \tag{8}$$

or

$$\frac{\theta_o^2 E_o}{2M} < 1 \tag{9}$$

and therefore

$$\frac{E}{E_o} > \frac{1}{1-\frac{\theta_o^2 E_o}{2M}} \tag{10}$$

To summarize: (i) if the parameters E_o, θ_o violate Eq. (9), there will be zero acceptance; (ii) if Eq. (9) is satisfied, there will be acceptance only for incident energy E above the value set by Eq. (10); (iii) when both Eqs. (9) and (10) are satisfied, the acceptance is as shown in Figure 2.

III. Integrated Cross Section

Next we must integrate the differential cross section, Eq. (1), over the allowed region of x and y as shown in Figure 2 to obtain an estimate of the total cross section. (Of course, we are now assuming that both Eqs. (9) and (10) are satisfied.) We first carry out the integral over y as follows:

$$\sigma_{E_o,\,\theta_o}(E) = \int_{\bar{x}_{min}}^{1} dx \int_{y_{min}(x)}^{y_{max}} dy \frac{d\sigma}{dxdy}$$

$$\simeq \frac{2\pi\alpha^2}{ME} \int_{x_{min}}^{1} \frac{dx}{x^2} F_2(x) \int_{y_{min}(x)}^{y_{max}} dy\left[\frac{2}{y^2} - \frac{2}{y} + 1\right]$$

$$= \frac{2\pi\alpha^2}{ME} \int_{x_{min}}^{1} \frac{dx}{x^2} F_2(x)\left[-\frac{2}{y} - 2\ln y + y\right]\Bigg|_{\left(1+\frac{2xM}{\theta_o^2 E_o}\right)^{-1}}^{1-\frac{E_o}{E}} \tag{11}$$

Here we have assumed

$$F_2(x, q^2) \simeq F_2(x), \tag{12}$$

which is expected to be a good approximation in the range of integration because the kinematic acceptance requires that $q^2 > EE_o\theta_o^2$ which for DUMAND means q^2 at least above 1000 GeV^2. Since both experimental indications and theoretical (QCD) predictions suggest that $F_2(x, q^2)$ decreases very slowly with q^2 beyond ~20 GeV^2 (if x is not near zero)Eq. (11) establishes an upper limit for the cross section is we use

$$F_2(x) = F_2(x, q^2_{min}) \tag{13}$$

and a lower limit if we use

$$F_2(x) = F_2(x, q^2_{max}) \tag{14}$$

where q^2_{min} and q^2_{max} are the limits of the range of q^2 for a given set of parameters (E_o, θ_o, E).

We have numerically evaluated Eq. (11) to obtain $\sigma_{E_o, \theta_o}(E)$ taking E_o = 2 TeV and a range of values for θ_o and E. The results are presented in Figure 3 where $\sigma_{E_o, \theta_o}(E)$ is plotted against E for values θ_o^2 from 10^{-5} to 4×10^{-4}. In Figure 4 $\sigma_{E_o, \theta_o}(E)$ is plotted against θ_o for E = 3 TeV to 100 TeV. In Figure 3 the solid lines are obtained with $F_2(x)$ equal to $\frac{1}{2}$ the measured μd structure function[3] for q^2 = 15- 30 GeV^2 and the dashed lines are obtained with $F_2(x)$ equal to the QCD prediction[4,5] for F_2 at q^2 = 10,000 GeV^2. The differences between the two sets of curves are very small and are thus neglected in Figure 4.

Figures 3 and 4 clearly show that the cross section is an extremely sensitive function of θ_o, the scattering angle cut-off. For instance, at all energies between 3-100 TeV $\sigma_{E_o, \theta_o}(E)$ drops by 2 to 3 orders of magnitude when θ is increased from 10 mr to 20 mr. The physical reason for this sharp drop can be traced to the fact that an increase in θ_o results in a smaller phase space with a larger x_{min} and q^2_{min}. This reduces σ in three ways: (i) the region of

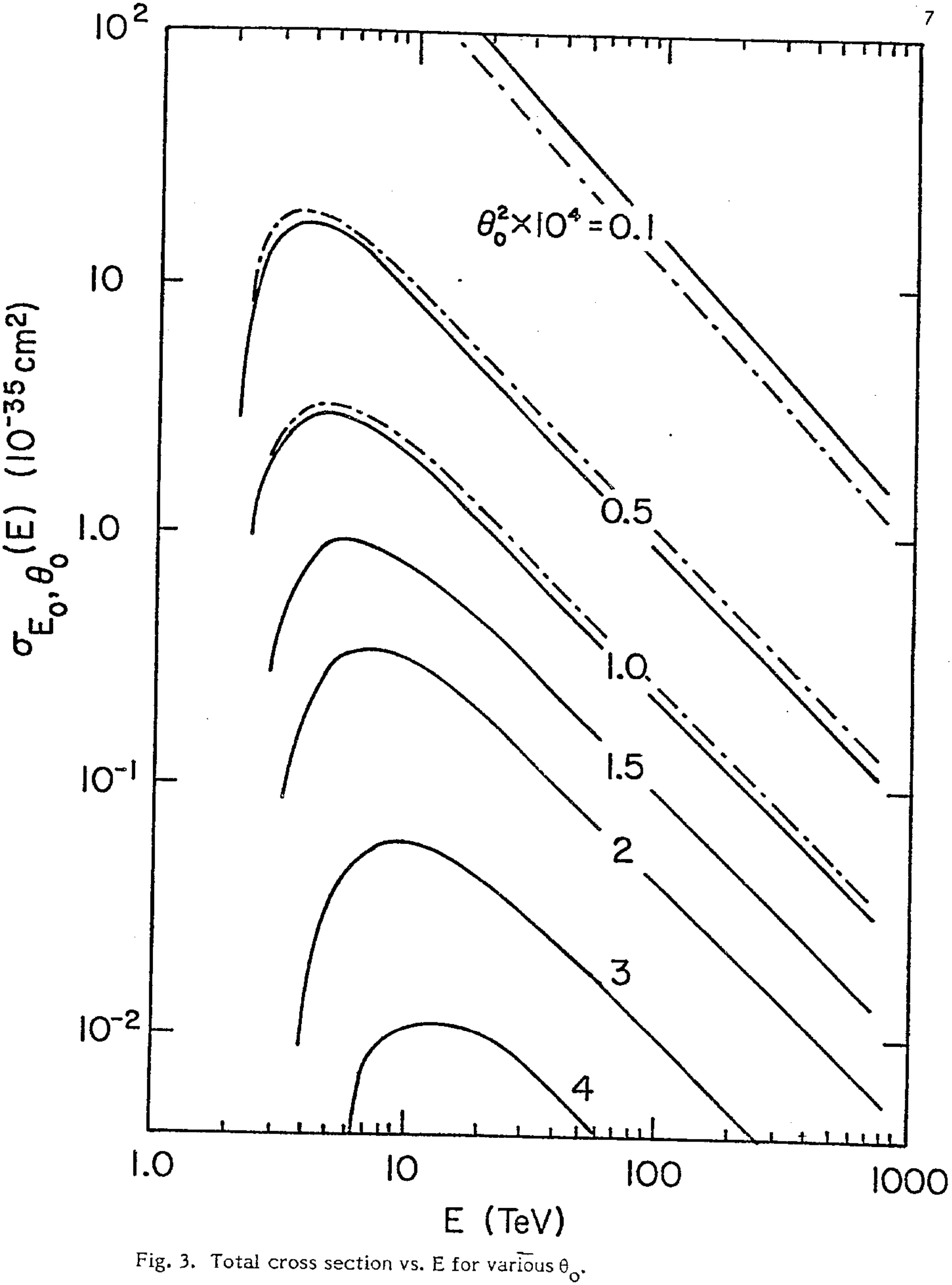

Fig. 3. Total cross section vs. E for various θ_o.

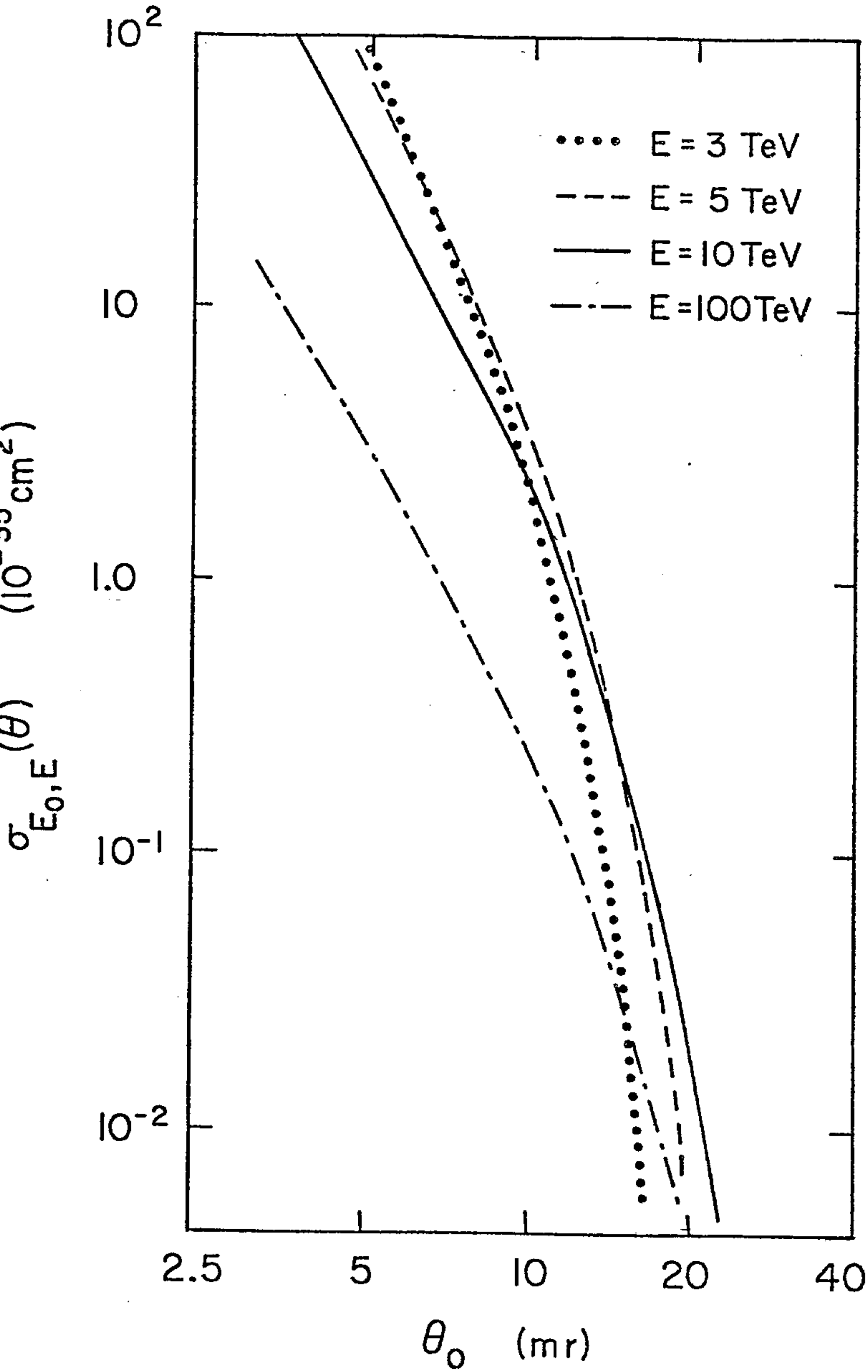

Fig. 4. Total cross section vs. θ_0 for various E.

integration shrinks; (ii) $F_2(x)$ is a rapidly decreasing function of x proportional to $(1-x)^n$ where $n > 3$ and therefore sensitive to changes in x_{min}; and (iii) the photon propagator factor $1/q^4$ in σ is very sensitive to changes in q^2_{min}.

IV. DUMAND counting Rate Estimate

To obtain a very crude estimate of the expected counting rate for deep inelastic muon scattering events in a 1 km^3 (10^9 ton) DUMAND array we shall assume a muon flux[6]

$$\Phi = 10 \text{ sec}^{-1} \text{ km}^{-2} = 10^{-9} \text{ sec}^{-1} \text{ cm}^{-2} \tag{15}$$

This corresponds to the estimated cosmic ray background and is clearly a gross overestimate in the TeV region. Again being overly optimistic we shall take $E_o = 2$ TeV and $\theta_o = 20$ mr so that at $E = 10$ TeV

$$\sigma_{E_o,\, \theta_o}(E) \sim 10^{-37} \text{ cm}^2 \tag{16}$$

Combining Eqs. (15) and (16) leads to a counting rate of about 1 event/year in the entire 1 km^3 DUMAND array. Clearly we must conclude that deep inelastic muon scattering is not a feasible experiment for DUMAND, unless some way can be found to significantly decrease θ_o in view of the extreme sensitivity of the cross section to this scattering angle cut off (see Figure 4).

Acknowledgement

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